Thursday July 20

Systems of Linear Equations Lesson #1:
Solving Systems of Linear Equations by Graphing

Overview of Unit

In this unit, we solve problems that involve systems of linear equations in two variables. We do this by inspection, by graphing, and algebraically using the method of substitution and the method of elimination.

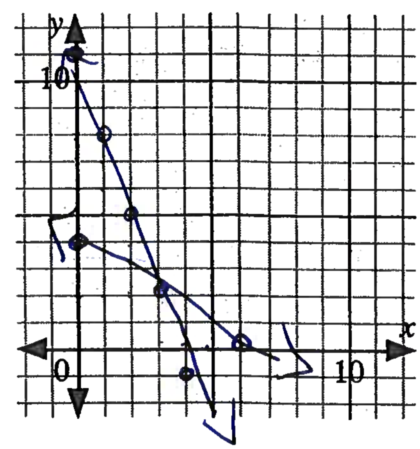
Exploring a Meaning of Two Intersecting Lines

The Smith family and the Harper family are going to a book fair which is raising money for charity. Mr. Smith pays an entry fee of \$11 for three adults and one child. Mrs. Harper pays an entry fee of \$12 for two adults and three children.

We can determine the cost of an adult ticket and the cost of a child ticket by forming two linear equations and graphing them.

Let \$$x$ be the entry fee for an adult ticket and let \$$y$ be the entry fee for a child. The information about the Smith family can be modelled by the equation $3x + y = 11$, and information about the Harper family can be modelled by the equation $2x + 3y = 12$.

a) Draw the graphs of the equations $3x + y = 11$ and $2x + 3y = 12$ on the grid without using technology.



$2x+3y=12$
$-2x \quad -2x$
$\frac{3y}{3} = \frac{-2x}{3} + \frac{12}{3}$
$y = -\frac{2}{3}x + 4$
slope $-\frac{2}{3}$ y-int 4

$3x+y=11$
$-3x \quad -3x$
$y = -\frac{3}{1}x + 11$

x y
(3, 2)

b) The graphs of the equations intersect at a point. State the coordinates of this point and explain what the coordinates represent in the context of the question.

adult ticket costs \$3
child ticket costs \$2

Check
$3(3) + 2 = 11$ ✓
$2(3) + 3(2) = 12$ ✓

Systems of Equations

In the exploration on the previous page, we worked with the equation $3x + y = 11$. There are many values for x and y which satisfy this equation, e.g. $x = 1$ and $y = 8$, or $x = 2$ and $y = 5$, or $x = 3$ and $y = 2$, etc.

We also worked with the equation $2x + 3y = 12$. There are also many values for x and y which satisfy this equation, e.g. $x = 0$ and $y = 4$, or $x = 3$ and $y = 2$, or $x = 4.5$ and $y = 1$, etc.

If we consider both of these equations simultaneously, there is only one solution, $x = 3$ and $y = 2$.

The equations $3x + y = 11$ and $2x + 3y = 12$, considered at the same time, are called a **system of equations**.

The **solution** to this system of equations is $x = 3$ and $y = 2$. This is because $x = 3$ and $y = 2$ **satisfy** each equation in the system.

Graphically, the solution to the system is the point of intersection of the two lines.

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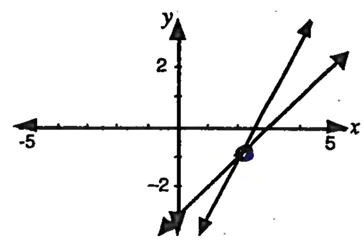
ss *Ex. #1*

A system of equations has been represented on the grid. The system has an integral solution.



a) State the solution $x =$ 2 , $y =$ ~~-2~~ 1

b) Write the solution as an ordered pair.

(x, y) (2, -~~2~~1)

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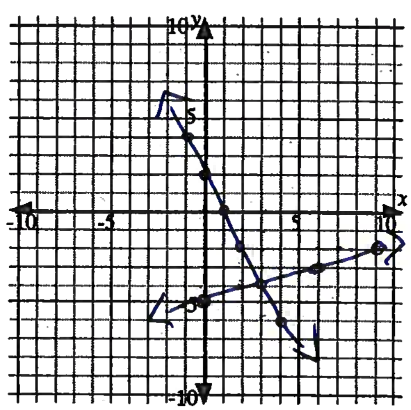
ss *Ex. #2*

Consider the system of equations $2x + y = 2$, $x - 3y = 15$.

a) Graph the system of equations without using technology.



$y = -2x + 2$

slope -2

y-int $+2$

$\frac{3y}{3} = \frac{x}{3} - \frac{15}{3}$

$y = \frac{1}{3}x - 5$

slope $\frac{1}{3}$ y int -5

b) State the solution to the system of equations.

(3, -4)

c) Algebraically verify the solution by replacing the values in the original equations.

$2x + y = 2$

$2(3) + (-4) = 2$

$6 - 4 = 2$ ✓

$x - 3y = 15$

$3 - 3(-4) =$

$3 + 12 = 15$ ✓

Complete Assignement Questions #1 and #2

Friday

Systems of Linear Equations Lesson #3: Solving Systems of Linear Equations by Inspection and by Substitution

Method of Inspection

In some simple cases, a system of linear equations can be solved by mentally trying different values for the variables until a correct solution is reached. This is called the method of inspection and is really only practical if the equations are very simple.

Class Ex. #1

The sum of two numbers is 14 and the difference between the numbers is 2. Form two equations in two variables and determine the numbers by inspection.

let x be the larger # let y be the smaller #

$x+y=14$ → inspection

$x-y=2$

13+1 ×
12+2 ×
11+3 ×
10+4 ×
9+5 ×
8+6 8−6=2 ✓

by inspection
$x=8$
$y=6$

Class Ex. #2

Solve the system $x + 2y = 12$ and $x + 3y = 17$ by inspection.

describe in words

- a # plus two times another # equals 12
- a # plus 3 times another # is 17

$x+2y=12$	$x+3y=17$
try $x=1$	try $x=1$
$1+2y=12$	$1+3y=17$
$y=11/2$	$y=16/3$
try $x=2$	try $x=2$
$2+2y=12$	$2+3y=17$
$y=5$	$y=5$

$x=2$
$y=5$

Complete Assignment Questions #1 - #3

Method of Substitution

If the equations are too complex to be solved by inspection, then algebraic procedures such as the method of substitution and the method of elimination (next lesson) may be used.

When using the method of substitution, there are four steps which are shown in the flowchart below.

Step 1		Step 2		Step 3		Step 4
Choose the simpler equation and express one variable in terms of the other.	→	Substitute the expression from step 1 into the other equation.	→	Solve the single variable equation.	→	Substitute the solution from step 3 into the equation in step 1 to find the value of the other variable.

• isolate the variable

Class Ex. #3

Consider the following system of equations: ① $x + 4y = 17$
② $2x - y = 7$

a) Solve the system using the method of substitution by rewriting the first equation in the form $x =$ .....

① $x+4y=17$ ② $2x-y=7$

Step 1 ✓ $x = 17-4y$ step2 → $2(17-4y)-y=7$

Step 3: solve for y

$34-8y-y=7$

$34-9y=7$

$-9y=7-34$

$-9y=-27$

$\boxed{y=3}$

Step 4

① $x+4y=17$

$x+4(3)=17$

$x+12=17$

$\boxed{x=5}$

b) Solve the system using the method of substitution by rewriting the first equation in the form $y =$ .....

c) Which method, a) or b), was simpler?

$\boxed{x=5 \quad y=3}$

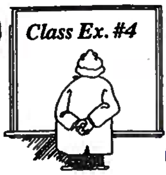
d) Verify that the solution satisfies both equations.

① $x+4y=17$ ② $2x-y=7$

$5+4(3)\overset{?}{=}17$ $2(5)-(3)\overset{?}{=}7$

$5+12=17$ ✓ $10-3=7$ ✓

e) Check the solution using a graphing calculator.



Class Ex. #4

Consider the following system of equations: $\boxed{4x + 3y = 0}$, $\boxed{8x - 9y = 5}$.

a) Solve and verify the system using the method of substitution.

① $4x + 3y = 0$

solve for y

$3y = -4x$

$\boxed{y = -\frac{4}{3}x}$

②

$8x - 9\left(\frac{-4}{3}x\right) = 5$

$8x + \frac{36}{3}x = 5$

$8x + 12x = 5$

$20x = 5$

$\boxed{x = \frac{5}{20}} = \boxed{\frac{1}{4}}$

$4\left(\frac{1}{4}\right) + 3y = 0$

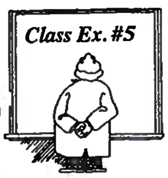
$1 + 3y = 0$

$3y = -1$

$\boxed{y = -\frac{1}{3}}$

$\boxed{\begin{array}{l} x = \frac{1}{4} \\ y = -\frac{1}{3} \end{array}}$

b) Check the solution using a graphing calculator.



Class Ex. #5

Consider the following system of equations: $5(2a - 3) + b = 5, \quad 6a - 2(b - 4) = 20.$

a) Solve the system using the method of substitution.

$5(2a - 3) + b = 5$

$10a - 15 + b = 5$

Solve for b

$b = 5 - 10a + 15$

$\boxed{b = -10a + 20}$

$6a - 2((-10a + 20) - 4) = 20$

$6a - 2(-10a + 16) = 20$

$6a + 20a - 32 = 20$
$\quad +32 \quad +32$

$26a = 52$

$\boxed{a = 2}$

$5(2(2) - 3) + b = 5$

$5(4 - 3) + b = 5$

$5 + b = 5$

$\boxed{b = 0}$

b) Verify algebraically that the solution satisfies both equations.

$5(2(2) - 3) + 0 \stackrel{?}{=} 5$

$5(1) + 0 = 5$

$5 = 5$ ✓

$6(2) - 2(0 - 4) \stackrel{?}{=} 20$

$12 + 8 = 20$

$20 = 20$ ✓

$\boxed{\begin{array}{l} a = 2 \\ b = 0 \end{array}}$

Complete Assignment Questions #4 - #10

Assignment #1-5, 8 Lunch 11:15-11:55

1. Solve the following linear systems by method of inspection.

   a) $x+y=9,\ x-y=1$ **b)** $x+y=12,\ x-y=0$ **c)** $x+y=4,\ x-y=6$

2. At the Little River Pow Wow, a vendor sells a salmon burger and two cans of cola for \$8. If two salmon burgers and two cans of cola sell for \$14, then determine the cost of

   a) a salmon burger **b)** a can of cola

3. Tickets are on sale for a music concert. Three adult tickets and two child tickets cost \$90. Three adult tickets and four child tickets cost \$120.

   a) Write a system of equations in two variables to represent the above information.

   x=adult ticket y=child ticket

   b) Determine the total cost of two adult tickets and three child tickets.

4. In each of the following systems:
   - solve the system using the method of substitution
   - verify the solution satisfies both equations
   - check the solution by graphing

   a) $y=x+2,\ \ 3x+4y=1$ **b)** $x-2y=10,\ \ x+5y+4=0$

5. Solve each of the following systems by substitution. Check each solution.

a) $4p + q = 0, \quad 7p + 4q = 3$

b) $6u - 3v + 4 = 0, \quad 3u = 3v - 5$

c) $2x - 5y = -7$

$\frac{1}{2}x - y = 3$

d) $2(x + 2) + y = 8$

$7x - 2(y - 3) + 24 = 0$

6. The straight line $px + qy + 14 = 0$ passes through the points $(-3, 1)$ and $(-4, 6)$.

a) Substitute the x and y-coordinates of the two points into the equation of the line to form two equations in p and q.

b) Solve this system of equations by substitution to determine the values of p and q and write the equation of the line.

c) Verify the equation in **b)** using the slope formula and the point-slope equation of a line formula.

7. Solve the following systems by substitution. Explain the results.

a) $y = 3x - 7$
$6x - 2y = 14$

b) $x = 3y + 2$
$2x - 6y = 5$

Multiple Choice

8. If $x + 2y = 10$ and $x - 2y = 2$, then $x + y$ is equal to

A. 8

B. 12

C. 13

D. -2

9. When solving a system of equations, one of which is $\frac{x}{2} - \frac{y}{3} = 1$, a substitution which can be made is

A. $x = \frac{1}{3}(2y + 1)$
B. $y = \frac{1}{2}(3x - 1)$
C. $x = \frac{1}{2}(3y + 6)$
D. $y = \frac{1}{2}(3x - 6)$

Numerical Response

10. If $s - 8t + 20 = 5s - 7t + 1 = 0$, then the value of $s + t$, to the nearest tenth, is ______.

(Record your answer in the numerical response box from left to right)

Answer Key

1. a) $x = 5, y = 4$ b) $x = 6, y = 6$ c) $x = 5, y = -1$

2. a) \$6 b) \$1

3. a) $3x + 2y = 90, 3x + 4y = 120$ b) \$85

4. a) $x = -1, y = 1$ b) $x = 6, y = -2$

5. a) $p = -\frac{1}{3}, q = \frac{4}{3}$ b) $u = \frac{1}{3}, v = 2$ c) $x = 44, y = 19$ d) $x = -2, y = 8$

6. a) $-3p + q + 14 = 0, -4p + 6q + 14 = 0$ b) $p = 5, q = 1$ $5x + y + 14 = 0$

7. a) There are an infinite number of solutions of the form $x = a, y = 3a - 7, a \in R$ because the equations are identical, (the resulting equation reduces to $0 = 0$).
b) There are no solutions since the graphs of the equations are parallel lines, (the resulting equation reduces to $4 = 5$).

8. A **9.** D **10.**

7	.	0	

Systems of Linear Equations Lesson #4:
Solving Systems of Linear Equations by Elimination

So far we have used three methods to solve systems of equations: graphing, inspection, and substitution. In this lesson we will learn another algebraic technique: the method of elimination. This method is particularly useful when the equations involve fractions.

Method of Elimination

In using the method of elimination, there are four steps which are shown below.

Step 1: If necessary, multiply each equation by a constant to obtain coefficients for x (or y) that are identical (except perhaps for the sign). →

Step 2: Add or subtract the two equations to eliminate one of the variables. →

Step 3: Solve the resulting equation to determine the value of one of the variables. →

Step 4: Substitute the solution into either of the original equations to determine the value of the other variable.

$$\begin{aligned} x+2y&=12 \\ -\quad x+3y&=17 \\ \hline 0-y&=-5 \\ \boxed{y}&\boxed{=5} \end{aligned}$$

$x+2(5)=12$

$x+10=12$

$\boxed{x=2}$

Class Ex. #1

Consider the system of equations:

$$\begin{aligned} 2x+7y&=13 \\ +\quad 3x-7y&=2 \\ \hline 5x&=15 \end{aligned}$$

$\boxed{x=3}$

a) Add the two equations. This will eliminate the variable y.

b) Use the equation in a) to determine the value of x and hence solve the system.

$2(3)+7y=13$

$6+7y=13$

$7y=7$

$\boxed{y=1}$

$\boxed{x=3,\ y=1}$

c) Verify the solution satisfies both equations.

$2(3)+7(1)\stackrel{?}{=}13$

$6+7=13$ ✓

$3(3)-7(1)\stackrel{?}{=}2$

$9-7=2$ ✓

Class Ex. #2

Consider the system of equations:

$$\begin{aligned} 2x+6y&=6 \\ -\quad 2x+3y&=4.5 \\ \hline 3y&=1.5 \end{aligned}$$

a) Subtract the two equations. This will eliminate the variable x.

$y=\frac{1.5}{3}=\boxed{0.5 \text{ or } \frac{1}{2}}$

b) Use the equation in a) to determine the value of y and hence solve the system.

$2x+6(0.5)=6$

$2x+3=6$

$2x=3$

$\boxed{x=3/2 \text{ or } 1.5}$

c) Verify the solution satisfies both equations.

$2(1.5)+6(0.5)\stackrel{?}{=}6$

$3+3=6$ ✓

$2(1.5)+3(0.5)\stackrel{?}{=}4.5$

$3+1.5=4.5$ ✓

Complete Assignment Questions #1 - #3

ass Ex. #3

Consider the system of equations: $2x + 3y = 4$
$(4x - y = 22) \times 3$ $\quad 12x - 3y = 66$

a) Does adding or subtracting the equations eliminate either of the variables?

b) Multiply the second equation by 3 and then add the two equations.

$$\begin{array}{r} 2x + 3y = 4 \\ +\ 12x - 3y = 66 \\ \hline \end{array} \quad = 14x = 70 \quad \boxed{x = 5}$$

c) Solve and verify the system.

$2(5) + 3y = 4$
$10 + 3y = 4$
$3y = -6$
$\boxed{y = -2}$ $\quad \boxed{x = 5, -2}$

d) Consider the original system. Multiply the first equation by an appropriate number which will eliminate x by addition or subtraction. Solve the system. $\times 2$

$$\begin{array}{r} 4x + 6y = 8 \\ -\ 4x - y = 22 \\ \hline 7y = -14 \end{array}$$

$\boxed{y = -2} \rightarrow 4x - (-2) = 22$
$4x + 2 = 22$
$4x = 20$
$\boxed{x = 5}$

'lass Ex. #4

Consider the system of equations: $(5a + 3b = 3) \times 7$
$(3a - 7b = 81) \times 3$

a) Choose appropriate whole numbers to multiply each equation so that the system can be solved by eliminating b.

b) Solve and verify the system by eliminating b.

$$\begin{array}{r} 35a + 21b = 21 \\ +\ 9a - 21b = 243 \\ \hline 44a \quad = 264 \end{array}$$

$\boxed{a = 6}$

$5(6) + 3b = 3$
$30 + 3b = 3$
$3b = -27$
$\boxed{b = -9}$

c) Choose appropriate whole numbers to multiply each equation so that the system can be solved by eliminating a. top $\times 3$ bottom $\times 5$

~~**d)**~~ Solve the system by eliminating a.

$x+y=\#$

Class Ex. #5

Solve the following system using elimination. $4x + 2y - 13 = 0,\quad 3x = 5y + 26$

$(4x+2y=13)\times 5$

$(3x-5y=26)\times 2$

$$\begin{array}{r} 20x+10y=65 \\ +\ \ 6x-10y=52 \\ \hline 26x \qquad =117 \end{array}$$

$x = \frac{117}{26} = \boxed{\frac{9}{2}}$ or 4.5

Class Ex. #6

Solve the following system using elimination.

$\frac{x-2}{3} - \frac{y+2}{5} = 2, \qquad \frac{3}{5}(x+1) - \frac{4}{5}(y-3) = \frac{21}{2}$

$3(4.5) = 5y+26$

$13.5 = 5y+26$

$-12.5 = 5y$

$\boxed{-2.5 = y}$

or $-\frac{5}{2}$

Complete Assignment Questions #4 - #12

Assignment

\#(1-5)ac, 6, 7

1. In each of the following systems:
   - solve the system using the method of elimination by adding the equations.
   - verify the solution satisfies both equations.

a) $8x - y = 10$, $4x + y = 14$

b) $x + 2y = 3$, $-x + 3y = 2$

c) $4a - 3b = 2$, $-4a - b = 6$

2. In each of the following systems:

- solve the system using the method of elimination by subtracting the equations.
- verify the solution satisfies both equations.

a) $7x + y = 15$
$3x + y = 3$

b) $5m + 3n = 10$
$5m - 2n = -15$

c) $4a - 3b = -18$
$-2a - 3b = -9$

3. Solve and verify each of the following systems using the method of elimination.

a) $-10p + 10q = 3$
$10p + 5q = 6$

b) $x + 4y = -0.5$
$5x + 4y = 2.3$

c) $4x + 2y - 31 = 0$
$-4x + 6y - 13 = 0$

4. Solve each of the following systems by elimination. Check each solution.

a) $2a + 5b = 16$
$a - b = 1$

b) $4x - 3y = 9$
$2x - 5y = 1$

c) $5x - 2y = 0.6$
$2x + y = 1.5$

5. Solve each of the following systems by elimination. Check each solution.

a) $2x + 4y = 7, \quad 4x - 3y = 3$

b) $5x = 8y, \quad 4x - 3y + 17 = 0$

c) $7e + 4f - 1 = 0, \quad 5e + 3f + 1 = 0$

d) $3x + 2y - 6 = 0, \quad 9x = 5y + 18$

6. Consider the system of equations $x - 2y + 1 = 0$, $2x + 3y = 12$. Solve the system by

a) elimination

b) substitution

Which method do you prefer?

7. Consider the system of equations: $11x + 3y + 2 = 0$, $11x - 5y - 62 = 0$.

Solve the system by

a) elimination

b) substitution

Which method do you prefer?

8. Solve each of the following systems by elimination. Explain the results.

a) $-2x + 6y - 1 = 0, \; 5x - 15y + 2.5 = 0$ b) $2x - 4y = 7, \; -7x + 14y = -21$

9. Solve each of the following systems by elimination.

a) $3x - \frac{1}{2}y = 5$
$\frac{1}{3}x + \frac{1}{4}y = 3$

b) $\frac{m}{2} - \frac{n-4}{4} = 2$
$\frac{3m}{4} - \frac{n}{5} = 5$

Multiple Choice

10. When b is eliminated from the equations $2x + b = 8$ and $5x + 2b = 2$, we obtain

A. $7x = 10$
B. $9x = 18$
C. $x = -14$
D. $3x = -6$

11. The solution to the systems of equations $x + y = 0$, $\frac{1}{2}x + \frac{1}{3}y = 1$ is

A. $x = 6, y = -6$
B. $x = 1, y = -1$
C. $x = 0, y = -0$
D. $x = -6, y = 6$

Numerical Response

12. If $\frac{1}{3}x + 5 = \frac{2}{3}y$ and $\frac{1}{2}x + \frac{1}{3}y = \frac{1}{3}$, then the value of $y - \frac{1}{2}x$, to the nearest tenth, is ______ .
(Record your answer in the numerical response box from left to right)

Answer Key

1. a) $x = 2,\ y = 6$ **b)** $x = 1,\ y = 1$ **c)** $a = -1,\ b = -2$

2. a) $x = 3,\ y = -6$ **b)** $m = -1,\ n = 5$ **c)** $a = -\frac{3}{2},\ b = 4$

3. a) $p = \frac{3}{10},\ q = \frac{3}{5}$ **b)** $x = 0.7,\ y = -0.3$ **c)** $x = 5,\ y = \frac{11}{2}$

4. a) $a = 3,\ b = 2$ **b)** $x = 3,\ y = 1$ **c)** $x = 0.4,\ y = 0.7$

5. a) $x = \frac{3}{2},\ y = 1$ **b)** $x = -8,\ y = -5$ **c)** $e = 7,\ f = -12$ **d)** $x = 2,\ y = 0$

6. $x = 3,\ y = 2$ **7.** $x = 2,\ y = -8$

8. a) There are an infinite number of solutions of the form $x = a, y = \frac{1}{6}(2a + 1), a \in R$ because the equations are identical (the resulting equation reduces to $0 = 0$).
b) There are no solutions since the graphs of the equations are parallel lines (the resulting equation reduces to e.g. $0 = 7$).

9. a) $x = 3,\ y = 8$ **b)** $m = 12,\ n = 20$ **10.** C **11.** A **12.**

7	.	5	