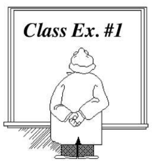
Trigonometry - Sine and Cosine Laws Lesson #2: The Sine Law

Trigonometry in Acute Angled and Obtuse Angled Triangles

In the last lesson, we reviewed trigonometry in right triangles using SOHCAHTOA.

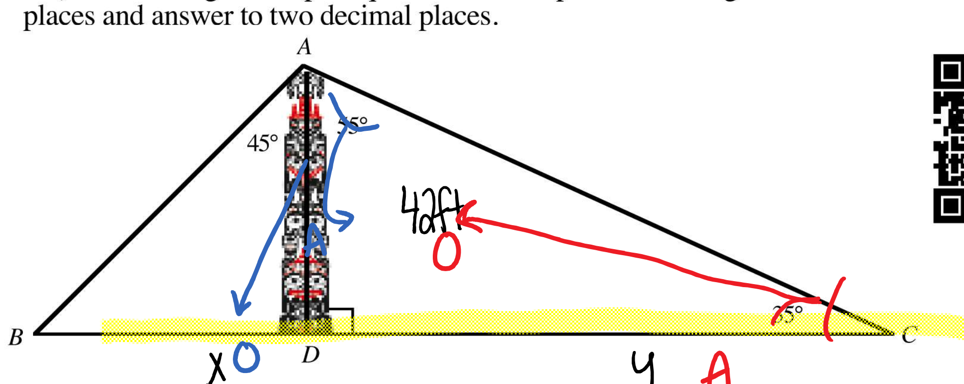
In the next three lessons, we focus on solving triangles which are not right angled and in which SOHCAHTOA is not valid.

In the next section of work we will determine the side of an acute angled triangle by
i) splitting it in two right triangles and using SOHCAHTOA as in Class Ex. #1
ii) using the Sine Law as in Class Ex. #2



To honor the 20th anniversary of the Gwaii Haanas Agreement, more than 400 people in 2013 participated in the monumental raising of a 42 foot Gwaii Haanas Legacy Pole. The poles design was inspired by the connections between the Haida Nation and all those who care for Gwaii Haana from mountain-top to seafloor. The pole tells the story of a ground breaking cooperative agreement between the Haida Nation and the Government of Canada to protect Gwaii Haanas.

Triangle *ABC* has three acute angles that represent the possible angles used when the pole is standing at 90 degrees to the ground. Master Carver, Jaalen Edenshaw and two assistants, will need to determine both the ground clearance needed and the rope required to raise the pole successfully. Use SOHCAHTOA to determine both the ground clearance (length of *BC*) and the length of rope required when the pole is standing vertical. Work to three decimal places and answer to two decimal places.



$\tan 35 = \frac{42}{y}$

$y = \frac{42}{\tan 35}$

$= 60\text{ft}$

59.982

$\tan 45 = \frac{x}{42}$

$x = 42\tan 45$

$= 42 \text{ ft}$

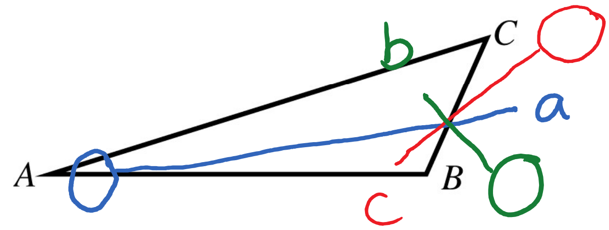
$BC = 60 + 42 = \boxed{102\text{ft}}$

A New Notation

Often, in trigonometry, it is convenient to use the following notation.

In triangle ABC, represent

the length of the side opposite angle A by a,
the length of the side opposite angle B by b,
and the length of the side opposite angle C by c.



upper case → angle

lower case → side

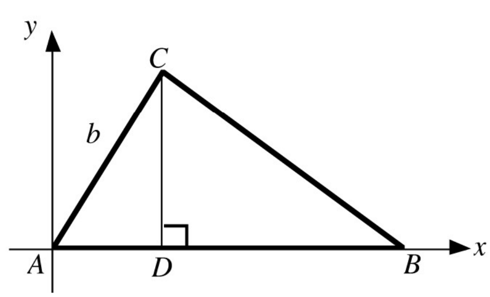
The Sine Law

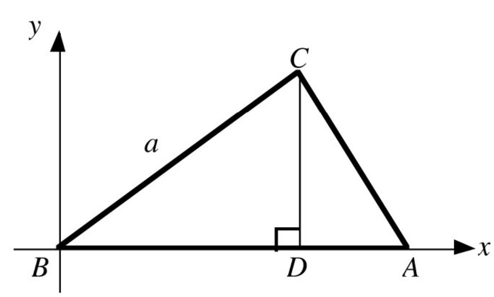
In every triangle ABC, $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ or $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$.

ratios

Proof of the Sine Law

The diagrams show the same triangle ABC placed with base AB on the x–axis.
In diagram i) the origin is at A, and in diagram ii) the origin is at B.
The line CD is drawn perpendicular to AB.





1. Complete the following work to show that $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$.

- In i) $\sin A = \dfrac{CD}{AC} = \dfrac{CD}{b}$ • In ii) $\sin B =$ $\quad =$

  $CD =$ $\qquad$ $CD =$

- It follows that $b \sin A \quad =$

- Dividing both sides by $\sin A \sin B$ gives the result _______________ .

2. Repeating the work above with AC placed on the x–axis would give

2. Repeating the work above with AC placed on the x–axis would give the result $\dfrac{a}{\sin A} = \dfrac{c}{\sin C}$.

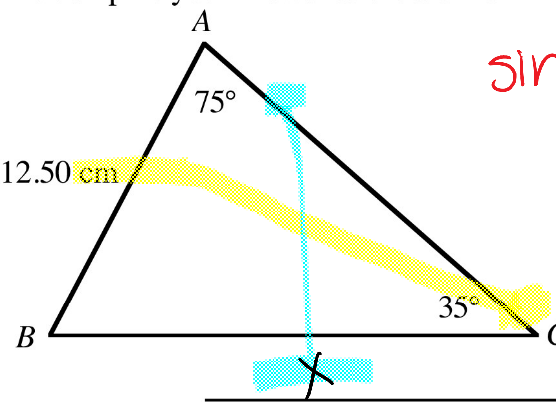
Hence $\quad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ or $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$.

Note

To use the sine law, we need to know **three** pieces of information.
This information must include both numerator and denominator of one of the three fractions, i.e. we need to know an angle and the measure of its opposite side.

Class Ex. #2

Triangle ~~ABC from Class Ex. #1 is shown.~~ Use the sine law to calculate the length of BC, and compare your answer to the SOHCAHTOA method.



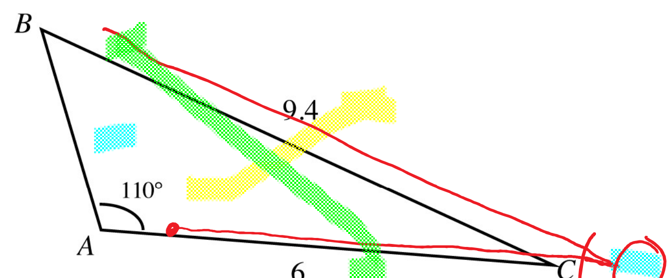
$$\sin 75 \cdot \frac{12.5}{\sin 35} = \frac{x}{\cancel{\sin 75}} \quad \cancel{\sin 75}$$

$$x = \frac{\sin 75 \cdot 12.5}{\sin 35}$$

$$\boxed{x = 21.05}$$

Class Ex. #3

Use the sine law in the triangle shown to determine the measure of $\angle ACB$ to the nearest degree.



put what you're solving for on top

$$\frac{\sin 110}{9.4} = \frac{\sin B}{6}$$

$$\sin B = \frac{6 \sin 110}{9.4}$$

$$B = \sin^{-1}\left(\frac{6\sin 110}{9.4}\right) \quad \boxed{B = 37°}$$

find angle C

$180 - 110 - 37 = 33°$

$\boxed{\angle ACB = 33°}$

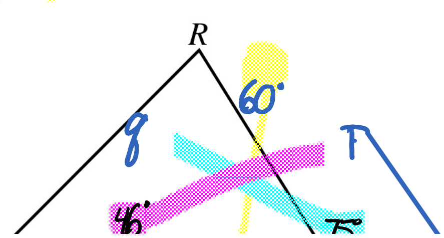
Class Ex. #4

A surveyor measures a base line PQ to be 440 m long. He takes measurements of a landmark R from P and Q, and finds that $\angle QPR = 46°$ and $\angle PQR = 75°$.

a) Calculate the perimeter of ΔPQR to the nearest metre.

1) Need R for ratio $\quad 180 - 46 - 75 = 60$

2) Solve for q $\quad \dfrac{440}{\sin 60} = \dfrac{q}{\sin 75}$



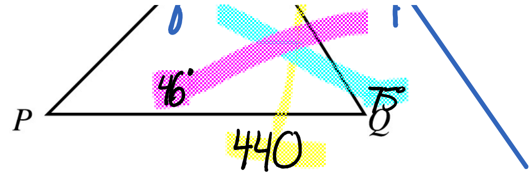
2) Solve for q $\frac{440}{\sin 60} = \frac{q}{\sin 75}$

$q = \frac{\sin 75 \cdot 440}{\sin 60}$

$q = 491$

3) Solve for p $\frac{440}{\sin 60} = \frac{p}{\sin 46}$

$p = \frac{\sin 46 \cdot 440}{\sin 60}$ $p = 365$

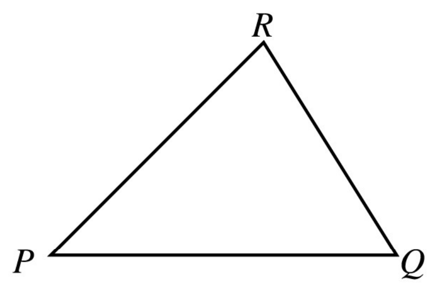


4) Perimeter: add all the sides together

$440 + 491 + 365$

$\boxed{= 1296\text{m}}$

b) Calculate the area of ΔPQR to the nearest hundred square metres.



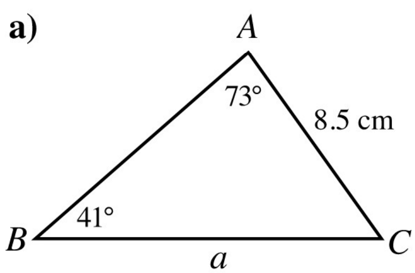
Complete Assignment Questions #1 - #9

#1-3

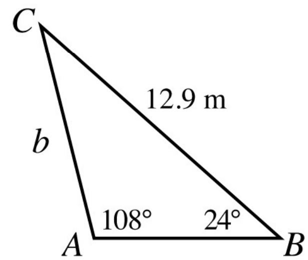
Assignment

1. Use the Sine Law to determine the length of the indicated side to the nearest tenth.

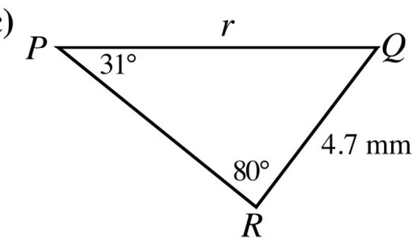
a)



b)

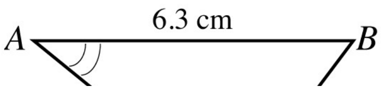


c)



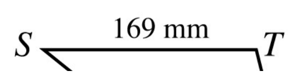
2. Use the Sine Law to determine the measure of the indicated angle to the nearest degree.

a)

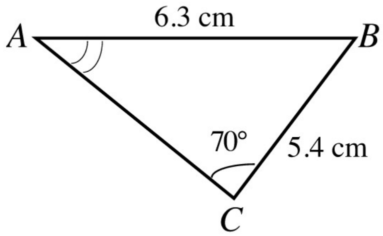


b)

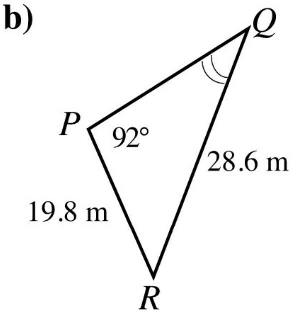
c)



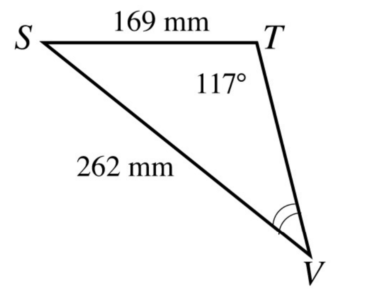
a)



b)

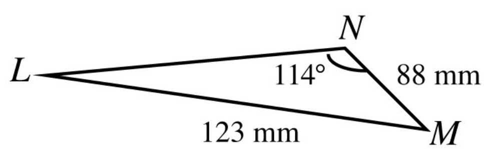


c)



3. In ΔABC, angle $A = 49°$, angle $B = 57°$, and $a = 8$. Calculate b to the nearest tenth.

4. In ΔLMN, angle $LNM = 114°$, $LM = 123$ mm, and $MN = 88$ mm.
Calculate $\angle LMN$, to the nearest degree.

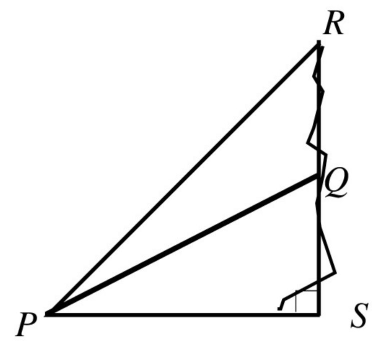


5. P and Q are two bases for a mountain climb.
PQ is 600 m and QR is a vertical stretch of a rock face

5. *P* and *Q* are two bases for a mountain climb.
PQ is 600 m and *QR* is a vertical stretch of a rock face.

The angle of elevation of *Q* from *P*, (i.e. angle *QPS*) is 31°.

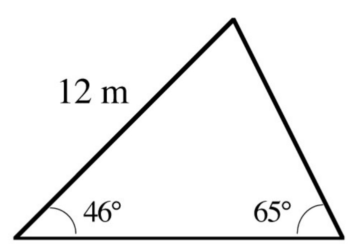
The angle of elevation of *R* from *P* is 41°.



a) Mark these measurements on the diagram and state the measures of angle *RPQ* and angle *PRQ*.

b) Use the sine law in ΔPQR to calculate the height of the vertical climb, *QR*, to the nearest metre.

6. Consider the triangle shown.



a) Use the sine law to calculate the lengths of the other two sides of the triangle to the nearest hundredth of a metre.

b) Three students are trying to determine the area of the triangle in the diagram. Each student is given a different formula with which to determine the area. The area of the triangle is 53.3 m^2.

Show how each student arrived at this answer.

Student #1: Draw a vertical line to represent the height of the triangle and

Show how each student arrived at this answer.

Student #1: Draw a vertical line to represent the height of the triangle and use the formula $A = \frac{1}{2}bh$, where b is the length of the base and h is the vertical height.

Student #2: Calculate the perimeter of the triangle and use Heron's formula $A = \sqrt{s(s-a)(s-b)(s-c)}$, where a, b, and c are the lengths of the three sides and s is the semi-perimeter of the triangle.

Student #3: Use the formula $A = \frac{1}{2}ab\sin C$, where a and b are the lengths of two sides, and angle C is the contained angle between the sides a and b.

Multiple Choice **7.** In triangle PQR, angle $P = 20°$, angle $R = 150°$, and $QR = 6$ m. The length of PQ is

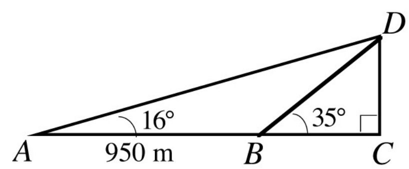
A. 4.1 m
B. 8.8 m
C. 15.2 m
D. 17.3 m

8. In ΔABC, $\angle A = 30°$, $BC = 10$ units, and $AC = 15$ units. If $\angle B$ is acute-angled, then $\angle C$ is

A. 19.4°
B. 48.6°
C. 101.4°
D. 130.6°

Numerical Response

9. From a point A, level with the foot of a hill, the angle of elevation of the top of the hill is 16°. From a point B, 950 metres nearer the foot of the hill, the angle of elevation of the top is 35°.
The height of the hill, DC, to the nearest metre, is ____ .



(Record your answer in the numerical response box from left to right.)

Answer Key

1. a) 12.4 cm **b)** 5.5 m **c)** 9.0 mm
2. a) 54° **b)** 44° **c)** 35°
3. 8.9
4. 25°
5. a) 10°, 49° **b)** 138 m
6. a) 9.52 m and 12.36 m
7. B
8. C
9.

4	6	1	