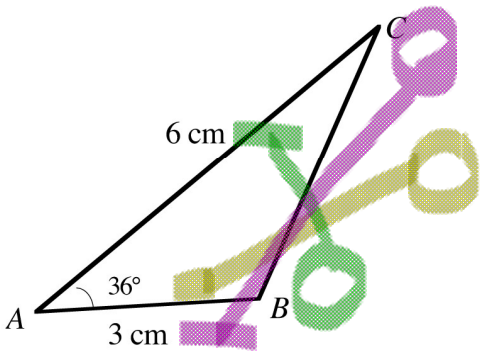
# Lesson 3: The Cosine Law

# Trigonometry - Sine and Cosine Laws Lesson #3:
## The Cosine Law

### *Introduction*

Consider triangle $ABC$ in which $\angle A = 36°$, $AB = 3$ cm and $AC = 6$ cm. What happens when you try to apply the sine law to determine the length of $BC$?

$$\frac{\sin 36}{a} = \frac{\sin B}{6} = \frac{\sin C}{3}$$




In the example above, where we are given the length of two sides and the contained angle, the sine law is **not** applicable.


We can find the length of $BC$ by making a right triangle $BCD$ in the diagram below and using SOHCAHTOA to determine the lengths of $CD$ and $AD$.

Determine the lengths of $CD$ and $AD$ to the nearest hundredth of a cm, and show how these lengths can be used to determine the length of $BC$ to the nearest tenth of a cm.

$\sin 36 = \frac{d}{6}$

$d = 6\sin 36$

$d = 3.5267$

$\cos 36 = \frac{c}{6}$

$c = 6\cos 36$

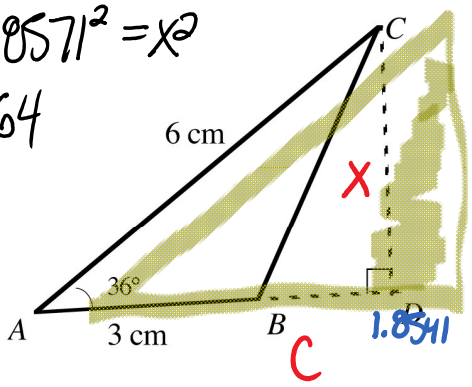
$c = 4.8541$

$4.8541 - 3 = 1.8541$

$3.5267^2 + 1.8571^2 = x^2$

$x^2 = 15.8864$

$x = 4.0$



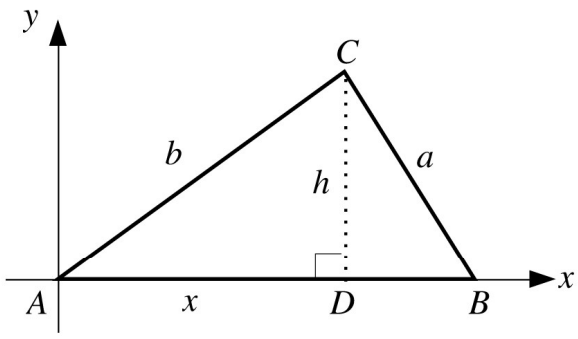


The method above is time consuming.
The length of $BC$ can be determined in one step by using the **cosine law**.

## *The Cosine Law*

In every triangle $ABC$, $a^2 = b^2 + c^2 - 2bc\cos A$.

## *Proof of the Cosine Law*




- The diagram shows triangle $ABC$ placed with base $AB$ on the $x$–axis and $A$ at the origin.
- The line $CD$ is drawn perpendicular to $AB$ and is $h$ units in length.
- $AD = x$ units so $DB = c - x$ units.

Complete the following work to show that $a^2 = b^2 + c^2 - 2bc\cos A$.

In $\Delta ADC$, $\cos A = \dfrac{AD}{AC} = \dfrac{x}{b}$

so $x =$

In $\Delta BDC$, $BC^2 = CD^2 + DB^2$

$a^2 = h^2 + (c - x)^2$

$a^2 = h^2 + c^2 - 2cx + x^2$

$a^2 = (h^2 + x^2) + c^2 - 2cx$

$a^2 = \quad + c^2 - 2c(\quad)$

$a^2 = b^2 + c^2 - 2bc\cos A.$

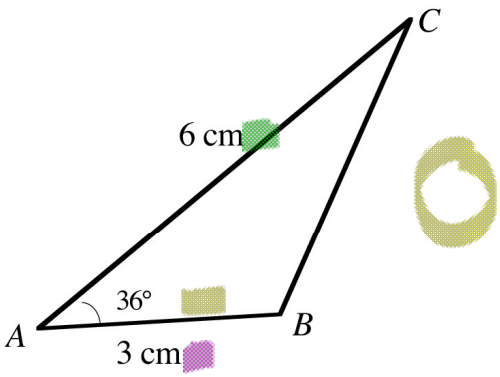
By placing $AC$ and then $BC$ on the $x$–axis, similar equations can be derived.

$b^2 = c^2 + a^2 - 2ca\cos B$ $\qquad$ $c^2 = a^2 + b^2 - 2ab\cos C$


This version of the cosine law can be used in any triangle if we are given the lengths of two sides and the contained angle (SAS).

**Class Ex. #2**

Consider the $\Delta ABC$ from Class Ex. #1 in which $\angle A = 36°$, $AB = 3$ cm, and $AC = 6$ cm. Determine the length of $BC$, to the nearest tenth of a cm, using the cosine law.




$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 6^2 + 3^2 - 2(6)(3)\cos 36$

$a^2 = 15.8753$

$a = 4.0$

what I'm solving for

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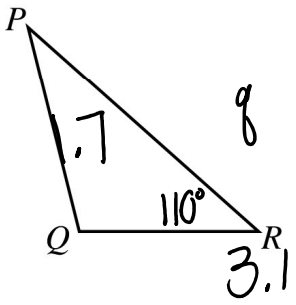
**Class Ex. #3**

Consider triangle $PQR$ shown.

**a)** Complete the cosine law for calculating side $q$. $q^2 =$ $r^2 + p^2 - 2rp\cos Q$

**b)** Calculate, to the nearest tenth of a cm, the length of the third side of $\Delta PQR$ if $QP = 1.7$ cm, $QR = 3.1$ cm, and $\angle PQR = 110°$.




$q^2 = 1.7^2 + 3.1^2 - 2(1.7)(3.1)\cos 110$

$q^2 = 16.1$

$q = 4.0$

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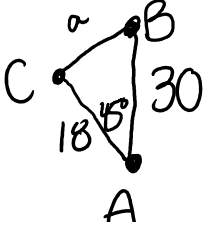
**Class Ex. #4**

Bellevue is 30 km north of Ayr and Churchville is 18 km northwest of Ayr. Calculate the distance between Bellevue and Churchville to the nearest km.




$a^2 = b^2 + c^2 - 2bc\cos A$

$= 18^2 + 30^2 - 2(18)(30)\cos 45$

$a^2 = 460.32$

$a = 21$ km

18 km

A

$a^2 = 460.32$

$a = 21$ km

**Complete Assignment Questions #1 - #4**





## *Alternative Form of the Cosine Law*

The equation $a^2 = b^2 + c^2 - 2bc\cos A$

can be rearranged to the form $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$.


This form of the cosine law can be used to determine any angle in a triangle when we are given the length of all three sides (SSS).

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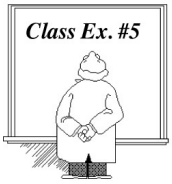


Complete the following for triangle $ABC$.

**a)** $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$ **b)** $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

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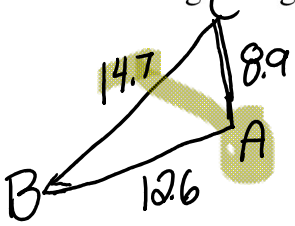
Determine the largest angle in $\Delta ABC$ if $a = 14.7$, $b = 8.9$, and $c = 12.6$.




$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\cos A = \dfrac{8.9^2 + 12.6^2 - 14.7^2}{2(8.9)(12.6)}$

$= (8.9^2 + 12.6^2 - 14.7^2) \div (2 \times 8.9 \times 12.6)$

$\cos A = 0.09756\ldots$

$A = \cos^{-1}(0.09756) = 84°$

#1-3, 5, 6

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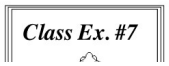
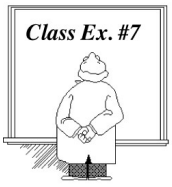
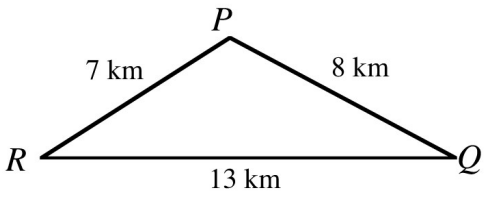



Two ships set sail from port, $P$, heading in different




Two ships set sail from port, $P$, heading in different directions. The first ship sails 7 km to $R$ and the second ship sails 8 km to $Q$. If the distance between $R$ and $Q$ is 13 km, determine the angle between the directions of the two ships.




**Complete Assignment Questions #5 - #11 and the Group Investigation.**





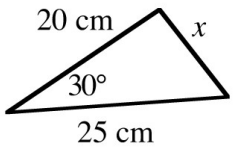
# Assignment

**1.** Complete the cosine law for triangle $STV$.

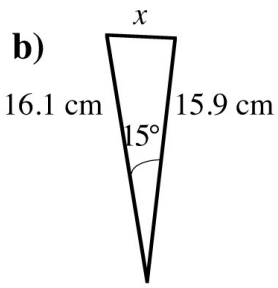
**a)** $s^2 =$ **b)** $v^2 =$

**2.** In each case, determine the length of $x$ to the nearest tenth of a cm.

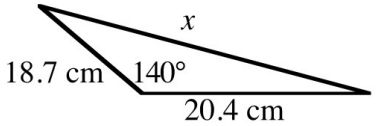
**a)**




**b)**

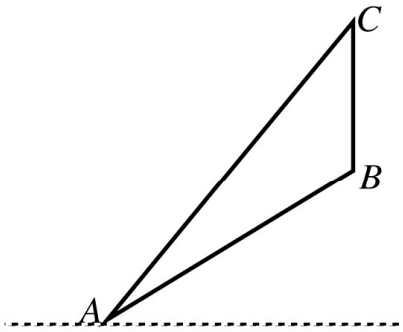



**c)**



**3.** In $\Delta ABC$, angle $A = 49°$, $b = 24$, and $c = 37$ .
Make a sketch of the triangle and calculate
$a$ to the nearest whole number.

**4.** In the diagram, $AB$ represents part of a road constructed on the incline of a hill. $BC$ represents a telephone pole 7.5 m tall at the side of the road. A guide wire attached to the top of the pole is joined to the ground at $A$. If $AB = 11.4$ m and $\angle ABC = 135°$, determine the length of the guide wire to the nearest 0.1 m.





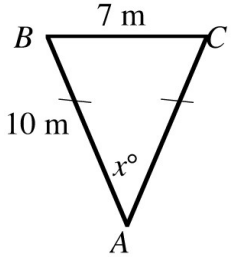



**5.** Complete the formula for the cosine law in triangle $DEF$.

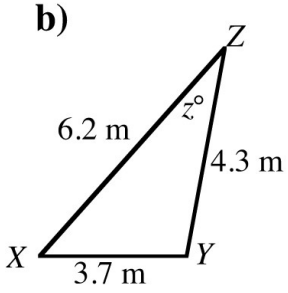
**a)** $\cos E =$ **b)** $\cos F =$

**6.** In each case, find the measure of the indicated angle to the nearest degree.

**a)**

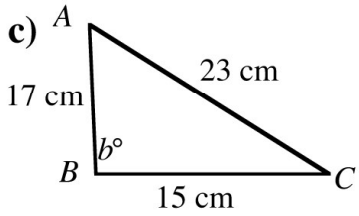



**b)**

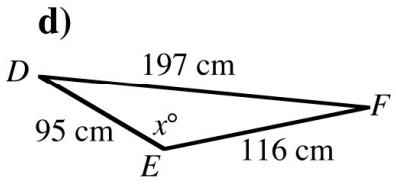


3.7 m

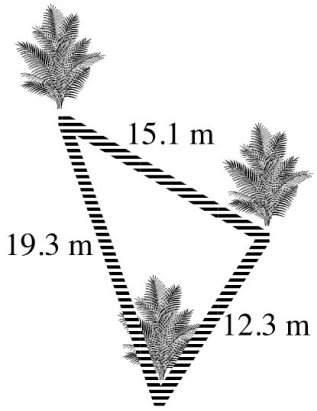
**c)**




**d)**



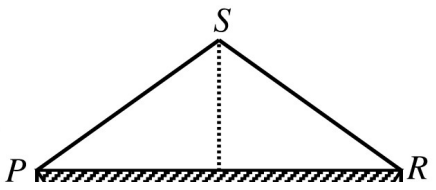

**7.** Anwar and Ingrid have three trees in their garden. The trees form a triangle as shown in the diagram. Determine, to the nearest degree, the smallest angle between the trees.





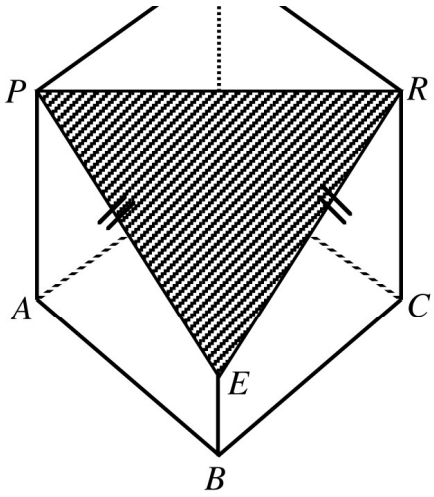



**8.** The solid in the diagram was formed by removing a corner from a cube of 24 cm. The length of *EB* is 6 cm.




**a)** Calculate, to the nearest tenth, the lengths of *PE* and *PR*.

**a)** Calculate, to the nearest tenth, the lengths of $PE$ and $PR$.

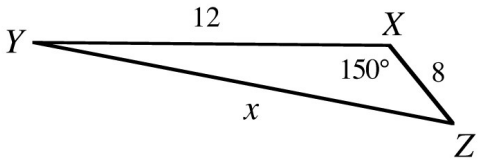



**b)** Calculate the measure of angle $PER$ to the nearest degree.

Multiple Choice

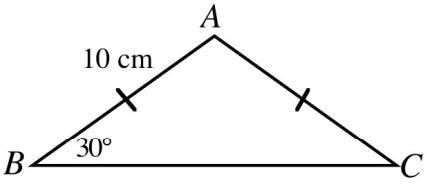
**9.** In the diagram, the value of $x^2$ is

**A.** 112 **B.** 304
**C.** $208 - 96\sqrt{3}$ **D.** $208 + 96\sqrt{3}$



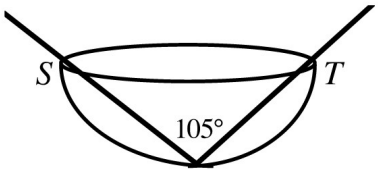

**10.** The length of $BC$ in cm is

**A.** $5\sqrt{3}$
**B.** 10
**C.** $10\sqrt{3}$
**D.** 20



Numerical Response

**11.** The diagram shows a glass bowl with two chop-sticks resting on the rim at points $S$ and $T$. The lengths of the parts of the chop-sticks inside the bowl are 9 cm and 11.5 cm, respectively.




The length of $ST$, to the nearest tenth of a cm, is _____ .

(Record your answer in the numerical response box from left to right.)

| | | | |
|---|---|---|---|
| | | | |

Group Investigation

The sines of the angles of a triangle are in the ratio 2:3:4. Determine the ratios of the cosines of the angles.

## *Answer Key*

**1.** **a)** $s^2 = t^2 + v^2 - 2tv \cos S$ **b)** $v^2 = s^2 + t^2 - 2st \cos V$

**2.** **a)** 12.6 cm **b)** 4.2 cm **c)** 36.7 cm **3.** 28 **4.** 17.5 m

**5.** **a)** $\cos E = \dfrac{d^2 + f^2 - e^2}{2df}$ **b)** $\cos F = \dfrac{d^2 + e^2 - f^2}{2de}$ **6.** **a)** 41° **b)** 36° **c)** 92° **d)** 138°

**7.** 40° **8.** **a)** $PE = 30.0$ cm, $PR = 33.9$ cm **b)** 69°

**5. a)** $\cos E = \frac{\quad}{2df}$ **b)** $\cos F = \frac{\quad}{2de}$ **6. a)** 41° **b)** 36° **c)** 92° **d)** 138°

**7.** 40° **8. a)** $PE = 30.0$ cm, $PR = 33.9$ cm **b)** 69°

**9.** D **10.** C **11.**

| | | | |
|---|---|---|---|
| 1 | 6 | . | 3 |

**Group Investigation** 14:11:–4