

## Trigonometry - Angles and Ratios Lesson #4: Solving Simple Trigonometric Equations

### Solving Trigonometric Equations with the Domain $0^\circ \leq \theta \leq 360^\circ$

We can use the concepts of reference angles and signs of the trigonometric ratio to solve equations of the form  $\sin \theta = a$ ,  $\cos \theta = a$ , or  $\tan \theta = a$ , where  $0^\circ \leq \theta \leq 360^\circ$ .

Use the following procedure to solve an equation such as  $\sin \theta = 0.5$ , where  $0^\circ \leq \theta \leq 360^\circ$ .

**Step 1:** Determine the quadrant(s) the angle will be in by looking at the sign of the ratio.

**Step 2:** Determine the reference angle (always between  $0^\circ$  and  $90^\circ$ ) and draw a rough sketch in the appropriate quadrant(s). To determine the reference angle, use

2nd sin or 2nd cos or 2nd tan

of the **absolute value** of the given quantity.  
^ **positive**

}  $\sin^{-1}( )$

**Step 3:** Determine the rotation angle(s) using the reference angle and the quadrant(s).



- Always check the given domain to determine which quadrants are valid in the calculation. Sometimes the domain is restricted to, for example,  $0^\circ \leq \theta \leq 180^\circ$ , or  $90^\circ \leq \theta \leq 180^\circ$ .



Class Ex. #1

Use the procedure above to solve  $\sin \theta = 0.5$ , where  $0^\circ \leq \theta \leq 360^\circ$

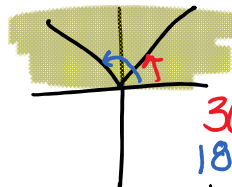
Sine is positive in Quad 1/2

$$\sin \theta = 0.5$$

$$\theta = \sin^{-1}(0.5)$$

$$\theta = 30^\circ$$

↑  
reference angle



$$180^\circ - 30^\circ = 150^\circ$$

$$\theta = 30^\circ, 150^\circ$$

S	A
T	C



Class Ex. #2

Find the measure of  $x$ , to the nearest degree, where  $0^\circ \leq x \leq 360^\circ$ .

a)  $\sin x = -0.8090$

b)  $\cos x = -0.8090$

c)  $\tan x = -2.4586$

i) Quad 3/4

ii) Reference angle  
 $x = \sin^{-1}(0.8090)$

$$x = 54^\circ$$

$$180 + 54 = 234^\circ$$

i) Quad 2/3

ii)  $x = \cos^{-1}(0.8090)$   
 $x = 36^\circ$

$$180 - 36 = 144^\circ$$

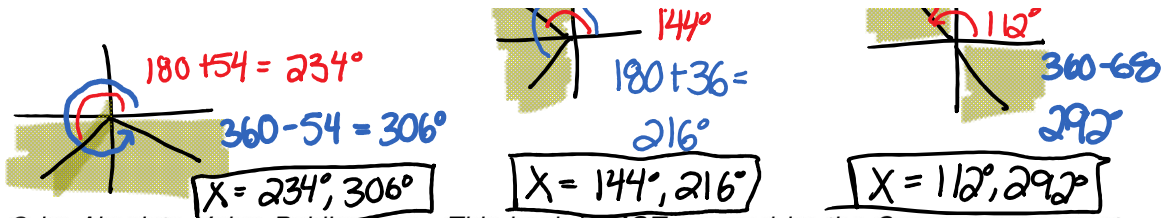
$$180 + 36 = 216^\circ$$

i) Quad 2/4

ii)  $x = \tan^{-1}(2.4586)$   
 $x = 68^\circ$

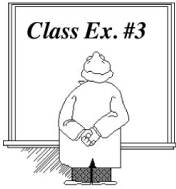
$$180 - 68 = 112^\circ$$

$$360 - 68 = 292^\circ$$



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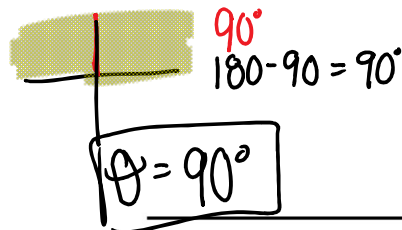
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Solve the following equations if  $0^\circ \leq \theta \leq 360^\circ$ .

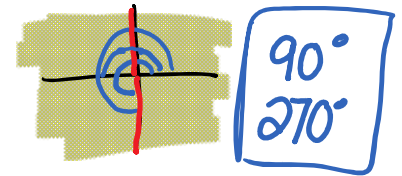
a)  $\sin \theta = 1$

Quad 1/2  
 $\theta = \sin^{-1}(1)$   
 $= 90^\circ$



b)  $\cos \theta = 0$

all Quad  
 $\theta = \cos^{-1}(0) = 90$

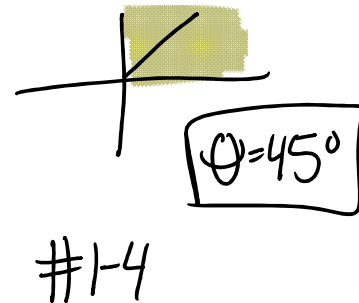


Solve the equation  $3 \tan \theta + 1 = 4$ ,  $0^\circ \leq \theta \leq 180^\circ$ .

$3 \tan \theta + 1 = 4$  \*solve for  $\theta$   
 $\frac{3 \tan \theta}{3} = \frac{3}{3}$

$\tan \theta = 1$   
 $\theta = \tan^{-1}(1)$   
 $\theta = 45^\circ$   
 ↑ reference angle

$3x + 1 = 4$   
 Quad 1 or 2  
 ↑  
 $\tan(+)$



**Complete Assignment Questions #1 - #4**

***Solving Trigonometric Equations Outside the Domain  $0^\circ \leq \theta \leq 360^\circ$***

Use the following procedure to solve equations where the domain is outside of  $0^\circ \leq \theta \leq 360^\circ$ .

Use the following procedure to solve equations where the domain is outside of  $0^\circ \leq \theta \leq 360^\circ$ .

Step 1: Solve the equation with the domain  $0^\circ \leq \theta \leq 360^\circ$  using the steps on the previous page.

Step 2: Using the concepts of coterminal angles, add or subtract  $360^\circ$  or multiples of  $360^\circ$ .

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Solve the equation  $\sqrt{3} \tan \theta + 1 = 0$ ,  $0^\circ \leq \theta \leq 720^\circ$ .



To the nearest whole number, solve the equation  $\cos x = -0.82$  where  $-360^\circ \leq x \leq 0^\circ$ .

**Complete Assignment Questions #5 - #9**

## ***Assignment***

# Assignment

1. Solve the following equations, where  $0^\circ \leq \theta \leq 360^\circ$ .

a)  $\cos \theta = \frac{1}{2}$

b)  $\sin \theta = -\frac{\sqrt{3}}{2}$

c)  $\tan \theta = -1$

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2. Find the measure of  $\theta$ , to the nearest degree, where  $0^\circ \leq \theta \leq 360^\circ$ .

a)  $\sin \theta = 0.6485$

b)  $\cos \theta = -0.8219$

c)  $\tan \theta = 0.4668$

d)  $6 \sin \theta = -1$

e)  $4 \cos \theta - 3 = 0$

f)  $\tan \theta + 5 = 0$

3. Determine the measure of  $A$  if  $0^\circ \leq A \leq 360^\circ$ .
- a)  $\tan A = 0$       b)  $\cos A = 1$       c)  $\sin A = -1$       d)  $\sin A = 0$

4. Given that  $(\tan \theta)^2$  can be written as  $\tan^2 \theta$ , solve the following equations if  $0^\circ \leq \theta \leq 360^\circ$ .
- a)  $\tan^2 \theta = 3$       b)  $\cos^2 \theta = \frac{3}{4}$

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5. Solve the following equations with the given domains.

a)  $\sin x = \frac{\sqrt{2}}{2}$ ,  $360^\circ \leq x \leq 720^\circ$       b)  $\tan x = \sqrt{3}$ ,  $-360^\circ \leq x \leq 0^\circ$

**c)**  $\cos x = \frac{\sqrt{3}}{2}, -360^\circ \leq x \leq 360^\circ$

**d)**  $\tan x + 1 = 0, 720^\circ \leq x \leq 1080^\circ$

**6.** Determine the measure of  $\theta$ , to the nearest degree, with the given domain.

**a)**  $\sin \theta = -0.29, -360^\circ \leq \theta \leq 360^\circ$

**b)**  $\cos \theta = -\frac{2}{3}, 0^\circ \leq \theta \leq 720^\circ$

**c)**  $3\tan \theta + 7 = 0, 360^\circ \leq \theta \leq 720^\circ$

**d)**  $\tan \theta = 0, -720^\circ \leq \theta \leq -360^\circ$

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**Multiple  
Choice**

7. The solution of the equation  $\cos x = 0.0999$  in the interval  $0^\circ \leq x \leq 360^\circ$  is

- A.  $84^\circ, 276^\circ$
- B.  $96^\circ, 264^\circ$
- C.  $96^\circ, 276^\circ$
- D.  $264^\circ, 276^\circ$

**Numerical  
Response**

8. For  $0^\circ < \theta < 180^\circ$ , the solution, to the nearest degree, of the equation  $\cos \theta = -\frac{1}{3}$  is \_\_\_\_ .

(Record your answer in the numerical response box from left to right.)

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9. If  $\sin x = -\frac{3}{8}$  and  $900^\circ \leq x \leq 1\,000^\circ$ , then the value of  $x$ , to the nearest degree, is \_\_\_\_ .

(Record your answer in the numerical response box from left to right.)

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**Answer Key**

1. a)  $60^\circ, 300^\circ$       b)  $240^\circ, 300^\circ$       c)  $135^\circ, 315^\circ$   
2. a)  $40^\circ, 140^\circ$       b)  $145^\circ, 215^\circ$       c)  $25^\circ, 205^\circ$       d)  $190^\circ, 350^\circ$       e)  $41^\circ, 319^\circ$       f)  $101^\circ, 281^\circ$   
3. a)  $0^\circ, 180^\circ, 360^\circ$       b)  $0^\circ, 360^\circ$       c)  $270^\circ$       d)  $0^\circ, 180^\circ, 360^\circ$   
4. a)  $60^\circ, 120^\circ, 240^\circ, 300^\circ$       b)  $30^\circ, 150^\circ, 210^\circ, 330^\circ$   
5. a)  $405^\circ, 495^\circ$       b)  $-300^\circ, -120^\circ$       c)  $-330^\circ, -30^\circ, 30^\circ, 330^\circ$       d)  $855^\circ, 1035^\circ$   
6. a)  $-163^\circ, -17^\circ, 197^\circ, 343^\circ$       b)  $132^\circ, 228^\circ, 492^\circ, 588^\circ$       c)  $473^\circ, 653^\circ$       d)  $-720^\circ, -540^\circ, -360^\circ$   
7. A      8. 

1	0	9	
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      9. 

9	2	2	
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