Quadratic Functions and Equations Lesson #4:
Equations and Intercepts from the Vertex and a Point

In the last lesson, we analyzed the graphs of quadratic functions with equations in standard form $y = a(x-p)^2 + q$. In this lesson, we determine the equation of a quadratic function from the graph. To do this we need the vertex of the parabola and a point on it. We will also learn how to find intercepts from the standard form of the equation.

$y = a(x-p)^2 + q$ ← up/down
↑ vertical stretch / reflection if −
↖ left/right
vertex (p, q) ↑ axis of sym ↑ max/min
point (x, y)

Determining the Equation from the Vertex and a Point

The following procedure will enable us to write quadratic functions in standard form if we are given the coordinates of the vertex and of another point on the parabola.

Step 1 Replace p and q with the coordinates of the vertex.

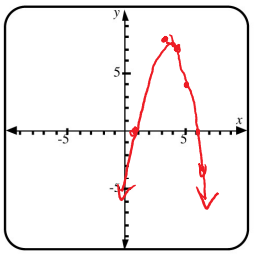
Step 2 Replace x and y with the coordinates of the point.

Step 3 Solve for a.

Step 4 Replace the values of a, p and q in the equation $y = a(x-p)^2 + q$.

Class Ex. #1

The graph of a quadratic function has vertex $(-2, 8)$ and passes through the point $(-1, 7)$.



a) Find the equation of the function in standard form $y = a(x-p)^2 + q$.

Step 1 $y = a(x+2)^2 + 8$

Step 2 $7 = a(-1+2)^2 + 8$

Step 3 solve for a

$7 = a + 8$

$-8 \quad -8$

$-1 = a$

$y = -1(x+2)^2 + 8$ standard form

b) Rewrite the equation in general form $y = ax^2 + bx + c$.

$y = -(x+2)(x+2) + 8$

$= -(x^2 + 4x + 4) + 8$

$y = -x^2 - 4x - 4 + 8$

$y = -x^2 - 4x + 4$ general form

yint: 2nd TRACE 1: value X = 0 ENTER y = 4

c) Use a graphing calculator to sketch the graph and determine the x and y-intercepts of the graph of the function. Answer to the nearest hundredth if necessary.

$x_{int} = 0.83, -4.83$ $y\text{-int} = 4$

Finding Intercepts from the Standard Form

We can use the equation of a quadratic function written in standard form to algebraically determine the x- and y-intercepts of the graph of the function.

Class Ex. #2 Determine, as exact values, the x and y intercepts of the graph of the function

We can use the equation of a quadratic function written in standard form to algebraically determine the x- and y-intercepts of the graph of the function.

Class Ex. #2

Determine, as exact values, the x and y-intercepts of the graph of the function $f(x) = 3(x-1)^2 - 9$.

y-int, $x=0$

$f(0) = 3(0-1)^2 - 9$

$= 3 - 9$

$y = -6$

$(0, -6)$

point on the graph

x-int, $y=0$

$0 = 3(x-1)^2 - 9$ solve for x

$+9 \quad +9$

$9 = 3(x-1)^2$

$\frac{9}{3} = \frac{3(x-1)^2}{3}$

$\pm\sqrt{3} = \sqrt{(x-1)^2}$

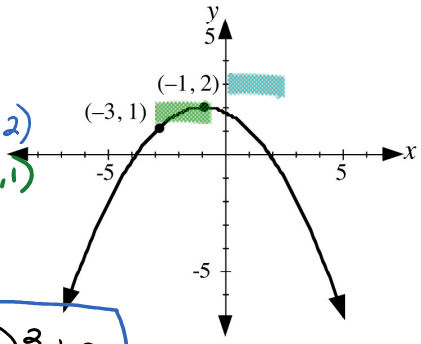
$\pm\sqrt{3} = x - 1$

$+1 \quad +1$

$1 \pm \sqrt{3} = x$ → $1+\sqrt{3} = 2.73$ and $1-\sqrt{3} = -0.73$

Class Ex. #3

The graph of a quadratic function is shown.
The maximum point is shown.



a) Find the equation of the function in standard form.

$y = a(x-p)^2 + q$ vertex $(p,q) = (-1,2)$

$y = a(x+1)^2 + 2$ point $(x,y) = (-3,1)$

$1 = a(-3+1)^2 + 2$ solve for a

$1 = a(-2)^2 + 2$

$1 = 4a + 2$

$-2 \quad -2$

$\frac{-1}{4} = \frac{4a}{4}$

$a = -\frac{1}{4}$

$y = -\frac{1}{4}(x+1)^2 + 2$

b) Find, algebraically, the x-intercepts and y-intercepts of the graph. Answer both as exact values and to the nearest hundredth.

x-int, $y=0$

$0 = -\frac{1}{4}(x+1)^2 + 2$

$-2 \quad -2$

$-2 = -\frac{1}{4}(x+1)^2$

$-4 \quad -4$

$\pm\sqrt{8} = \sqrt{(x+1)^2}$

$\pm\sqrt{8} = x+1$

$-1 \quad -1$

$x = -1 \pm \sqrt{8}$

x-int: $-1+\sqrt{8} = 1.83$

$-1-\sqrt{8} = -3.83$

y-int, $x=0$

$y = -\frac{1}{4}(0+1)^2 + 2$

$y = -\frac{1}{4} + \frac{8}{4}$

$y = \frac{7}{4} = 1.75$

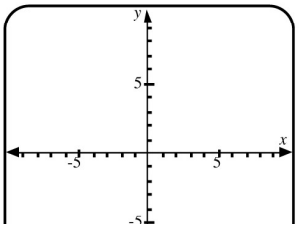
c) State the domain, range and equation of the axis of symmetry.

#1, 2ac, 3a, 4ac, 5 ✗

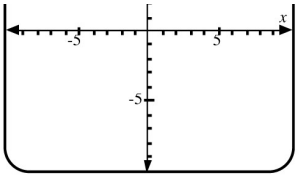
Complete Assignment Questions #1 - #12

Assignment

1. The graph of a quadratic function has vertex $(3, -4)$ and passes through the point $(4, 1)$.



a) Find the equation of the function in standard form.



b) Rewrite the equation in general form.

c) Use a graphing calculator to sketch the graph and determine the x and y-intercepts of the graph of the function. Answer to the nearest hundredth if necessary.

2. In each case, write an equation in standard form for the parabola with the given vertex and passing through the given point.

a) vertex $(7, -6)$, point $(9, -4)$

b) vertex $(-2, 5)$, point $(-4, 21)$

c) vertex $(-1, 0)$, point $(-5, -12)$

d) vertex $(3, -8)$, y-intercept is 10

3. The graph of a quadratic function has a vertex at $\left(\frac{5}{3}, 1\right)$ and one x-intercept is $\frac{2}{3}$.

a) Determine the equation of the function in standard form.

b) Determine the equation of the function in general form.

c) State the other x-intercept.

d) State the domain and range of the function.

e) State the equation of the axis of symmetry of the graph.

4. Determine, as exact values, the x and y-intercepts of the graph of the following functions.

a) $f(x) = (x - 4)^2 - 16$

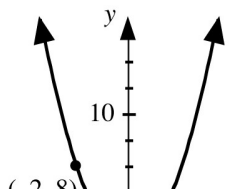
b) $f(x) = -3(x + 2)^2 + 3$

c) $f(x) = 2(x - 6)^2 - 6$

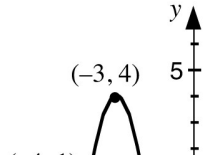
d) $f(x) = -\frac{1}{4}(x + 1)^2 + 5$

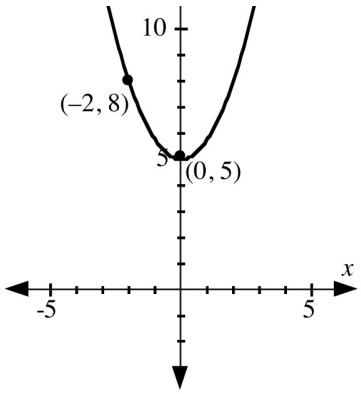
5. Write an equation of the form $y = a(x - p)^2 + q$ for each parabola.

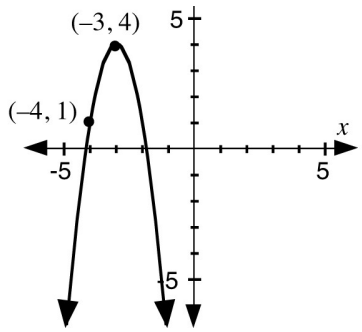
a)



b)







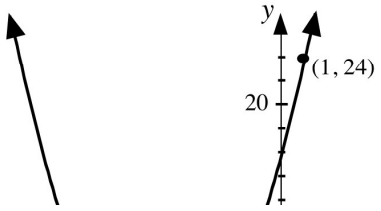
6. A function of the form $p(x) = ax^2 + q$ has two x-intercepts, one of which is 9.
Determine the other x-intercept and explain how you arrived at your answer.

7. The parabola with equation $y = a(x - 2)^2 + q$ passes through the points $(-2, 5)$ and $(4, -1)$.
Determine the coordinates of the vertex of the parabola.

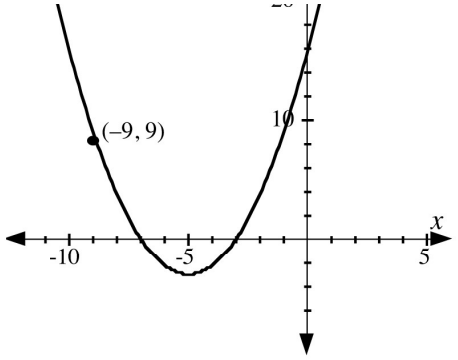
8. The graph of the function with equation $y = a(x + 5)^2 + q$ passes through the points $(-6, 2)$ and $(-3, 20)$.
Determine whether the function has a maximum or minimum value and state the value.

9. The graph of a quadratic function is shown.
The equation of the axis of symmetry is $x = -5$.

a) Find the equation of the graph of the function in standard form.



in standard form.



b) Find, algebraically, the x-and y-intercepts of the graph.

c) State the domain and range.

Multiple Choice **10.** The parabola with equation $y = a(x - p)^2 + q$ has a maximum value of 8. The line $x = 2$ is the axis of symmetry of the parabola. If the graph passes through the origin, then the value of a is

A. 2

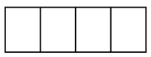
B. $\frac{1}{32}$

C. –2

D. $-\frac{1}{32}$

Numerical Response **11.** The graph of the function $g(x) = -2(x - 3)^2 + q$ passes through the point $(-5, -2)$. The value of q, to the nearest whole number, is _____ .

(Record your answer in the numerical response box from left to right.)



12. The graph of a function of the form $f(x) = a(x + 2)^2 - 7$ has two x-intercepts, one of which is –6.5. The other x-intercept, to the nearest tenth, is _____ .
(Record your answer in the numerical response box from left to right.)

Answer Key

1. **a)** $y = 5(x - 3)^2 - 4$ **b)** $y = 5x^2 - 30x + 41$ **c)** x-intercepts are 2.11, 3.89; y-intercept is 41

2. **a)** $y = \frac{1}{2}(x - 7)^2 - 6$ **b)** $y = 4(x + 2)^2 + 5$ **c)** $y = -\frac{3}{4}(x + 1)^2$ **d)** $y = 2(x - 3)^2 - 8$

3. **a)** $y = -\left(x - \frac{5}{3}\right)^2 + 1$ **b)** $y = -x^2 + \frac{10}{3}x - \frac{16}{9}$ **c)** $\frac{8}{3}$
d) Domain: $\{x \mid x \in R\}$, Range: $\{y \mid y \le 1, y \in R\}$ **e)** axis of symmetry $x = \frac{5}{3}$

4. **a)** x - intercepts 0 and 8; y -intercept 0
b) x - intercepts –3 and –1; y -intercept –9
c) x - intercepts $6 + \sqrt{3}$ and $6 - \sqrt{3}$; y -intercept 66
d) x - intercepts $-1 + 2\sqrt{5}$ and $-1 - 2\sqrt{5}$; y -intercept $\frac{19}{4}$

5. **a)** $y = \frac{3}{4}x^2 + 5$ **b)** $y = -3(x + 3)^2 + 4$

6. The vertex of the parabola is on the y-axis so the x-intercepts are an equal distance on either side of $x = 0$. If one x-intercept is 9, the other must be –9.

7. (2, –3) **8.** minimum value of –4

9. **a)** $y = \frac{3}{4}(x + 5)^2 - 3$ **b)** x-intercepts are –7 and –3, y-intercept $\frac{63}{4}$
c) Domain: $\{x \mid x \in R\}$, Range: $\{y \mid y \ge -3, y \in R\}$

10. C **11.**

1	2	6	

12.

2	.	5	