Foundation & Pre-Calculus of Math 10

Lesson # 5 - Common Factors

In Lessons 2 - 4 we were concerned with multiplying polynomial expressions. We focused on:

a) a monomial by a polynomial eg) $2x(x+5) = 2x^2+10x$

b) a binomial by a binomial to form a binomial. eg) $(x-5)(x+5) = x^2-5x+5x-25$ x^2-25

c) a binomial by a binomial to form a trinomial eg) $(2x+3)(x+4) = 2x^2+8x+3x+12$ $2x^2+11x+12$

In these examples we **expanded** a product of polynomials to form a sum or differences of monomials.

In the next three lessons we focus on the opposite process. We want to write a sum or difference of monomials as a product of polynomials. This process is called **factoring**.

We will be concerned with three major types of factoring, some of which you have met in previous courses.

a) Factoring by removing a common factor eg) $2x^2+10x = 2x(x+5)$

b) factoring differences of squares eg) $x^2-25 = (x+5)(x-5)$

c) factoring trinomials eg) $x^2+4x+3 = (x+3)(x+1)$

eg) $2x^2+11x+12 = (2x+3)(x+4)$

ex. 1) Find the GCF for the following:

1. $-40a^3b, -20a^2b^3, -10a^2b^2\ 12ab,\ 15a^2b$

$ab(-40a^2, -20ab^2, -10ab, 12, 15a)$

2. $18x^4y^2, -24x^3y^2$

6 | 3 1

$\uparrow$ $6x^3y^2(3x, -4)$

3. $a^3bc^2,\ 2ac^7$

$ac^2(a^2b, 2c^5)$

4. $-40a^3b, -20a^2b^3, -10a^2b^2$

$-10a^2b$

Ex. 2) Factor:

a) $6x^2 - 14x$

$2x(3x - 7)$

Check

$6x^2 - 14x$

b) $10x + 15$

$5(2x + 3)$

c) $8abc - 12ab\ 15x^2y^3 - 25xy - 10xyz$

$8abc - 12ab$

$4ab(2c - 3)$

$15x^2y^3 - 25xy - 10xyz$

$5xy(3xy^2 - 5 - 2z)$

BINOMIAL COMMON FACTORS

Binomial Common Factors

In certain circumstances the greatest common factor may be a binomial rather than a monomial.

$-26x^4y - 39x^3y^2 + 52x^2y^3 - 13xy^4$

$-13xy(2x^3 + 3x^2y - 4xy^2 + y^3)$

Factoring by Grouping

Sometimes polynomials in four terms can be factored by removing the GCF from a pair of terms followed by a binomial common factor. This method is called **factoring by grouping**.

Examples:

1) $(x^2+3x)+(6x+18)$

$x(x+3)+6(x+3)$

$(x+6)(x+3)$

2) $(8x^2-2x)+(12x-3)$

$2x(4x-1)+3(4x-1)$

$(4x-1)(2x+3)$

3) $(8a^2-4a)-(10a+5)$

$(8a^2-4a)-(10a-5)$

$4a(2a-1)-5(2a-1)$

$(2a-1)(4a-5)$

4) $6a^2-9a-2a+3$

$(6a^2-9a)-(2a-3)$

$3a(2a-3)-1(2a-3)$

$(2a-3)(3a-1)$

5) $pg+pr-sg-sr$

$(pg+pr)-(sg+sr)$

$p(g+r)-s(g+r)$

$(g+r)(p-s)$

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Score: _____/20

Section A odd #'s

Section A: Factor each of the following by removing a common Factor! (1 mark)

1) $2x + 2y$

2) $3x + 4x$

3) $15a - 3a$

4) $2x^3 - 10x^2 + 12x$

5) $-3y^4 - 6y^3 + 21y^2$

6) $36a^7x^4 - 42a^5x^2$

7) $42a^2b^2 + 7ab + 6b$

8) $3x^2 + 12x + 9$

9) $2ax - 2bx - 2cx$

10) $4ab^2 - ab^2c$

All

Section B: Factor by the Grouping Method (1 mark)

11) $6xy + 4x + 15y + 10$

12) $xy + 2x + 3y + 6$

13) $5xy - 25x - 3y + 15$

14) $110xy + 30x - 99y - 27$

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15) $ab + 4a + 5b + 20$

16) $2ab + a + 6b + 3$

17) $1 + ab + a + b$

$a(ab - a) + (1 - b)$

18) $35a + 21ab + 12b + 20$

$7a(5 + 3b) + 4(5 + 3b)$

19) $6xa - 4x + 15a - 10$

20) $24 - 6a + 8b - 2ab$

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Lesson 6 – Factoring Trinomials: $x^2 + bx + c$

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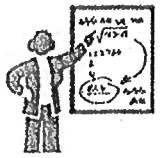
Factor by inspection:

~~x^2+3x~~ x^2+4x+3

~~x^2+3x+4~~

When we factor polynomial expressions in the form of $x^2 + bx + c$ we need find two integers which have a product of c and sum equal to b. If no two integers exist then the polynomial expression cannot be factored.

Class Ex. 3:



Complete the following tables.

Sum	Product	Integers
+12	+20	2, 10
+8	+15	~~2~~ 3, 5
-9	18	-6, -3
9	20	4, 5

Sum	Product	Integers
-15	14	-14, -1
-1	-6	-3, +2
2	-15	-3, 5
-26	48	-24, -2

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Factoring Trinomials: $x^2 + bx + c$

1. Expand:

   a) $(x+5)(x+2)$

   b) $(x-5)(x-5)$

   c) $(x+2)(x+3)$

2. Factoring Trinomials: $x^2 + bx + c$

   Many trinomials can be written as the product of two binomials.

$$x^2 + bx + c$$

Factor:

$x^2 + 11x + 24$

Find two integers which have a product of 24 and sum of 11

Example 1

When your middle term and last term are both positive: this means both binomials are positive.

$x^2 + 10x + 9$ $\quad + \times \quad (1,9)$

$(x+1)(x+9)$

$x^2 + 12x + 20$

$(x+2)(x+10)$

Example 2

When your middle term is negative and your last term is positive: this means both binomials factors are negative.

$x^2 - 10x + 16$ $(-2, -8)$

$+ \quad +$

$(x-2)(x-8)$

$x^2 - 6x + 9$ $(-3, -3)$

$(x-3)(x-3)$ $(x-3)^2$

$x^2 - 25x + 24$ $(-24, -1)$

$(x-24)(x-1)$

$x^2 - 9x + 14$ $(-7, -2)$

$(x-7)(x-2)$

$(2x+4)(2x+4)$

$(2x+4)^2$

Example 3

When your middle term is positive and the last term is negative: this means on binomial factor is positive and the other binomials factor is negative.

$x^2 + 4x - 21$ $(7, -3)$ $(x-3)(x+7)$

$x^2 + 3x - 40$ $(-5, 8)$ $(x-5)(x+8)$

$x^2 + 21x - 100$ $(25, -4)$

$(x+24)(x-4)$

$(x+25)(x-4)$

$x^3 + 1$

$x^3 + 21x^2 - 100x$

$x(x^2 + 21x - 100)$

Example 4

When the middle term and last term are negative: this means that one binomial factor **is** positive and one binomial factor is negative.

$x^2 - 7x - 18$ $(-9, 2)$

$(x-9)(x+2)$

$x^2 - 2x - 80$ $(-10, 8)$

$(x-10)(x+8)$

$x^2 - x - 30$ $(-6, 5)$

$(x-6)(x+5)$

Name:_____ Foundations of Math 10 Honours Score:_____/35

Date:_____ Assignment 6

1) $x^2+8x+12=$_____

2) $x^2+12x+27=$_____

3) $x^2+4x+3=$_____

4) $x^2-8x+12=$_____

5) $a^2+2x-63=$_____

6) $y^2+2y-15=$_____

7) $x^2+x-12=$_____

8) $x^2+6x-16=$_____

9) $x^2+5x-36=$_____

10) $20+9x+x^2=$_____

11) $x^2+x-132=$_____

12) $x^2-x-132=$_____

13) $x^2-x-42=$_____

14) $x^2+3x-10=$_____

15) $x^2+5x-14=$_____

16) $a^2-4a-21=$_____

17) $a^2-2a-15=$_____

18) $a^2-a-56=$_____

19) $r^2-2r-99=$_____

20) $x^2+13x+12=$_____

21) $x^2+27x+50=$_____

22) $f^2-10f+21=$_____

23) $p^2-16p+28=$_____

24) $y^2+17y+42=$_____

25) $x^2-4x-12=$_____

26) $x^2-9x-10=$_____

27) $x^2+3x-18=$_____

28) $x^2+10x+16=$_____

29) $x^2-2x-15=$_____

30) $x^2+x-30=$_____

31) $x^2+13xy+30y^2=$_____

32) $x^2+71xy-72y^2=$_____

33) $x^2+6x-27=$_____

34) $x^2-7x-8=$_____

35) $x^2-10x+25=$_____

add #'s only

Name: Key Foundations of Math 10 Honours Score: _____/35

Date: _____ Assignment 6

1) $x^2+8x+12=$ $(x+6)(x+2)$

2) $x^2+12x+27=$ _____

3) $x^2+4x+3=$ $(x+3)(x+1)$

4) $x^2-8x+12=$ _____

5) $a^2+2x-63=$ $(x+9)(x-7)$

6) $y^2+2y-15=$ $(x+5)(x-3)$

7) $x^2+x-12=$ $(x+4)(x-3)$

8) $x^2+6x-16=$ _____

9) $x^2+5x-36=$ $(x+9)(x-4)$

10) $20+9x+x^2=$ _____

11) $x^2+x-132=$ $(x+12)(x-11)$

12) $x^2-x-132=$ _____

13) $x^2-x-42=$ $(x-7)(x+6)$

14) $x^2+3x-10=$ _____

15) $x^2+5x-14=$ $(x+7)(x-2)$

16) $a^2-4a-21=$ $(a-7)(a+3)$

17) $a^2-2a-15=$ $(a-5)(a+3)$

18) $a^2-a-56=$ _____

19) $r^2-2r-99=$ $(r+9)(r-11)$

20) $x^2+13x+12=$ _____

21) $x^2+27x+50=$ $(x+25)(x+2)$

22) $f^2-10f+21=$ _____

23) $p^2-16p+28=$ $(p-2)(p-14)$

24) $y^2+17y+42=$ _____

25) $x^2-4x-12=$ $(x-6)(x+2)$

26) $x^2-9x-10=$ _____

27) $x^2+3x-18=$ $(x+6)(x-3)$

28) $x^2+10x+16=$ _____

29) $x^2-2x-15=$ $(x-5)(x+3)$

30) $x^2+x-30=$ _____

31) $x^2+13xy+30y^2=$ $(x+3y)(x+10y)$

32) $x^2+71xy-72y^2=$ $(x+72y)(x-y)$

33) $x^2+6x-27=$ $(x+9)(x-3)$

34) $x^2-7x-8=$ _____

35) $x^2-10x+25=$ $(x-5)(x-5)$

$(x-5)^2$

Lesson #7

Foundations and Pre – Calculus Mathematics 10

Factoring: $ax^2 + bx + c$

Factoring ax^2+bx+c using the decomposition method

Review # 1 Factor by the grouping method!

poly → a) $6x^2 + 3x + 8x + 4$

binomial → $3x(2x+1) + 4(2x+1)$

$= (2x+1)(3x+4)$

Factored ☺

b) $12x^2 + 3x + 8x + 4$ (16x)

$3x(4x+1) + 4(2x+1)$

$(4x+1)(3x+4)$

The method of factoring $ax^2 + bx + c$ is done by breaking the middle term (b value) into two integers whose product is ac and whose sum is b. This is called the "method of decomposition"☺

In order to factor $6x^2 + 11x + 4$:

Step 1: Find two numbers that multiply to 24 and add to 11.

$6 \cdot 4$

Step 2: Rewrite the question, decomposing the middle term into the two numbers from step 1:

$6x^2 + 11x + 4$

$6x^2 + 3x + 8x + 4$

$3x(2x+1) + 4(2x+1)$

×	+
24	11

$8 \cdot 3$

$= (2x+1)(3x+4)$

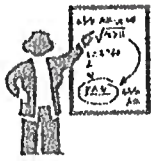
Step 3: Now factor by the grouping method!

could have written $6x^2+8x+3x+4$

$2x(3x+4)+1(3x+4)$

$=(3x+4)(2x+1)$

Class Ex. 1: Factor!



a) $3x^2+8x+4$

x	+
12	8
6, 2	

$3x^2+6x+2x+4$

$3x(x+2)+2(x+2)$

$=(x+2)(3x+2)$

b) $2x^2+7x+6$

x	+
12	7
6·2	
12·1	
3·4	

$2x^2+4x+3x+6$

$=2x(x+2)+3(x+2)$

$=(x+2)(2x+3)$

c) $5x^2+7x+2$

x	+
10	7

$5x^2+2x+5x+2$

$=x(5x+2)+1(5x+2)$

$=(5x+2)(x+1)$

d) $6x^2+17x-3$

x	+
-18	17
-1·18	
2·9	
3·6	

$6x^2+18x-x-3$

$6x(x+3)-1(x+3)$

$(x+3)(6x-1)$

ex. $ab+b$

$b(a+1)$

e) $3a^2-2a-8$

x	+
-24	-2
-6·4	
8·3	
2·12	
1·24	

$=3a^2-6a+4a-8$

$=3a(a-2)+4(a-2)$

$=(a-2)(3a+4)$

f) $12a^2-8a+1$

x	+
12	-8
-2·-6	
3·4	

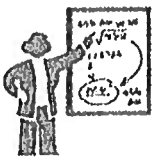
$12a^2-6a-2a+1$

$6a(2a-1)-1(2a-1)$

$(2a-1)(6a-1)$

Factoring Trinomials of the form

Class Ex. 2:



Factor.

a) $2x^2 - 5xy + 2y^2$

b) $2x^2 - 7xy - 15y^2$

Name:_____ **Foundations of Math 10**
Date:_____ **Assignment # 7**

Instructions: Show work on a separate piece of paper neatly and clearly for full marks to be given

Factor ax^2+bx+c (where a $\neq 0$)

1) $2x^2+7x+3=$ _____

2) $2x^2+5x+3=$ _____

3) $2x^2+11x+5=$ _____

4) $2x^2-5x+3=$ _____

5) $5x^2+6x+1=$ _____

6) $5x^2+9x-2=$ _____

7) $2x^2-15x+7=$ _____

8) $7x^2+8x+1=$ _____

9) $3x^2-2x-5=$ _____

10) $5x^2-12x+7=$ _____

11) $3x^2+14x+15=$ _____

12) $2x^2+5x+3=$ _____

13) $3a^2-23a-8=$ _____

14) $3a^2+a-2=$ _____

15) $6x^2-13x+6=$ _____

16) $2x^2+x-1=$ _____

17) $9x^2-24x+16=$ _____

18) $8y^2+2y-3=$ _____

19) $9x^2-x-10=$ _____

20) $12r^2+13r-4=$ _____

Review

$1x^2 + bx + c$

ex. $x^2 + 7x + 12$

$(x+3)(x+4)$

x	+
12	7

$1 \cdot 12$
$2 \cdot 6$
$3 \cdot 4$

$x^2 - x + 12$

$(x+3)(x-4)$

x	+
−12	−1

$1 \cdot 12$
$2 \cdot 6$
$3, 4$
$-1 \cdot 12$
$3 \cdot -4$
$-3 \cdot 4$