Wed.

Foundations & Pre-calculus of Math 10

Unit 5 Lesson 5 - Slope-Point Form of the Equation of a Linear Function

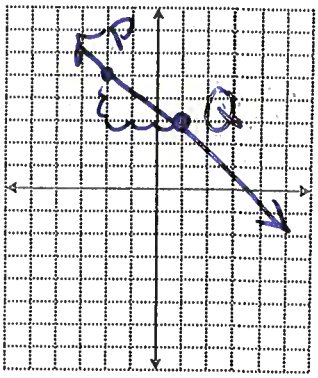
Investigation # 1

When we know the slope of a line and the coordinates of a point on the line, we use the property that the slope of line is constant to determine the equation for the line.

The line has a slope of $-\frac{2}{3}$ and passes through $P(-2,5)$. We use any other point on the line to write an equation for the slope m. Let's call this other point $Q(x,y)$

x_1, y_1

$$\frac{y_2-y_1}{x_2-x_1}=m$$



$$\frac{y-5}{x-(-2)}=\frac{-2}{3}=\frac{y-5}{x+2}=\frac{-2}{3}$$

$$y-5=\frac{-2}{3}(x+2)$$

The equation in this form is called slope-point form:
both the slope and the coordinates of a point on the line can be identified from the equation.

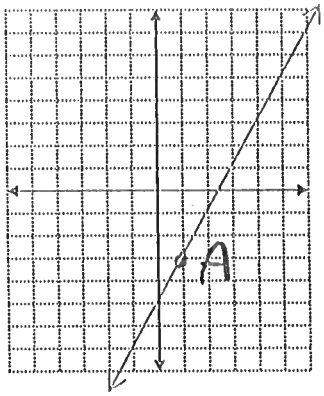
SLOPE POINT FORM OF THE EQUATION OF A LINEAR FUNCTION

The equation of a line that passes through $P(x_1,y_1)$ and has a slope m is:

$$y-y_1=m(x-x_1)$$

$y-y_1=m(x-x_1)$

1. The graph of $y+3=2(x-1)$ is shown on the grid.



a) Determine the slope of the graph of $y+3=2(x-1)$

$m=2$

b) List the coordinates of point A on the line.

$(1,-3)$

c) Compare your answers in a) and b) with the numbers in the equation.

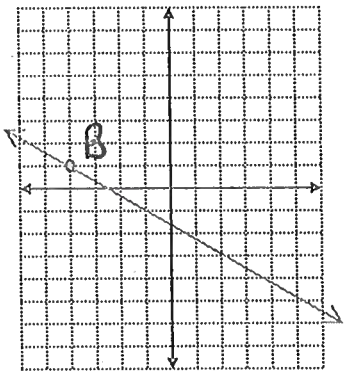
*same but opposite

$y+3=2(x-1)$

-3 $\quad +1$

$(1,-3)$

2. The graph of $y-1=-\frac{2}{3}(x+4)$ is shown on the grid



a) Determine the slope of the graph.

$-\frac{2}{3}$

b) List the coordinates of point B on the line.

$(-4,1)$

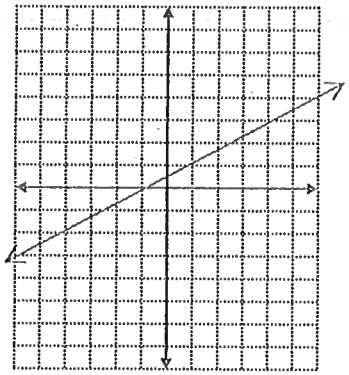
c) Compare your answers in a) and b) with the numbers in the equation.

- The point –slope form of the equation of a line is $y-y_1=m(x-x_1)$ where m is the slope of the line, and (x_1,y_1) represents a point on the line.
- To determine the equation of a line in higher level math classes, the point-slope equation $y-y_1=m(x-x_1)$ is used more frequently than the slope - intercept equation $y=mx+b$

- Note: **The slope-point equation is used when we have the slope of a line and the coordinates of any point on the line.** When using this method to determine the equation of a line it is usual to give the final equation in General form $Ax + By + C = 0$ or in slope - intercept form $y = mx + b$.

Investigation 2

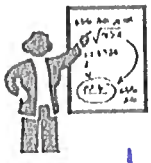
Writing an equation using a point on the Line and its Slope



a) Write an equation in slope-point form for this line.

b) Write the equation in part (a) in slope – intercept form. What is the y intercept of this line?

Class Ex. 1: State the equation, **in slope-point form**, of the line through the given point and with the given slope.



a) $m=3,\ (6,5)$

$y-y_1=m(x-x_1)$

$y-5=3(x-6)$

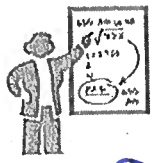
b) $m=-4,\ (1,1)$

$y-1=-4(x-1)$

c) $m=\frac{1}{2},\ (-9,-8)$

$y+8=\frac{1}{2}(x+9)$

Class Ex.2: In each case, the slope of a line and a point on the line are given. Determine the equation of the line in slope intercept form, $y=mx+b$



a) $m=5,\ (-5,2)$

① Point-slope form

$y-2=5(x+5)$

② Solve for y to change to $y=mx+b$

$y-2=5(x+5)$

$y-2=5x+25$ (+2 +2)

$y=5x+27$

b) $m=-7,\ (-3,4)$

① $y-4=-7(x+3)$

② $y-4=-7x-21$

$y=-7x-17$

Class Ex.3: A line passes through the following points $(-6,0)$, $(-2,-3)$ Determine the equation in slope intercept form.

① Determine slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\frac{-3-0}{-2-(-6)}$

$\frac{-3}{-2+6} = \boxed{\frac{-3}{4}}$

② Use the slope and any point for $y-y_1=m(x-x$

$y-0=\frac{-3}{4}(x+6)$

③ convert $y=mx+b$ $y=\frac{-3}{4}x-\frac{18}{4} \rightarrow \boxed{y=\frac{-3}{4}x-\frac{9}{2}}$

In each case state the slope of the line, and write the coordinates of a point on the line.

a) $y+11=\frac{1}{7}(x-4)$

$m=\frac{1}{7}$

$(4,-11)$

b) $y-9=-\frac{5}{3}(x-7)$

$m=\frac{-5}{3}$

$(7,9)$

c) $y=-3(x-6)$

$m=-3$

$(6,0)$

d) $y-9=-11(x+3)$

$m=-11$

$(-3,9)$

Investigation 3 Graphing a Linear Function Given its Equation in Slope-Point Form

a) Describe the graph of the linear function with this equation:

$$y-2=\frac{1}{3}(x+4)$$

b) Graph the equation

Solution:

a) Compare the given equation with the equation in slope-point form.

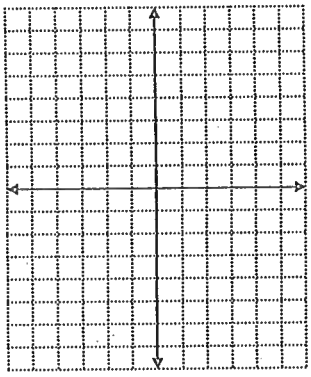
$$y-y_1=m(x-x_1)$$

$$y-2=\frac{1}{3}(x+4)$$

To match the slope – point form, rewrite the given equation so the operations are subtraction

The graph passes through P( ) and has a slope _____.

b) Graph



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Date:_____ Slope –Point Form – Assignment #5

1. State the equation, in slope point form, of the line through the given point and with the given slope.

a) $(9,3)$, 4

b) $(8,-2)$, -3

c) $(-5,7)$, 1

d) $(0,3)$, $\frac{1}{2}$

e) $(-7,0)$, $\frac{1}{4}$

f) $\left(-\frac{1}{2},-\frac{5}{4}\right)$, $\frac{6}{5}$

2. Write the following equations in slope – intercept form $y=mx+b$

a) $y+1=8(x-2)$

b) $y-3=-2(x-7)$

c) $y-9=-11(x+3)$

d) $y+3=7(x+12)$

#1all, 2all, 3ace 4all, 5a

y=mx+b slope intercept ~~point form~~

3. Find the equation, in slope ~~intercept~~ for, of the line through the given point, with the given slope.

a) $m=6$, $(2,4)$

b) $m=2$, $(2,-1)$

c) $m=-2$, $(0,4)$

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Date:_____ Slope –Point Form – Assignment 4

d) $m=\frac{1}{2},\ (-6,2)$

c) $m=1,\ (-7,-7)$

f) $m=3,\ (6,1)$

g) $m=\frac{1}{4},\ (2,-5)$

4. The slope – point equation of a line is given. State the slope and the coordinates of the point which was used to write the equation.

a) $y-9=-\frac{11}{3}(x+3)$

b) $y+3=\frac{1}{2}x$

c) $y=3(x+12)$

d) $y-9=-\frac{5}{3}x$

e) $y=\frac{2}{5}x$

5. Determine the equation of the line given the two points it runs through.

a) $(-1,1),\ (-4,3)$

b) $(5,-3),\ (3,-8)$

Lesson 6: The General Form of an Equation

Recall: a linear equation of form $y = mx + b$ has slope m and y-intercept b

General Form of the Equation of a Line → $Ax + By + C = 0$

The **general form** of the equation of a line is an equation where all the terms are collected to the left side of the equation.
The general form of an equation is written as $Ax + By + C = 0$, where A, B, and C are expressed as **integers** if possible, and A is usually positive.

The general form of the equation of a line allows us to write equations for oblique lines, horizontal lines, and vertical lines.

In some texts, the form $Ax + By + C = 0$ is referred to as **standard form**.

$Ax + By = C$

Ex. 1)

Convert the following equations from slope y-intercept form, $y = mx + b$, to general form $Ax + By + C = 0$, where A, B, and C are integers.

a) $y = 5x - 8$

$-y \quad -y$

$5x - y = 8$

$5x - y - 8 = 0$

b) $\left(y = \frac{2}{3}x + 7\right)^3$

$3y = 2x + 21$

$-3y \quad -3y$

$2x - 3y + 21 = 0$

c) $\left(y = -\frac{1}{4}x + \frac{3}{5}\right)^{20}$

$20y = -5x + 12$

$+5x \quad +5x \quad -12$

$5x + 20y - 12 = 0$

Ex. 2) $y = mx + b$

Determine the slope and y-intercept of the graph of the following lines.

a) $2x - 5y + 25 = 0$

$+5y \quad +5y$

$2x + 25 = 5y$

$\frac{5y}{5} = \frac{2x}{5} + \frac{25}{5}$

$y = \frac{2}{5}x + 5$

slope $= \frac{2}{5}$ y int $= 5$

b) $6x + 2y + 15 = 0$

$-6x \quad +15 \quad -6x + 15$

$\frac{2y}{2} = \frac{-6x}{2} + \frac{15}{2}$

$y = -3x + \frac{15}{2}$

slope $= -3$ y int $= \frac{15}{2}$

Ex. 3)

The lines $3x - 4y + 8 = 0$ and $5x - ky - 6 = 0$ have the same y-intercept. Determine the value of k.

slope + y-int

$6x - 2y = 12$

$+2y \quad +2y$

$6x = 12 + 2y$

$-12 \quad -12$

$\frac{2y}{2} = \frac{6x}{2} - \frac{12}{2} \Rightarrow y = 3x - 6$

slope $= 3$ y-int -6

Ex. 4) $-\frac{1}{2}$

Which of the following lines is/are perpendicular to the line $4x - 2y + 9 = 0$?

i) $6x + 3y - 1 = 0$ ii) $x + 2y - 12 = 0$ iii) $5x + 10y = 0$

① $4x - 2y + 9 = 0$

$+2y \quad +2y$

$\frac{2y}{2} = \frac{4x}{2} + \frac{9}{2}$

$y = \boxed{2x} + \frac{9}{2}$

slope $= \frac{2}{1}$

i) $\frac{3y}{3} = \frac{-6x}{3} + \frac{1}{3}$

$y = \frac{-2x}{1} + \frac{1}{3}$

no!!

ii) $\frac{2y}{2} = \frac{-x}{2} + \frac{12}{2}$

$-\frac{1}{2}$ yes!!

iii) $\frac{10y}{10} = \frac{-5x}{10}$

$-\frac{1}{2}$

yes!!

Ex. 5)

Write the equation of a line which is perpendicular to $2x + 5y - 7 = 0$ and has the same y-intercept as $2x + y - 6 = 0$.

Answer in slope y-intercept form and in general form.

slope

$2x + 5y - 7 = 0$

$\frac{5y}{5} = \frac{-2x}{5} + \frac{7}{5}$

$y = \frac{-2}{5}x + \frac{7}{5}$

slope $= \frac{-2}{5}$

$\frac{5}{2}$

y-int

$2x + y - 6 = 0$

$y = -2x + 6$

y int $= +6$

slope int.

$y = mx + b \Rightarrow \boxed{y = \frac{5}{2}x + 6}$

General

$2(y) = 2\left(\frac{5}{2}x\right) + 2(6)$

$2y = 5x + 12$

$-2y \quad -2y$

$5x - 2y + 12 = 0$

Ex. 6)

Use the following information to answer Class Ex. #6.

A student made the following statements about the line with equation $2y = 5x + 12$.

Statement 1: The line has a slope of 5.
Statement 2: The line is parallel to $10x - 4y + 13 = 0$.
Statement 3: The line passes through $(-2, 1)$.

Which of the above statement(s) is/are true?

A. 1 and 2 only
B. 1 and 3 only
C. 2 and 3 only
D. some other combination of statements 1, 2, and 3

#1 all #2 all #Nope# 3a,c #4 #5 #9,10,11

Assignment

1. Convert the following equations from slope y-intercept form $(y = mx + b)$ to general form $(Ax + By + C = 0)$, where A, B, and C are integers.

a) $y = 7x - 3$ b) $y = -2x + 9$ c) $y = mx + b$

d) $y = -\frac{3}{4}x + 5$ e) $y = \frac{2}{3}x + \frac{1}{6}$ f) $y = \frac{5}{3}x - \frac{1}{4}$

2. Determine the slope and y-intercept of the graph of the following lines.

a) $x + y - 11 = 0$ b) $3x - 2y + 30 = 0$ c) $8x - 3y - 3 = 0$

d) $3x + 6y - 7 = 0$ e) $8y = 4x + 32$ f) $4x + 3y = 12$

3. Determine the slope, y-intercept, and x-intercept of the graph of the following lines.
   a) $2x + y - 6 = 0$ b) $5x - 2y + 20 = 0$ c) $4x - 5y - 3 = 0$

4. Write the equation, in general form, of a line parallel to $2x - 3y + 9 = 0$ and with the same y-intercept as $22x - 3y - 18 = 0$.

5. Write the equation, in general form, of a line perpendicular to $3x - 2y + 5 = 0$ and with the same y-intercept as $3x - y + 18 = 0$.

6. Consider the lines $x - 2y + 1 = 0$ and $4x + ky - 8 = 0$.

a) Determine the value of k if the lines have the same slope.

a) Determine the value of k if the lines have the same y-intercept.

7. Consider the lines $3x - 5y - 15 = 0$ and $ax + 2y - 6 = 0$.

a) Determine the value of a if the lines have the same slope.

a) Determine the value of a if the lines have the same x-intercept.

8. Match each equation on the left with the correct characteristic of the graph of the equation on the right. Each characteristic may be used once, more than once, or not at all.

Equation		Characteristic	
i)	$6x - 2y + 5 = 0$	A.	Slope $= -\frac{1}{3}$
ii)	$2x - 5y = 0$	B.	y-intercept $= -\frac{5}{2}$
iii)	$x + 3y + 6 = 0$	C.	Passes through $(-10, -4)$
iv)	$x - 4y + 10 = 0$	D.	Slope $= 0$
v)	$2x - y - 5 = 0$	E.	y-intercept $= \frac{5}{2}$
		F.	Perpendicular to $y = \frac{5}{2}x - 3$
		G.	x-intercept $= \frac{5}{2}$

9. The slope of the line with equation $6x + 5y - 1 = 0$ is

A. $-\frac{6}{5}$

B. $-\frac{5}{6}$

C. $\frac{6}{5}$

D. $\frac{1}{5}$

10. Which line has a y-intercept of 1?

A. $x + 5y + 1 = 0$

B. $x + 3y + 3 = 0$

C. $x - 2y + 2 = 0$

D. $2y = 3x + 1$

11. The slope of a line perpendicular to the line $x + 3y + 8 = 0$ is

A. -8

B. $-\frac{1}{3}$

C. $\frac{1}{3}$

D. 3

12. The line $2y + 3x + 6 = 0$ intersects the y-axis at P. The slope of the line joining P to $Q(6, -2)$ is

A. $-\frac{5}{6}$

B. $\frac{1}{6}$

C. $-\frac{1}{6}$

D. $-\frac{2}{3}$

13. The lines with equations $ay = 4x + 9$ and $y = 5x - 7$ are perpendicular. The value of a is

A. $\frac{4}{5}$

B. $-\frac{4}{5}$

C. $-\frac{5}{4}$

D. -20

Answer Key

1. a) $7x - y - 3 = 0$ b) $2x + y - 9 = 0$ c) $mx - y + b = 0$
   d) $3x + 4y - 20 = 0$ e) $4x - 6y + 1 = 0$ f) $20x - 12y - 3 = 0$

2. a) slope $= -1$, y–int $= 11$ b) slope $= \frac{3}{2}$, y–int $= 15$ c) slope $= \frac{8}{3}$, y–int $= -1$
   d) slope $= -\frac{1}{2}$, y–int $= \frac{7}{6}$ e) slope $= \frac{1}{2}$, y–int $= 4$ f) slope $= -\frac{4}{3}$, y–int $= 4$

3. a) slope $= -2$, y–int $= 6$, x–int $= 3$
   b) slope $= \frac{5}{2}$, y–int $= 10$, x–int $= -4$
   c) slope $= \frac{4}{5}$, y–int $= -\frac{3}{5}$, x–int $= \frac{3}{4}$

4. $2x - 3y - 18 = 0$ 5. $2x + 3y - 54 = 0$

6. a) -8 b) 16 7. a) $-\frac{6}{5}$ b) $\frac{6}{5}$

8. i) E ii) C iii) A iv) E v) G

9. A 10. C 11. D 12. B

13. D

Example #6 Parallel to $4x + 7y - 3 = 0$

y-int $6x - 4y + 8 = 0$

slope

$$\frac{7y}{7} = \frac{-4x}{7} + \frac{3}{7}$$

slope $= \frac{-4}{7}$

y-int

$$\frac{4y}{4} = \frac{6x}{4} + \frac{8}{4}$$

y int $= 2$

$$(y)^7 = \left(-\frac{4}{7}\right)^7 x \; (+2)^7 \Rightarrow 7y = -4x + 14$$

$$4x + 7y - 14 = 0$$