

Prime Factorization and Exponents Lesson #4: Combining the Exponent Laws

Using Factors To Combine the Exponent Laws

Part One:

Three students are attempting to simplify the following expression:

$$3x^2 \times 5x^3$$

$$15x^5$$

Their answers are shown below.

Harry $\Rightarrow 8x^5$

Janet $\Rightarrow 15x^6$

Laura $\Rightarrow 15x^5$

Explain using factors which student is correct.

Part Two:

Use factors to explain why $6a^6 \div 3a^2 = 2a^4$.

$$6 \div 3 = 2 \quad 2a^{6-2} \rightarrow 2a^4$$

Class Ex. #1



State the simplified form of the following.

a) $(-7a^8)(6a^{12})$

b) $3a^4 \times b^5 \times 6a^3$

c) $-16n^5 \div (-2n)$

$$-42a^{8+12}$$

$$-42a^{20}$$

$$18a^{4+5+3}$$

$$18a^{12}$$

$$8n^{5-1}$$

$$8n^4$$

d) $\frac{20y^5}{5y^5}$

e) $\frac{30b^{14}}{45b^{10}}$

$$4y^{5-5} = 4$$

$$\frac{2}{3} b^{14-10}$$

$$\frac{2}{3} b^4$$

$$\frac{2}{3} b^4$$

Class Ex. #2



Simplify the following.

a) $x^5y^8x^3y^4$

b) $\frac{x^5y^8}{x^3y}$

c) $(-3bc)(b^3c^2)(-4b^2c)$

d) $\frac{10e^8f^{12}}{4e^4f^7}$

$$x^{5+3}y^{8+4}$$

$$x^8y^{12}$$

$$x^{5-3}y^{8-1}$$

$$x^2y^7$$

$$-3 \times 1 \times -4 = 12$$

$$b^1 + 3 + 2 = b^6$$

$$c^2 + 1 + 2 = c^5$$

$$= 12b^6c^5$$

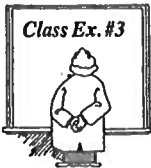
$$\frac{5e^4f^5}{2}$$

Complete Assignment Questions #1 - #4

Combining the Exponent Laws

$2(a^4)^3 \quad (2a^2)^3$

The following examples use two or more of the exponent laws in their solution.



Simplify.

a) $(3x^2)^3$

$3^3 \times x^6$
 $27x^6$

b) $(-2a^2b^3)^2$

$(-2)^2 a^4 b^6$
 $4a^4b^6$

c) $\frac{x^3 x^5}{x^2 x}$

x^8
 $\frac{x^8}{x^3} = x^{8-3} = x^5$

d) $(-\frac{2a}{y^3})^3$

$(-2)^3 a^3$
 $\frac{-8a^3}{y^9}$



Simplify the following.

Bedmas

a) $-(-n^2)^5$

$-1(n^2)^5$
 $= -1(-1)^5 n^{5 \cdot 2}$
 $= (-1)(-1) n^{10}$
 $+1n^{10}$

b) $(\frac{4y^3 \times 3x^6}{6x^5})^4$

$\frac{12x^6 y^3}{6x^5}$
 \downarrow
 $(2x^1 y^3)^4$
 $2^4 x^4 y^{12}$
 $16x^4 y^{12}$

c) $\frac{16(x^3 y^5)^2}{(2x^2)^3}$

$16x^6 y^{10}$
 $\frac{16x^6 y^{10}}{8x^6}$
 $2y^{10}$

d) $(5ab^6)^2 (4a^2b)$

$(5^2 a^2 b^{12})(4a^2 b)$
 $(25a^2 b^{12})(4a^2 b)$
 $100a^4 b^{13}$



Write in simplest form.

a) $(-a)^6 \div (-a)^4$

$a^6 \div a^4$
 a^2

b) $-a^6 \div (-a)^4$

$-a^6 \div a^4$
 $-a^2$

c) $-a^7 \div (-a)^3$

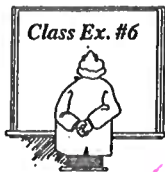
$-a^7 \div -a^3$
 a^4

Complete Assignment Questions #5 - #12

Extension

In higher level mathematics courses, you may meet variable bases and variable exponents including binomial exponents.

Use the exponent laws to simplify the following.



a) $\frac{b^{4x+y}}{b^{x-2y}}$

b) $\frac{x^{5a+7b} \cdot x^{3a+b}}{x^a \cdot x^{2a-7b}}$

$b^{(4x+y) - (x-2y)}$
 $b^{4x+y-x+2y} \Rightarrow b^{3x+3y}$

TOP $x^{(5a+7b)+(3a+b)}$
 x^{8a+8b}

bottom $x^{a+2a-7b}$

x^{3a-7b}

$\frac{x^{8a+8b}}{x^{3a-7b}} \Rightarrow x^{5a+15}$

Complete Assignment Question #13

#(1-6) all, 7ac, (8,9) all
Assignment

1. Simplify the following.

a) $3a^3 \times 3a^4$

b) $(10b^7)(3b^8)$

c) $3a^3 \cdot 5a^3$

d) $(-2x^4)(12x^9)$

e) $\left(-\frac{1}{2}e^7\right)(-14e^8)$

f) $0.4c^3 \times 0.5c$

2. Simplify.

a) $12x^4 \div 6x^2$

b) $(81e^9) \div (9e^8)$

c) $\frac{21d^6}{7d^2}$

d) $\frac{-80d^{80}}{8d^8}$

e) $(-10e^{10}) \div (-5e^5)$

f) $\frac{12f^6}{12f^5}$

3. Write in simplest form.

a) $(3a^2b^3)(5a^4b^8)$

b) $x^9y^0x^2y^4$

c) $\frac{6x^4y^7}{2x^3y^2}$

d) $\frac{5x^4y^7}{x^3y^2}$

e) $\frac{4f^{12}d^3}{12f^4d}$

f) $(7b^4c)(bc^2)(-2b^2c^6)$

4. Simplify.

a) $\frac{10e^8f^8}{15e^4f^2}$

b) $(2p^3)(4p^7)(-2p)$

c) $(-2xy)(x^2y^3)(-3xy)$

d) $(-8b^6c) \div (2b^3c)$

e) $(-10t^8y^6) \div (-2t^7y^3)$

f) $(4x^5z^7) \div (-16xz^6)$

5. Write in simplest form.

a) $(-a^2b^3)^4$

b) $(-a^2b^3)^5$

c) $\left(\frac{b^4}{a^3}\right)^3$

d) $\frac{c^5 \times c^2}{c^4 \times c}$

6. Simplify.

a) $(3ab^2)^4$

b) $(-4a^5c^2)^4$

c) $(-2m^3n^4)^5(m^2n^3)$

d) $(-4x^2y^3)^3(8xy^8)$

e) $(a^3b^4c^5)(3abc^2)^3$

7. Write each expression in simplest form without brackets.

a) $\left(\frac{2d^5 \times d^4}{4d^3}\right)^3$

b) $\left(\frac{-16a^5b^3 \cdot 2a^2b^6}{8ab^7}\right)^3$

c) $\left(\frac{-5k^3 \cdot k^2}{k}\right)^2 \left(\frac{(-k)^5 \cdot k^2}{5k^2}\right)$

8. Write in a simpler form and evaluate.

a) $\frac{6^6 \times 6}{6^4}$

b) $(-3^3)^2$

c) $\left(\frac{2^{10}}{2^5}\right)^3$

d) $\frac{(0.7)^8}{(0.7)^4 \times (0.7)^2}$

e) $-5^6 \times 5^2$

f) $(-5)^6 \times (-5)^2$

g) $-10^{10} \div (-10)^8$

h) $\frac{-10^{10}}{-10^8}$

9. Write each expression in simplest form without brackets.

a) $(-x)^{12} \div (-x)^6$

b) $(-a)^6 \div (-a^4)$

c) $-p^{10} \div (-p)^2$

d) $c^5 \div (-c)^2$

e) $-(-t)^4 \div (-t)^3$

f) $-(-t^4) \div (-t)^3$

**Multiple
Choice**

10. The simplified form of $\frac{1}{36}(2x^3)^2(-3yx^2)$ is

A. x^8y^2

B. $-\frac{1}{3}x^8y$

C. $-\frac{1}{3}x^7y$

D. $-\frac{1}{6}x^6y$

11. The expression $\frac{6(x^3y^5)^2}{(3xy)^4}$ is equivalent to the expression

A. $\frac{4}{9}x^2y^6$

B. $2x^5y^6$

C. $2x^2y^6$

D. $\frac{2}{27}x^2y^6$

Numerical Response

12. If the expression $\frac{4x^{-4}}{8x^{-3}}$ is written in the form ax^b , then the value of $a - b$ is _____.

(Record your answer in the numerical response box from left to right)

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Extension

13. Simplify each expression.

a) $a^{x+y}a^{2x+3y}$

b) $\frac{m^{x+9}}{m^3}$

c) $\frac{a^{3m+2}}{a^{m-3}}$

d) $\frac{x^{2y+7} \cdot x^{3y+2}}{x^{y+8}}$

Answer Key

1. a) $9a^7$ b) $30b^{15}$ c) $15a^6$ d) $-24x^{13}$ e) $7e^{15}$ f) $0.2c^4$

2. a) $2x^2$ b) $9e$ c) $3d^4$ d) $-10d^{72}$ e) $2e^5$ f) f

3. a) $15a^6b^{11}$ b) $x^{11}y^4$ c) $3xy^5$ d) $5xy^5$ e) $\frac{1}{3}f^8d^2$ f) $-14b^7c^9$

4. a) $\frac{2}{3}e^4f^6$ b) $-16p^{11}$ c) $6x^4y^5$ d) $-4b^3$ e) $5ty^3$ f) $-\frac{1}{4}x^4z$

5. a) a^8b^{12} b) $-a^{10}b^{15}$ c) $\frac{b^{12}}{a^9}$ d) c^2

6. a) $81a^4b^8$ b) $256a^{20}c^8$ c) $-32m^{17}n^{23}$ d) $-512x^7y^{17}$ e) $27a^6b^7c^{11}$

7. a) $\frac{d^{18}}{8}$ b) $-64a^{18}b^6$ c) $-5k^{13}$

8. a) $6^3 = 216$ b) $3^6 = 729$ c) $2^{15} = 32768$ d) $(0.7)^2 = 0.49$
 e) $-5^8 = -390625$ f) $(-5)^8 = 390625$ g) $-10^2 = -100$ h) $10^2 = 100$

9. a) x^6 b) $-a^2$ c) $-p^8$ d) c^3 e) t f) $-t$

10. B 11. D 12.

1	.	5	
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13. a) a^{3x+4y} b) m^{x+6} c) a^{2m+5} d) x^{4y+1}

July 6

Prime Factorization and Exponents Lesson #5: Integral Exponents

The Negative Exponent

a) Complete the patterns below.

$$10^3 = 1000 \quad \swarrow \div 10$$

$$10^2 = 100 \quad \swarrow \div 10$$

$$10^1 = 10$$

$$10^0 = 1 \quad \swarrow \div 10$$

$$10^{-1} = \frac{1}{10} = \frac{1}{10^1}$$

$$10^{-2} = \frac{1}{100}$$

$$10^{-3} = \frac{1}{1000}$$

$$3^3 = 27$$

$$3^2 = 9$$

$$3^1 = 3$$

$$3^0 =$$

$$3^{-1} =$$

$$3^{-2} =$$

$$3^{-3} =$$

$$a^0 = 1$$

$$a^{-1} = \frac{1}{a}$$

$$a^{-2} = \frac{1}{a^2}$$

$$a^{-3} = \frac{1}{a^3}$$

b) Write the following with positive exponents.

i) $10^{-7} = \frac{1}{10^7}$

ii) $3^{-5} = \frac{1}{3^5}$

iii) $a^{-n} = \frac{1}{a^n}$

Using the Exponent Laws to Define the Negative Exponent

Consider the expression $5^4 \div 5^7$.

a) Evaluate the expression as an exact value using a calculator.

$$0.008 = \frac{1}{125}$$

b) Complete the following to evaluate the expression.

$$5^4 \div 5^7 = \frac{\cancel{5 \cdot 5 \cdot 5 \cdot 5}}{\cancel{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}} = \frac{1}{5^3} = \frac{1}{125}$$

c) Use the quotient law to complete the following.

$$5^4 \div 5^7 = 5^{4-7} = 5^{-3}$$

d) The results in a) to c) are examples of a general rule when a base is raised to a negative exponent. Complete: $a^{-p} = \frac{1}{a^p}$

e) Write the following with positive exponents and evaluate.

i) $2^{-1} = \frac{1}{2}$

ii) $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

iii) $4^{-3} = \frac{1}{4^3} = \frac{1}{64}$

The Negative Exponent in the Denominator

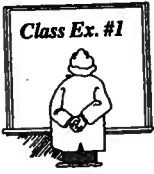
Use the rule for division of fractions to show that $\frac{1}{4^{-3}} = 4^3$. Use a calculator to confirm.

Negative Exponent Law

A base (not including zero) raised to a negative exponent has the following properties:

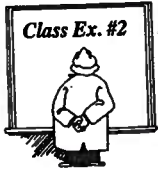
$a^{-n} = \frac{1}{a^n}, a \neq 0$ and $\frac{1}{a^{-n}} = a^n, a \neq 0$

$6^{-7} \cdot 6^5 = 6^{-7+5} = 6^{-2}$



Simplify, express with positive exponents, and evaluate without using a calculator.

a) $4^5 \times 4^{-3} = 4^{5-3} = 4^2 = 16$
 b) $3^2 \times 3^{-5} = 3^{2-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$
 c) $\frac{1}{2^{-5}} = 2^5 = 32$
 d) $\frac{6^{-7}}{6^{-5}} = \frac{6^{-7}}{6^{-5}} = 6^{-7-(-5)} = 6^{-2} = \frac{1}{6^2} = \frac{1}{36}$
 e) $(2^3)^{-1} = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$



Identify the following as true or false.

a) $\frac{8^3}{8^{-1}} = 8^4$ TRUE
 $8^3 \cdot 8^1 = 8^4$
 b) $\frac{8^3}{4^{-1}} = 2^4$ FALSE
 $8^3 \cdot 4 = 2048$
 $2^4 = 16$
 c) $a^{-3} = \frac{1}{a^3}$ TRUE



Explain why $2p^{-3} \neq \frac{1}{2p^3}$.

$2p^{-3} = \frac{2}{p^3}$



a) $a^{-4} \times a^{-3}$
 $a^{-4+(-3)}$
 $= a^{-7} = \boxed{\frac{1}{a^7}}$

b) $6x^2 + 2x^7$
 $= 3x^{2-7}$
 $= 3x^{-5} = \boxed{\frac{3}{x^5}}$

c) $\frac{y^6}{2y^{-5}}$
 $= \frac{1}{2} y^{6-(-5)}$
 $= \frac{1}{2} y^{11} = \boxed{\frac{y^{11}}{2}}$

d) $(-2x)^{-3}$
 $\frac{1}{(-2x)^3} = \frac{1}{-8x^3}$

e) $\frac{8a^{-5}}{4b^{-3}}$
 $= \boxed{\frac{2b^3}{a^5}}$

f) $\frac{(5p)^{-2}}{5q^4}$
 $\frac{1}{5q^4(5p)^2}$
 $= \frac{1}{5q^4 \cdot 25p^2} = \boxed{\frac{1}{125p^2q^4}}$

Simplifying a Fractional Base with a Negative Exponent

Consider the expression $\left(\frac{2}{3}\right)^{-4}$.

a) Complete the following $\left(\frac{2}{3}\right)^{-4} = \frac{1}{\left(\frac{2}{3}\right)^4} = \frac{1}{16} = 1 \times \frac{1}{16}$

b) Evaluate $\left(\frac{3}{2}\right)^4$.

c) Classify the following statement as true or false. $\left(\frac{2}{3}\right)^{-4} = \left(\frac{3}{2}\right)^4$

d) Suggest a quick method for evaluating $\left(\frac{5}{2}\right)^{-3}$ without using a calculator.

In general, $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ $a, b \neq 0$.

flip fraction to solve neg. exponent
 $\left(\frac{5}{2}\right)^{-3} = \left(\frac{2}{5}\right)^3 = \frac{8}{125}$

Complete Assignment Questions #1 - #15

Assignment

(1-13) every 2nd letter
ACEGI

1. Write the following with positive exponents.

a) x^{-3} b) y^{-9} c) 4^{-1} d) $\frac{1}{a^{-5}}$ e) $\frac{1}{6^{-2}}$

2. Without using a calculator show that $\frac{3}{5^{-2}} = 75$.

3. Simplify, express with positive exponents, and evaluate without using a calculator.

a) $4^3 \times 4^{-4}$ b) $3^0 \times 3^{-3}$ c) $\frac{1}{7^{-2}}$ d) $\frac{10^{-3}}{10}$ e) $(3^2)^{-2}$

4. Express with positive exponents.

a) n^2m^{-5} b) $c^{-2}x^{-5}$ c) $16h^{-1}$ d) $\frac{2}{3}b^{-8}$ e) $(y^{-4})^{-2}$

f) $\frac{t^{-5}}{4}$ g) $\frac{1}{4x^{-9}}$ h) $\frac{4}{x^{-9}}$ i) $\frac{a^2}{b^{-7}}$ j) $\frac{a^{-2}}{b^7}$

5. Evaluate the following without using a calculator.

a) -3^{-2} b) $(-3)^{-2}$ c) $-7^2 \cdot 8^{-2}$ d) $(-8.3)^0$ e) $[-(3.9)^0]^{-2}$

6. Use a calculator to find the exact value of the following.

a) -4^{-4} b) $(-7)^{-3}$ c) $(0.75)^{-3}$ d) $(-0.025)^{-2}$ e) $\left(\frac{4}{7}\right)^{-3}$

7. State whether the following are true or false.

a) $6x^{-3} = \frac{6}{x^3}$ b) $5a^{-4} = \frac{1}{5a^4}$ c) $\frac{4}{b^{-6}} = 4b^6$ d) $\frac{x^{-3}}{2} = \frac{2}{x^3}$

e) $\frac{1}{5y^{-1}} = 5y$ f) $\frac{1}{\frac{1}{4}p} = \frac{1}{4}p^{-1}$ g) $(3x)^5 = \frac{1}{(3x)^{-5}}$ h) $\frac{1}{\left(\frac{1}{7}a\right)^{-2}} = 49a^2$

8. Simplify and write the answer with positive exponents.

a) $x^{10} \cdot x^{-5}$ b) $m^5 \div m^8$ c) $b^{-1} \cdot b^{-3}$ d) $-w^0 \div w^5$

9. Simplify and write the answer with positive exponents.

a) $a^8 \times a^{-10}$ b) $10x^2 \div 2x^{-1}$ c) $\frac{6y^{-6}}{2y^{-4}}$ d) $\frac{2a^{-5}}{4b^6}$

e) $-7x^{-2}$ f) $-(7x)^{-2}$ g) $(-7x)^{-2}$ h) $\frac{(-7x)^{-2}}{-7x^{-2}}$

10. Simplify each expression, writing the answer with positive exponents.

a) $a^{-3}a^{-3}$ b) $(5b^8b^{-12})(-10b^3b^{-12})$ c) $(-7x^3x^{-5})(x^2x^{-3})$

d) $(-2a^3)^{-3} \cdot 3a^{12}$ e) $\frac{16a^6b^{-3}}{-4a^6b^3}$ f) $(-3a^5b^{-3}c^0)^{-2}$

11. Simplify. Write the final answer with positive exponents.

a) $\frac{32a^2b^{-4}}{4a^{-8}b^{-2}} \times \frac{-8a^{-2}}{-3b^{-3}}$

b) $\frac{10(p^3q^2r^0)^{-3}}{(8p^{-3}q^5r^3)^{-2}}$

c) $(-2x^5y^3z^8)^{-2}(-2x^2y^{-8}z^{12})^3$

d) $(5a^3b^2)(-2a^{-2}b)^{-3} + (-5a^8b^{-9})^{-2}$

12. Evaluate the following without using a calculator.

a) $\left(\frac{2}{3}\right)^{-3}$

b) $\left(\frac{1}{5}\right)^{-2}$

c) $\left(\frac{8}{5}\right)^{-1}$

d) $\left(\frac{3}{2}\right)^{-4}$

13. Simplify. Write the final answers with positive exponents.

a) $\left(\frac{c}{d}\right)^{-3}$

b) $\left(\frac{x}{4}\right)^{-3}$

c) $\left(\frac{p^2}{r^4}\right)^{-3}$

d) $\left(\frac{a^{-2}}{b^{-5}}\right)^{-3}$

e) $\left(\frac{-12x^{-3}}{6y^{-8}}\right)^{-1}$

f) $\left(\frac{12x^3y^{-1}}{-8x^{-1}y^5}\right)^{-2}$

Multiple
Choice14. The value of $\frac{1^{-3} + 3^0}{2^{-1}}$ is

- A. 1
B. 4
C. 8
D. 12

15. Which of the following statements are true?

i) $3a^{-3} = \frac{1}{3a^3}$ ii) $8x^4 \div 4x^7 = \frac{1}{2x^3}$ iii) $\frac{1}{2a} = 2a^{-1}$

- A. i) only
B. ii) only
C. iii) only
D. none of the statements are true

Answer Key

1. a) $\frac{1}{x^3}$ b) $\frac{1}{y^9}$ c) $\frac{1}{4}$ d) a^5 e) 6^2
2. $\frac{3}{5^{-2}} = 3 \times 5^2 = 3 \times 25 = 75$
3. a) $\frac{1}{4^1} = \frac{1}{4}$ b) $\frac{1}{3^3} = \frac{1}{27}$ c) $7^2 = 49$ d) $\frac{1}{10^4} = \frac{1}{10\,000}$ e) $\frac{1}{3^4} = \frac{1}{81}$
4. a) $\frac{n^2}{m^5}$ b) $\frac{1}{c^2x^5}$ c) $\frac{16}{h}$ d) $\frac{2}{3b^8}$ e) y^8
- f) $\frac{1}{4t^5}$ g) $\frac{x^9}{4}$ h) $4x^9$ i) a^2b^7 j) $\frac{1}{a^2b^7}$
5. a) $-\frac{1}{9}$ b) $\frac{1}{9}$ c) $-\frac{49}{64}$ d) 1 e) 1
6. a) $-\frac{1}{256}$ b) $\frac{1}{343}$ c) $\frac{64}{27}$ d) 1600 e) $\frac{343}{64}$
7. a) T b) F c) T d) F e) F f) F g) T h) F
8. a) x^5 b) $\frac{1}{m^3}$ c) $\frac{1}{b^4}$ d) $-\frac{1}{w^5}$

9. a) $\frac{1}{a^2}$

b) $5x^3$

c) $\frac{3}{y^2}$

d) $\frac{1}{2a^5b^6}$

e) $-\frac{7}{x^2}$

f) $-\frac{1}{49x^2}$

g) $\frac{1}{49x^2}$

h) $-\frac{1}{343}$

10. a) $\frac{1}{a^6}$

b) $-\frac{50}{b^{13}}$

c) $-\frac{7}{x^3}$

d) $-\frac{3}{8}a^3$

e) $-\frac{4}{b^6}$

f) $\frac{b^6}{9a^{10}}$

11. a) $\frac{64}{3}a^8b$

b) $\frac{640q^4r^6}{p^{15}}$

c) $-\frac{2z^{20}}{x^4y^{30}}$

d) $-\frac{125a^{25}}{8b^{19}}$

12. a) $\frac{27}{8}$

b) 25

c) $\frac{5}{8}$

d) $\frac{16}{81}$

13. a) $\frac{d^3}{c^3}$

b) $\frac{64}{x^3}$

c) $\frac{r^{12}}{p^6}$

d) $\frac{a^6}{b^{15}}$

e) $-\frac{x^3}{2y^8}$

f) $\frac{4y^{12}}{9x^8}$

14. B

15. D