Prime Factorization and Exponents Lesson #1:
Prime Factors

July 5 #1

Overview

In this unit we look at prime factorization of whole numbers and applications including greatest common factor, least common multiple, square root, and cube root. We review powers with whole number exponents where the base is numerical. We also extend these concepts to powers where the base is variable and where the exponents are integers using inquiry, analyzation, and reasoning.

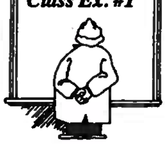
Factors

The whole number 6 is exactly divisible by the whole numbers 1, 2, 3, and 6.

The numbers 1, 2, 3, and 6 are the **factors** of the whole number 6.

The number 6 has four factors.

Class Ex. #1



In each case, state the number of factors of the given whole number.

a) 1 — one

b) 2 — 1, 2 two

c) 3 — 1, 3 two

d) 4 — 1, 2, 4 three

e) 12 — 1, 2, 3, 4, 6, 12 six

Factor Pairs

We say **2** and **3** are a **factor pair** of **6** because $2 \times 3 = 6$.
A **factor pair** is a set of two whole numbers which when multiplied result in a specific product.

i) State another factor pair of 6.

We can use a diagram listing all the factors of a number to help determine all possible factor pairs of that number.

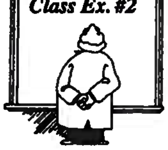
The diagram opposite shows there are four factor pairs of 64.

8 and 8 are a factor pair because $8 \times 8 = 64$.

ii) List the other factor pairs.

1 2 4 8 16 32 64

Class Ex. #2



List the factors of the following and determine all the factor pairs for each.

a) 15 — 1, 3, 5, 15

b) 16 — 1, 2, 4, 8, 16

[redacted]

Prime and Composite Numbers

A **prime number** is defined as a whole number which has exactly two distinct factors. The two factors are always 1 and the number itself, e.g. 3, 7.

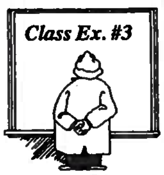
A **composite number** is a whole number which has more than two factors, e.g. 10, 18.

The number 1 has only one factor and is neither prime nor composite.

In this course the number 0 is defined to have no factors.

- Complete the list of the first ten prime numbers:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29....



Classify the following whole numbers as prime or composite.

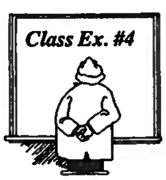
a) 46 composite b) 37 prime c) 39 composite 3 & 13 are factors d) 101 prime e) 103 prime

Prime Factors

The **prime factors** of a whole number are the factors of the number which are prime.

For example The factors of 6 are 1, 2, 3, and 6.

The prime factors of 6 are 2 and 3.



a) State the factors of 12.

1, 2, 3, 4, 6, 12

b) State the prime factors of 12.

2, 3

c) Express 12 as a **product of prime factors**.

$12 = 2 \times 3 \times 2$

Complete Assignment Questions #1 - #9

Prime Factorization

Every composite number can be expressed as a product of prime factors. Expressing a whole number as a product of prime factors is called the **prime factorization** of the number.

The prime factorization of small numbers like 12 can probably be done mentally. For larger numbers, a division table or a tree diagram can be used.

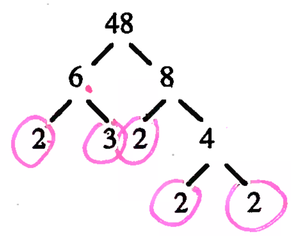
The diagrams below illustrate these techniques for the prime factorization of 48.

- Prime numbers must be used as divisors when using a division table.
- Any divisor may be used when using a tree diagram.

division table

2	48
2	24
2	12
2	6
3	3
	1

tree diagram



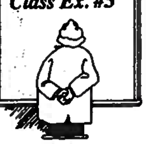
$48 = 2 \times 2 \times 2 \times 2 \times 3$

or using powers

$48 = 2^4 \times 3$

$5 \times 5 \times 5$

Class Ex. #5

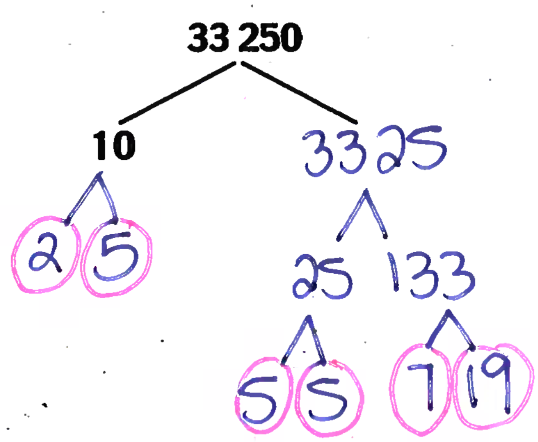


Two students, Tantoo and Craig, work together to confirm that the prime factorization of 33 250 is $2 \times 5^3 \times 7 \times 19$. Tanto uses a division table and Craig uses a factor tree. Complete their work to verify that the prime factorization of 33 250 is $2 \times 5^3 \times 7 \times 19$.

Tantoo Division Table

2	33 250
5	16625
5	3325
5	665
7	133
19	19
	1

Craig Tree Diagram



$2 \times 5 \times 5 \times 5 \times 7 \times 19 = 2 \times 5^3 \times 7 \times 19$

Complete Assignment Questions #10 - #17

[redacted]

#2, 5, 7, 8, 10, 11, 12

/31

Assignment

1. In each case, determine the number of factors of the given whole number.

   a) 8 b) 11 c) 17 d) 33 e) 45

2. List the factors of the following numbers and determine all the factor pairs.

   a) 21 b) 22

   c) 25 d) 36

3. In some of the parts of question #2, the number of factor pairs is exactly one half of the number of factors and in other parts it is not. Explain.

4. State the numbers in question #1 which are

   a) prime b) composite

5. Classify the following whole numbers as prime or composite.

   a) 30 b) 41 c) 43 d) 57

   e) 59 f) 121 g) 133 h) 169

6. Twin primes are defined to be consecutive odd numbers that are both prime (e.g. 5 and 7). List the seven other twin primes less than 80.

7. a) State the factors of 20.

b) State the prime factors of 20.

c) Express 20 as a product of prime factors.

8. State the prime factors of

a) 15 b) 24 c) 45 d) 66

9. Explain why the numbers 0 and 1 have no prime factors.

10. Use a division table to express the prime factorization of the following using powers where appropriate.

a) 140

b) 330

c) 1 911

d) 1 925

[redacted]

11. Use a tree diagram to determine the prime factorization of the following:

a) 390

b) 546

c) 3 705

d) 6 762

12. In each case write the number as a product of prime factors.

a) 189

b) 685

c) 4 235

d) 7 980

Multiple Choice

13. Which of the following numbers is **not** a prime factor of 1 925?

A. 5
B. 7
C. 11
D. 13

14. How many of the numbers in the list 2, 3, 9, 13 are **not** prime factors of 2 592?

A. 4
B. 3
C. 2
D. 1

Numerical Response

15. The sum of all of the prime factors of 373 065 is _______.

(Record your answer in the numerical response box from left to right)

#2, 5, 7, 8, 10, 11, 12

16. There is only one set of prime triplets: three consecutive odd numbers which are all prime. If the prime triplets are a, b, and c, then the value of abc is _______.

(Record your answer in the numerical response box from left to right)

17. The number 686 can be expressed as a product of prime factors in the form $p \times q^r$. The value of $p+q+r$ is ______.

(Record your answer in the numerical response box from left to right)

Answer Key

1.a) 4 **b)** 2 **c)** 2 **d)** 4 **e)** 6

2.a) factors: 1, 3, 7, 21
factor pairs: 1 and 21, 3 and 7

b) factors: 1, 2, 11, 22
factor pairs: 1 and 22, 2 and 11

c) factors: 1, 5, 25
factor pairs: 1 and 25, 5 and 5

d) factors: 1, 2, 3, 4, 6, 9, 12, 18, 36
factor pairs: 1 and 36, 2 and 18, 3 and 12, 4 and 9, 6 and 6

3.
- If the whole number is not a perfect square, then each factor has another factor which forms a factor pair thus the number of factor pairs is exactly one half the number of factors.
- If the whole number is a perfect square, then the square root of the number is a factor which does not have a different factor to form a factor pair. Therefore the number of factors will be an odd number and the number of factor pairs will not be half that number.

4. a) 11, 17 **b)** 8, 33, 45

5.a) composite **b)** prime **c)** prime **d)** composite **e)** prime **f)** composite **g)** composite **h)** composite

6. 3 and 5, 11 and 13, 17 and 19, 29 and 31, 41 and 43, 59 and 61, 71 and 73.

7.a) 1, 2, 4, 5, 10, 20 **b)** 2, 5 **c)** $2 \times 2 \times 5$ **8. a)** 3, 5 **b)** 2, 3 **c)** 3, 5 **d)** 2, 3, 11

9. 0 is defined to have no factors and 1 has only one factor. Since a prime number has exactly two distinct factors neither 0 or 1 can be a prime number.

10.a) $2^2 \times 5 \times 7$ **b)** $2 \times 3 \times 5 \times 11$ **c)** $3 \times 7^2 \times 13$ **d)** $5^2 \times 7 \times 11$

11.a) $2 \times 3 \times 5 \times 13$ **b)** $2 \times 3 \times 7 \times 13$ **c)** $3 \times 5 \times 13 \times 19$ **d)** $2 \times 3 \times 7^2 \times 23$

12.a) $3^3 \times 7$ **b)** 5×137 **c)** $5 \times 7 \times 11^2$ **d)** $2^2 \times 3 \times 5 \times 7 \times 19$

13. D **14.** C

15.

6	2		

16.

1	0	5	

17.

1	2		

Prime Factorization and Exponents Lesson #2:
Applications of Prime Factors

July 5th

Review

Express the numbers 48 and 72 as products of prime factors.

48: 4, 12; 2, 2, 3, 4; 2, 2

$2\times2\times2\times2\times3=$

$2^4\times3=48$

72: 2 | 72, 2 | 36, 2 | 18, 3 | 9, 3 | 3, 1

$2\times2\times2\times3\times3$

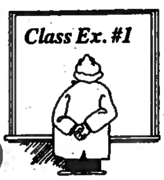
$2^3\times3^2$

$=72$

Greatest Common Factor

The **greatest common factor** (GCF) of a set of whole numbers is the largest whole number which divides exactly into each of the members of the set.

For example, the GCF of 8, 16, and 20 is 4.



State the greatest common factor of

a) 15, 25, and 35   5   **b)** 18 and 20   2   **c)** 36 and 54   18

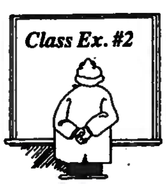
In the example above parts a) and b) were fairly simple to do, but part c) was more complicated because each number had a large number of factors.

In cases like this we can use prime factorization to determine the GCF.

From the review $48 = 2\times2\times2\times2\times3$ and $72 = 2\times2\times2\times3\times3$.

To determine the GCF of 48 and 72 we find the product of each prime factor (including repeats) which is common to both prime factorizations.

GCF of 48 and 72 is $2\times2\times2\times3 =$ ______.



Use prime factorization to determine the greatest common factor of the given whole numbers.

a) 90 and 225

5 | 90 225, 3 | 18 45, 3 | 6 15, 2 5

$5\times3\times3=45$

b) 154 and 198

154: 2, 77; 11, 7
198: 2, 99; 11, 9; 3, 3

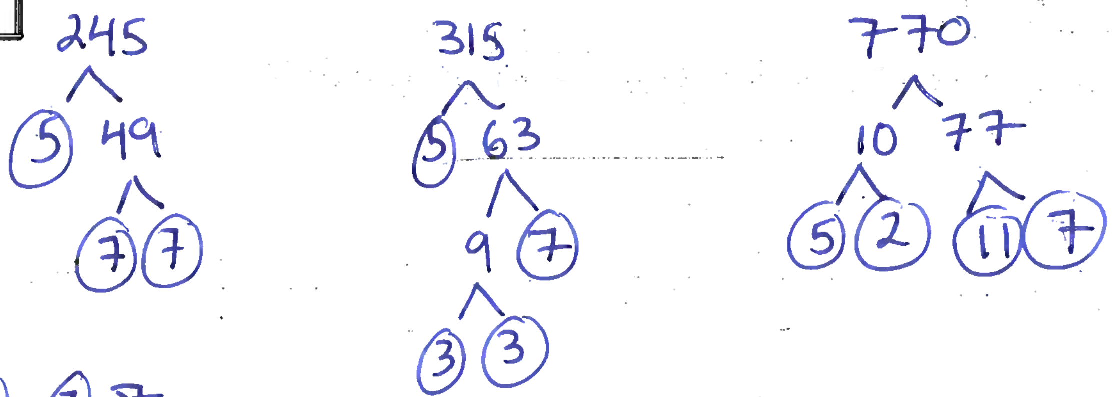
154: 2 × 11
198: 2 × 3 × 3 × 11

$11\times2=22$

[redacted]

Class Ex. #3

Use prime factorization to determine the GCF of 245, 315, and 770.



245: 5 7 7
315: 3 3 5 7
770: 2 5 7 11

$5 \times 7 = 35$

Lowest Common Multiple

Multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, ...

Multiples of 8 are 8, 16, 24, 32, 40, 48, 56, ...

Common multiples of 6 and 8 are 24, 48, ...

The **lowest common multiple** (LCM) of 6 and 8 is 24.

Class Ex. #4

Determine the lowest common multiple of the following:

a) 5 and 7 LCM: 35 **b)** 10, 15, and 20 **c)** 10, 12, and 14

5: 5, 10, 15, 20, 25, 30, 35, 40

7: 7, 14, 21, 35, 42

In the example above, parts a) and b) were fairly simple to do, but part c) was more complicated. Prime factors can be used to simplify the solution.

$10 = 2 \times 5$

$12 = 2 \times 2 \times 3$

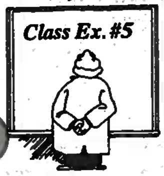
$14 = 2 \times 7$

To determine the LCM, take all the prime factors of one of the numbers and multiply by any additional factors in the other numbers.

Take 2 and 5 from 10, another 2 and 3 from 12, and 7 from 14.

$2 \times 5 \times 2 \times 3 \times 7 =$ 420, the LCM of 10, 12, and 14.

[redacted]



Class Ex. #5

Use prime factorization to determine the LCM of

a) 15 and 35

5 | 15, 35
3, 7

5 x 3 x 7 = 105

b) 126 and 441

126 → 63, 2; 63 → 7, 9; 9 → 3, 3

441 → 7, 63; 63 → 9, 7; 9 → 3, 3

126 : 2, 3, 3, 7
441 : 3, 3, 7, 7

2x3x3x7x7 = 882

$2 \times 3^2 \times 7^2 = 882$

c) 22, 154, and 198

2 | 22, 154, 198
11 | 11, 77, 99
1, 7, 9

2 x 7 x 9 x 11 = $\boxed{1386}$

Complete Assignment Questions #1 - #7

Prime Factorization of a Perfect Square

Perfect squares of whole numbers include 1, 4, 9, 16, 25, 36, 49, etc.

Every perfect square has two **square roots**: one positive and one negative. The square root which is positive is called the **principal square root**.

The principal square root of each number above is 1, 2, 3, 4, 5, 6, 7, etc.

In this lesson, where we are dealing only with whole numbers, we will use the term square root to mean the principal square root.

Complete the prime factorization of the following perfect squares: 9, 36, 49, and 64.

9 → 3, 3

36 → 4, 9; 4 → 2, 2; 9 → 3, 3

49 → 7, 7

64 → 2, 32; 32 → 2, 16; 16 → 8, 2; 8 → 4, 2; 4 → 2, 2

9 = 3 × 3

36 = 2 × 2 × 3 × 3

49 = 7 × 7

64 =

The square root of 9 is 3

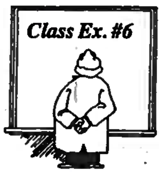
The square root of 36 is 2 × 3 = 6

The square root of 49 is

The square root of 64 is 2 × 2 × 2 = 8

The prime factorization of a perfect square will involve sets of factors which each occur two times (or a multiple of two times).

If the prime factorization of a number does not result in sets of factors which each occur two times (or a multiple of two times), then we can say that the number is **not** a perfect square.



Consider the number 44 100.

a) Use a calculator to find the square root of 44 100.

b) **Explain** how we can use the prime factorization of 44 100 to show that 44 100 is a perfect square. Verify your calculator answer by this method.

a) $\sqrt{44100}$

$= 210$

44 100

4410 10

441 10 2 5

147 3 5 2

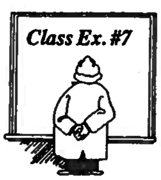
49 3

7 7

2 2 3 3 5 5 7 7

$2 \times 3 \times 5 \times 7$

$= 210$



In each case use prime factorization to determine if the number is a perfect square. If the number is a perfect square, state the square root of the number.

a) 1 225

5 245

5 49

7 7

5 5 7 7

$5 \times 7 = 35$

b) 3 042

2 1521

3 507

3 169

13 13

2 3 3 13 13

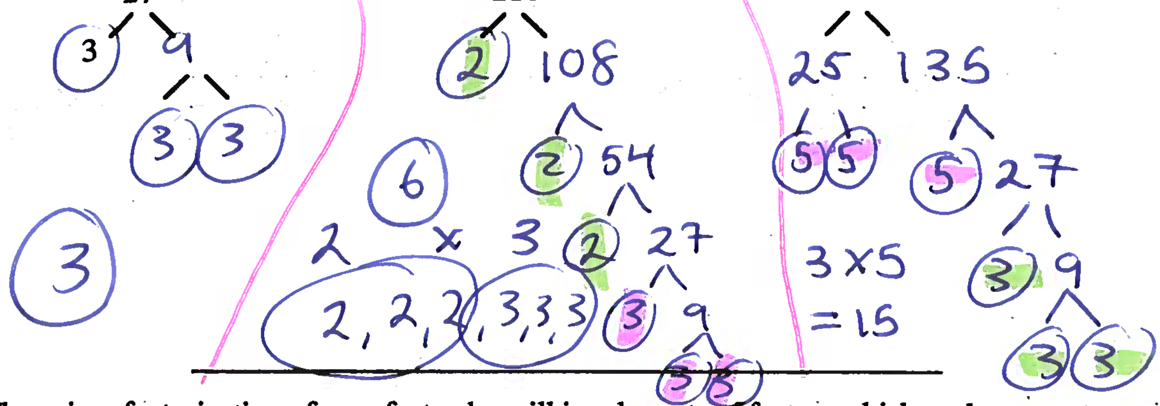
not a perfect square

Prime Factorization of a Perfect Cube

Perfect cubes of whole numbers include 1, 8, 27, 64, 125, etc.
The **cube root** of each number above is 1, 2, 3, 4, 5, etc.

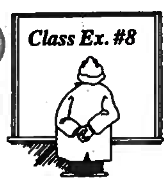
Complete the prime factorization of the following perfect cubes: 27, 216, and 3 375.

27 216 3375



The prime factorization of a perfect cube will involve sets of factors which each occur three times (or a multiple of three times).

If the prime factorization of a number does not result in factors which each occur three times (or a multiple of three times), then we can say that the number is not a perfect cube.



In each case use prime factorization (division table) to determine whether the number is a perfect cube. If the number is a perfect cube, state the cube root of the number.

a) 6 912

b) 970 299

Complete Assignment Questions #8 - #15

Assignment

1-7 (ACE), 8-10

/23

1. State the greatest common factor of

a) 14 and 21 **b)** 30 and 40 **c)** 12, 30, and 54

2. Use prime factorization to determine the GCF of

a) 150 and 420 **b)** 126 and 189 **c)** 294 and 385

3. Use prime factorization to determine the greatest common factor of

a) 483 and 575 **b)** 180 and 504 **c)** 1 700 and 1 938

d) 663 and 910 **e)** 84 and 231 **f)** 525 and 850

4. Determine the GCF of 66, 495, and 2 541

5. State the lowest common multiple of

a) 4 and 6 **b)** 3 and 9 **c)** 9 and 15 **d)** 40, 60, and 100

6. Use prime factorization to determine the LCM of

a) 14 and 30 **b)** 28 and 60 **c)** 10 and 115

d) 18 and 63 **e)** 55 and 143 **f)** 125 and 175

[redacted]

7. Determine the LCM of

a) 72 and 252

b) 6, 10, and 42

c) 12, 30, and 105

$3 \,|\, 12,\ 30,\ 105$

$\quad 4,\ 10,\ 35$

LCM: $3 \times 4 \times 10 \times 35 = 4200$

GCF: 3

8. In each case use prime factorization to determine whether the number is a perfect square. If the number is a perfect square, state the square root of the number.
Verify your answer by using a calculator to determine the square root.

a) 216

b) 11 025

c) 882

d) 1 225

[redacted]

9. Consider the number 74 088.

a) Use a calculator to find the cube root of 74 088.

b) **Explain** how we can use the prime factorization of 74 088 to show that 74 088 is a perfect cube. Verify your calculator answer by this method.

10. In each case use prime factorization to determine whether the number is a perfect cube. Verify your answer by using a calculator to determine the cube root.

a) 216 **b)** 11 025

11. **Explain** how you could use prime factorization to determine if a particular whole number is both a perfect square and a perfect cube.

Multiple Choice

12. The greatest common factor of 399 and 462 is

A. 3
B. 7
C. 19
D. 21

[redacted]

13. The greatest common factor of two whole numbers x and y is 10. Which of the statements A, B, C, or D below is false?

A. x and y must be even numbers.
B. The product xy must be divisible by 100.
C. x and y are both divisible by 5.
D. Neither x nor y can be a prime number.
Answer **E** if none of the statements is false.

Numerical Response

14. The lowest common multiple of 35, 231, and 275 is ______.

(Record your answer in the numerical response box from left to right)

15. A new children's encyclopedia has 950 pages. Each page contains two background colours for illustrations. Page 8 and every 8th page thereafter has green as one of the background colours. Page 18 and every 18th page thereafter has orange as one of the background colours.
How many pages in the book have both green and orange as background colours?

(Record your answer in the numerical response box from left to right)

Answer Key

1a) 7 **b)** 10 **c)** 6 **2a)** 30 **b)** 63 **c)** 7
3a) 23 **b)** 36 **c)** 34 **d)** 13 **e)** 21 **f)** 25
4. 33 **5a)** 12 **b)** 9 **c)** 45 **d)** 600
6a) 210 **b)** 420 **c)** 230 **d)** 126 **e)** 715 **f)** 875
7a) 504 **b)** 210 **c)** 420 **8a)** no **b)** yes, 105 **c)** no **d)** yes, 35
9a) 42 **b)** If the prime factorization of 74 088 has factors which each appear three times, then 74 088 is a perfect cube.
10a) yes, 6 **b)** no
11. If the prime factorization of the number has factors which each appear six times, then the number will be both a perfect square and a perfect cube.
12. D **13.** E **14.** | 5 | 7 | 7 | 5 | **15.** | 1 | 3 | | |

[redacted]

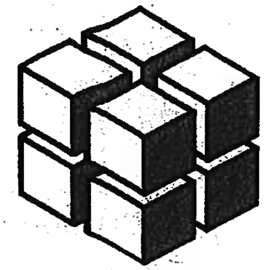
1-5 (a, c, e, etc.) 7+8 (all) 12-15(a, c, e, et

Prime Factorization and Exponents Lesson #3:
Powers with Whole Number Exponents

In this lesson we review numbers written as powers, and the exponent laws applied to powers with numerical bases and whole number exponents.
We extend the work to consider bases which are variable.

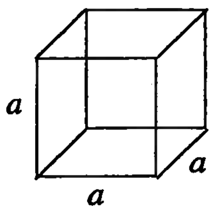
Exponents

In mathematics, exponents are used as a short way to write repeated multiplication.



The number of small cubes in the diagram can be calculated by the **repeated multiplication** $2 \times 2 \times 2$.
This can be written in **exponential form** as 2^3.

Exponents can also be used with variables.

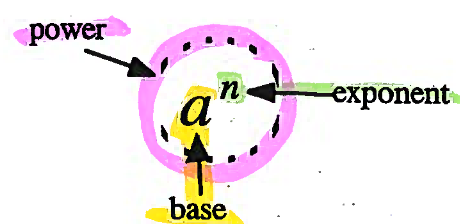


The volume of the cube to the left can be determined by repeated multiplication $a \times a \times a$ or in exponential form a^3.

Powers

A **power** is a number written in exponential form.

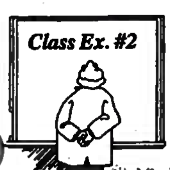
It consists of a **base** and an **exponent**.



State the base and the exponent in each of the following powers.

a) 4^5 base: 4 exponent: 5 b) $(-3)^6$ base: -3 exponent: 6 c) x^y base: x exponent: y

A number that multiplies a variable is called a **coefficient.**
In the expression $7p^3$ the coefficient is 7.

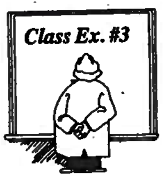


State the coefficient in each of the of the following.

a) $8x^2$ 8 b) $-3z^9$ -3 c) $\frac{a^8}{7}$ $\frac{1}{7}$ ÷

[redacted]

Note that written as a repeated multiplication $7p^3 = 7 \times p \times p \times p$,
whereas $(7p)^3 = 7p \times 7p \times 7p = 7 \times p \times 7 \times p \times 7 \times p = 7 \times 7 \times 7 \times p \times p \times p = 343p^3$.



Write each of the following as a repeated multiplication.

a) $3a^4b$

$3 \times \underbrace{a \times a \times a \times a} \times b$

b) $3ab^4$

$3 \cdot a \cdot b \cdot b \cdot b \cdot b$

c) $3(ab)^4$

$3 \times ab \times ab \times ab \times ab$

$3 \times a \times a \times a \times a \times b \times b \times b \times b$

d) $(3ab)^4$

$3ab \cdot 3ab \cdot 3ab \cdot 3ab$

$3 \cdot 3 \cdot 3 \cdot 3 \cdot a \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b$

$81 \cdot a \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b$

Evaluating Powers

$10^3 = 10 \times 10 \times 10 =$ 1000

$3^4 = 3 \times 3 \times 3 \times 3 =$ 81

$(-6)^2 = (-6) \times (-6) =$ 36

$-6^2 = -(6 \times 6) =$ −36

The Zero Exponent

Complete the patterns below by adding one more row.

$10^4 = 10000$ $\quad$ $3^4 = 81$

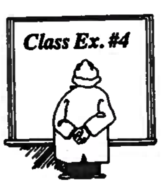
$10^3 = 1000$ $\quad$ $3^3 = 27$

$10^2 = 100$ $\quad$ $3^2 = 9$

$10^1 = 10$ $\quad$ $3^1 = 3$

$10^0 =$ 1 $\quad$ $3^0 = 1$

The results above are examples of a general rule when a base is raised to the exponent zero.
Complete: $a^0 =$ 1.



Evaluate the following. a) 6^0 b) $(-9)^0$ c) -9^0 d) $2(6^2)^0$

a) 1

b) 1

c) −1

d) $2(1)$
$= 2$

Complete Assignment Questions #1 - #7

The Exponent Laws

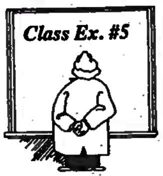
The exponent laws with whole number exponents and numerical bases were covered in previous math courses.

The chart below extends the exponent laws to bases which are variables.

Complete the table as a review of the exponent laws.

Numerical Bases	Variable Bases	Exponent Laws
$8^3 \times 8^2 = (8 \cdot 8 \cdot 8)(8 \cdot 8)$ $= 8^5$ or 8^{3+2}	$a^3 \times a^2 = (a \cdot a \cdot a)(a \cdot a)$ $= a^5$ or a^{3+2}	**Product Law** $(a^m)(a^n) = a^{m+n}$
$8^3 \div 8^2 = \dfrac{\cancel{8} \cdot \cancel{8} \cdot 8}{\cancel{8} \cdot \cancel{8}}$ $= 8^1$ or 8^{3-2}	$a^3 \div a^2 = \dfrac{a \cdot a \cdot a}{a \cdot a}$ $= a^1$ or a^{3-2}	**Quotient Law** $a^m \div a^n = \dfrac{a^m}{a^n} = a^{m-n}$ $(a \neq 0)$
$(8 \cdot 7)^3 = (8 \cdot 7)(8 \cdot 7)(8 \cdot 7)$ $= (8 \cdot 8 \cdot 8)(7 \cdot 7 \cdot 7)$ $= 8^3 \cdot 7^3$	$(a \cdot b)^3 = (a \cdot b)(a \cdot b)(a \cdot b)$ $= (a \cdot a \cdot a)(b \cdot b \cdot b)$ $= a^3 b^3$	**Power of a Product Law** $(ab)^m = a^m b^m$
$\left(\dfrac{8}{7}\right)^3 = \left(\dfrac{8}{7}\right)\left(\dfrac{8}{7}\right)\left(\dfrac{8}{7}\right)$ $= \dfrac{8^3}{7^3}$	$\left(\dfrac{a}{b}\right)^3 = \left(\dfrac{a}{b}\right)\left(\dfrac{a}{b}\right)\left(\dfrac{a}{b}\right)$ $= \dfrac{a^3}{b^3}$	**Power of a Quotient Law** $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$ $(b \neq 0)$
$(8^3)^2 = (8^3)(8^3)$ $= (8 \cdot 8 \cdot 8)(8 \cdot 8 \cdot 8)$ $= 8^6$ or $8^{3\times 2}$	$(a^3)^2 = (a^3)(a^3)$ $= (a \cdot a \cdot a)(a \cdot a \cdot a)$ $= a^6$ or $a^{2\times 3}$	**Power of a Power Law** $(a^m)^n = a^{mn}$

[redacted]



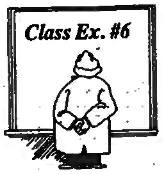
Use the exponent laws to simplify and then evaluate.

a) $3^4 \cdot 3^2$ $= 3^{4+2}$
$3^6 = 3\cdot3\cdot3\cdot3\cdot3\cdot3$
$= 729$

b) $\frac{(-2)^5}{(-2)^3}$ $(-2)^{5-3}$
$(-2)^2 = 4$

c) $(5^2)^3$ $= 5^{2\cdot3}$
5^6
$= 15625$

x^4
$3x^46$



Use the exponent laws to simplify.

a) $(a)^4(a)^3$ $= a^7$

b) x^6x^1 $= x^7$

c) $b^7 \times b^0 \times b^4$ $= b^{11}$

d) $\frac{x^8}{x^4}$ $= x^4$

e) $\frac{y^{10}}{y^2}$ $= y^8$

f) $(a^4)^3$ $= a^{12}$

g) $(y^5)^5$ $= y^{25}$

h) $\left(\frac{x}{y}\right)^9$ $\frac{x^9}{y^9}$

i) $\left(\frac{c}{4}\right)^2$ $\frac{c^2}{4^2} = \frac{c^2}{16}$

j) $(st)^6$ s^6t^6

k) $(2a)^5$
$= 2^5a^5$
$= 32a^5$

l) $(-3pq)^4$ $= (-3)^4p^4q^4$
$= 81p^4q^4$

Complete Assignment Questions #8 - #22

Assignment (1-5)acegi, 7, 8, (12-15)acegi

1. State the base and the exponent in each of the following powers.

a) 8^3 b) k^{15} c) 2^x d) $(-x)^4$ e) $\left(\frac{3}{4}\right)^6$

2. State the coefficient in each of the following.

a) $5x^7$ b) $-6z^2$ c) a^3 d) $\frac{y^3}{4}$ e) $\frac{5y^9}{8}$

3. Write each of the following as a repeated multiplication.

a) c^4 b) $5x^3$ c) $(ab)^2$ d) $(-5)^3$ e) s^2t

f) $2\left(\frac{5}{4}\right)^3$ g) $(4a)^3$ h) $3cd^2$ i) $3(cd)^2$ j) $(3cd)^2$

[redacted]

4. Evaluate.

a) 3^8 b) -5^2 c) $(-5)^2$

d) $(-5)^3$ e) -5^3 f) $\left(\frac{3}{5}\right)^3$

5. Evaluate without using a calculator.

a) -10^2 b) $(-10)^2$ c) -10^3 d) $(-10)^3$

6. Explain why -8^0 and $(-8)^0$ have different values.

7. Evaluate without using a calculator.

a) 32^0 b) -1^0 c) $\left(-\frac{1}{2}\right)^0$ d) $\frac{1}{2}(4)^0$ e) $\frac{1}{2}(4^2)^0$

8. Write in a simpler form and evaluate.

a) $9^3 \cdot 9^6$ b) $(7^2)^3$ c) $\frac{8^{15}}{8^{13}}$ d) $\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)^3$

e) $\frac{1.5^7}{1.5^5}$ f) $(-3^3)^2$ g) $(-5)^6 \times (-5)^2$ h) $4^3 \cdot 4^4 \cdot 4^2$

9. Explain using factors why $(x^2)(x^3) \neq (x^2)^3$.

10. Use the Product Law to simplify.

a) $a^4 \times a^2$ b) $m^6 \cdot m^3$ c) $(s^5)(s^5)$ d) x^6x^5 e) $y^{10} \times y^2$

11. Use the Quotient Law to simplify.

a) $\frac{t^8}{t^2}$ b) $x^6 \div x^4$ c) $\frac{p^{10}}{p^9}$ d) $d^{18} \div d^9$ e) $p^8 \div p$

[redacted]

12. Use the Power of a Product Law to simplify.

a) $(xy)^5$ b) $(mn)^4$ c) $(3x)^3$ d) $(10z)^3$ e) $\left(\frac{1}{2}c\right)^2$

f) $(2b)^4$ g) $(-x)^3$ h) $(-3y)^4$ i) $(-4pq)^2$ j) $(-4pq)^3$

13. Use the Power of a Quotient Law to simplify.

a) $\left(\frac{x}{y}\right)^2$ b) $\left(\frac{a}{b}\right)^6$ c) $\left(\frac{5}{c}\right)^4$ d) $\left(\frac{b}{5}\right)^3$ e) $\left(\frac{z}{y}\right)^{10}$

14. Use the Power of a Power law to simplify.

a) $(p^2)^2$ b) $(h^4)^5$ c) $(b^4)^3$ d) $(s^9)^{10}$ e) $(z^7)^3$

15. State the value of x in each of the following.

a) $(a^3)(a^x) = a^9$ b) $b^x \cdot b^4 = b^8$ c) $c^x \div c^4 = c^{12}$

d) $\frac{d^{10}}{d^x} = d^2$ e) $(e^x)^5 = e^{15}$ f) $(f^7)^x = f^7$

16. Use the exponent laws to simplify.

a) $\frac{x^{12}}{x^3}$ b) $(xy)^7$ c) $(t^3)^3$ d) t^3t^3

e) $y^4 \times y^8$ f) $\left(\frac{a}{b}\right)^{11}$ g) $\left(\frac{d}{2}\right)^3$ h) $(2st)^6$

17. Simplify.

a) $g^{12}g^3$ b) $\frac{a^7}{a^5}$ c) $(3bc)^4$ d) $\left(\frac{5}{y}\right)^2$

e) $(-a)^4$ f) $\left(-\frac{1}{3}pq\right)^4$ g) $(a^3)(a^4)(a^5)$ h) $\frac{x^6}{x^0}$

[redacted]

18. Simplify.

a) $\dfrac{y^7}{y^7}$ b) $(-ab)^5$ c) $(m^6)(m^6)$ d) $(r^0)^3$

e) $c^3c^4c^5c^6$ f) $(-ab)^6$ g) $\dfrac{1}{a^3a^5}$ h) $2(xy)^3$

19. After marking an exponents quiz a teacher recorded the most common errors made by students. In each case, identify the error made by the students and, where possible, provide the correct simplification.

a) $2^3 \times 2^4 = 2^{12}$ b) $(4^3)^2 = 4^9$

c) $3^4 \times 3^5 = 9^9$ d) $3^2 \times 2^3 = 6^5$

e) $(-5a^2b)^3 = -5a^6b^3$ f) $\left(\frac{1}{2}pq\right)\left(\frac{1}{2}pq\right) = p^2q^2$

Matching **20.** Match each item in column 1 on the left with the equivalent item in column 2 on the right. Each item in column 2 may be used once, more than once, or not at all.

Column 1		Column 2	
i)	$(-a^2)^3$	**A.**	a^4
ii)	$(-a^3)^2$	**B.**	a^5
iii)	$a^3 \times a^2$	**C.**	a^6
iv)	$a^8 \div a^2$	**D.**	a^{24}
v)	$a^{30} \div a^6$	**E.**	$-a^5$
vi)	$-(a^2)^3$	**F.**	$-a^6$
vii)	$-(a^3)^2$		

Use the following information to answer the next question.

$$(2^3)^p = 2^{12} \qquad \frac{4^{10}}{4^q} = 4^2$$

$$2^r \cdot 2^r = 2^{16} \qquad (3^s)^2 = 1$$

Numerical Response **21.** Write the value of p in the first box.
Write the value of q in the second box.
Write the value of r in the third box.
Write the value of s in the fourth box.
(Record your answer in the numerical response box from left to right)

22. If $(a^n)\,(a^n)\,(a^n) = a^{27}$, where n is a whole number, then the value of n is ______ .

(Record your answer in the numerical response box from left to right)

[redacted]

Answer Key

1. a) base 8, exponent 3 b) base k, exponent 15 c) base 2, exponent x
   d) base $-x$, exponent 4 e) base $\frac{3}{4}$, exponent 6

2. a) 5 b) -6 c) 1 d) $\frac{1}{4}$ e) $\frac{5}{8}$

3. a) $c\times c\times c\times c$ b) $5\times x\times x\times x\times x$ c) $a\times a\times b\times b$ d) $(-5)\times(-5)\times(-5)$
   e) $s\times s\times t$ f) $2\times\frac{5}{4}\times\frac{5}{4}\times\frac{5}{4}$ g) $4\times4\times4\times a\times a\times a$
   h) $3\times c\times d\times d$ i) $3\times c\times c\times d\times d$ j) $3\times3\times c\times c\times d\times d$

4. a) 6561 b) -25 c) 25 d) -125 e) -125 f) $\frac{27}{125}$

5. a) -100 b) 100 c) -1000 d) -1000

6. $-8^0=-1$ since the exponent applies only to the base 8.
   $(-8)^0=1$ since the exponent applies to the base -8.

7. a) 1 b) -1 c) 1 d) $\frac{1}{2}$ e) $\frac{1}{2}$

8. a) $9^9=387\,420\,489$ b) $7^6=117\,649$ c) $8^2=64$ d) $\left(\frac{2}{3}\right)^4=\frac{16}{81}$
   e) $1.5^2=2.25$ f) $3^6=729$ g) $(-5)^8=390\,625$ h) $4^9=262\,144$

9. $(x^2)(x^3)=(x\times x)(x\times x\times x)=(x\times x\times x\times x\times x)=x^5$
   $(x^2)^3=x^2\times x^2\times x^2=x\times x\times x\times x\times x\times x=x^6$
   Therefore $(x^2)(x^3)\neq(x^2)^3$.

10. a) a^6 b) m^9 c) s^{10} d) x^{11} e) y^{12}

11. a) t^6 b) x^2 c) p d) d^9 e) p^7

12. a) x^5y^5 b) m^4n^4 c) $27x^3$ d) $1000z^3$ e) $\frac{1}{4}c^2$
    f) $16b^4$ g) $-x^3$ h) $81y^4$ i) $16p^2q^2$ j) $-64p^3q^3$

13. a) $\frac{x^2}{y^2}$ b) $\frac{a^6}{b^6}$ c) $\frac{625}{c^4}$ d) $\frac{b^3}{125}$ e) $\frac{z^{10}}{y^{10}}$

14. a) p^4 b) h^{20} c) b^{12} d) s^{90} e) z^{21}

15. a) 6 b) 4 c) 16 d) 8 e) 3 f) 1

16. a) x^9 b) x^7y^7 c) t^9 d) t^6 e) y^{12} f) $\frac{a^{11}}{b^{11}}$ g) $\frac{d^3}{8}$ h) $64s^6t^6$

17. a) g^{15} b) a^2 c) $81b^4c^4$ d) $\frac{25}{y^2}$ e) a^4 f) $\frac{1}{81}p^4q^4$ g) a^{12} h) x^6

18. a) $y^0=1$ b) $-a^5b^5$ c) m^{12} d) $r^0=1$ e) c^{18} f) a^6b^6 g) $\frac{1}{a^8}$ h) $2x^3y^3$

[redacted]

19. a) The student multiplied 3 and 4, instead of adding 3 and 4. The correct simplification is $2^{3+4} = 2^7$.
b) The student squared 3 instead of multiplying 3 and 2. The correct simplification r is $4^{3\times 2} = 4^6$.
c) The student multiplied the bases together. The correct simplification is 3^9.
d) The student multiplied the bases together. The exponent laws are only valid when the bases are the same. No simplification.
e) The student did not cube the 5. The correct simplification is $-125a^6b^3$.
f) The student added $\frac{1}{2}$ and $\frac{1}{2}$ instead of multiplying $\frac{1}{2}$ and $\frac{1}{2}$. The correct simplification is $\frac{1}{4}p^2q^2$.

20. i) F ii) C iii) B iv) C v) D vi) F vii) F

21.

4	8	8	0

22.

9			