

Lesson 2: Analyzing Quadratic Functions - Part One

Quadratic Functions and Equations Lesson #2: Analyzing Quadratic Functions - Part One

Quadratic Function

A **quadratic function** is a function which can be written in the form

$$f(x) = ax^2 + bx + c, \text{ where } a, b, c \in R, \text{ and } a \neq 0$$

or in equation form as

$$y = ax^2 + bx + c, \text{ where } a, b, c \in R, \text{ and } a \neq 0$$

Quadratic Equation

A **quadratic equation** is an equation which can be written in the form

$$ax^2 + bx + c = 0, \text{ where } a, b, c \in R, \text{ and } a \neq 0.$$

The roots of the quadratic equation $ax^2 + bx + c = 0$ are the zeros of the related quadratic function $f(x) = ax^2 + bx + c$.

General and Standard Forms

A quadratic function can be written in **general** or **standard** form.

General Form: $f(x) = ax^2 + bx + c$, or $y = ax^2 + bx + c$, where $a, b, c \in R$, and $a \neq 0$.

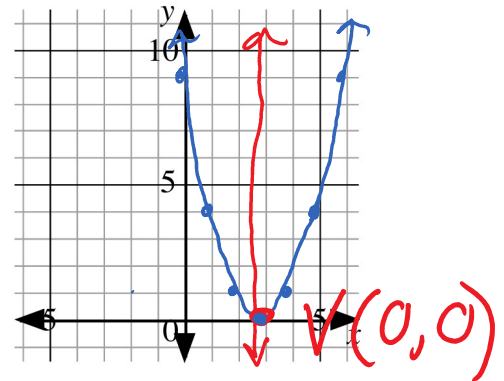
Standard Form: $f(x) = a(x - p)^2 + q$, or $y = a(x - p)^2 + q$, where $a, p, q \in R$, and $a \neq 0$.

In this unit we will study both the general form and standard form, beginning with the standard form in this lesson.

Analyzing the Graph of the Function with Equation $y = x^2$

- Graph the function with equation $y = x^2$ by completing the table of values. Join the points with a smooth curve. The graph of this function is called a parabola.

x	-3	-2	-1	0	1	2	3
y		9	4	1	0	1	4



- The **axis of symmetry** is the “mirror” line which splits the parabola in half. State the equation of the axis of symmetry for this parabola.
- The **vertex** of a parabola is where the axis of symmetry intersects the parabola. The vertex can represent a minimum point or maximum point depending on whether the parabola opens up or down.

Label the vertex (V) on the graph and state its coordinates.

(0,0)

- The maximum or minimum **value** of a quadratic function occurs at the vertex and is represented by the y-coordinate of the vertex. Complete the following:

The minimum value of the function with equation $y = x^2$ is 0.

- State the domain and range of the function with equation $y = x^2, x \in R$.

Domain: $x \in \mathbb{R}$ Range: $y \geq 0$



The following investigations can be completed as a class lesson or as an individual assignment. The process used in these explorations will be further developed in grade 12 mathematics.

Analyzing the Function with Equation $y = a(x - p)^2 + q$, $a = 1$

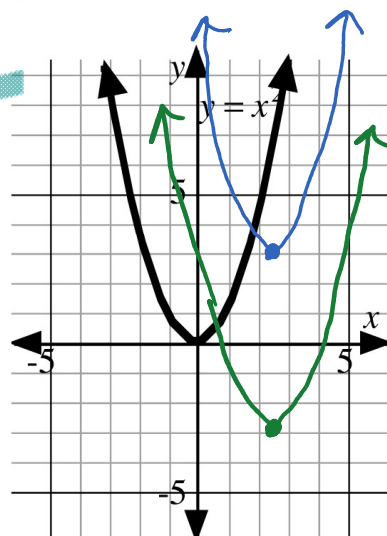
The next three investigations help us explore some general **transformations** on the graph of $y = x^2$ and the relationship they have to the standard form $y = a(x - p)^2 + q$, where $a = 1$.

A **transformation** is an operation which moves (or maps) a figure from an original position to a new position.

In each investigation, use a graphing calculator to sketch the equations.

Investigation #1

Analyzing the Graph of $y = x^2 + q$



The graph of $y = f(x) = x^2$ is shown.

a) Write an equation which represents each of the following:

• $y = f(x) + 3$

• $y = f(x) - 3$

$y = x^2 + 3$

$y = x^2 - 3$

b) Use a graphing calculator to sketch $y = f(x) + 3$ and $y = f(x) - 3$ on the grid.

c) Complete the following chart.

Function	Equation Representing Function	Vertex	Max/Min Value	Equation of Axis of Symmetry	Description of Transformation
$y = f(x)$	$y = x^2$	$(0, 0)$	min, 0	$x = 0$	no transformation
$y = f(x) + 3$	$y = x^2 + 3$	$(0, 3)$	min 3	$x = 0$	vertical translation 3 units up
$y = f(x) - 3$	$y = x^2 - 3$	$(0, -3)$	min -3	$x = 0$	vertical translation 3 units down
$y = f(x) + q$	$y = x^2 + q$	$(0, q)$	min q	$x = 0$	vertical translation q units up

d) What is the effect of the **parameter**, q , on the graph of $y = x^2 + q$?

the graph undergoes a vertical translation

the graph undergoes a vertical translation
 q units up

e) Compared to the graph of $y = x^2$, the graph of $y = x^2 + q$ results in a vertical translation (or shift) of q units.

If $q > 0$, the parabola moves up If $q < 0$, the parabola moves down

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280 Quadratic Functions and Equations Lesson #2: Analyzing Quadratic Functions - Part One

Investigation #2

Analyzing the Graph of $y = (x - p)^2$

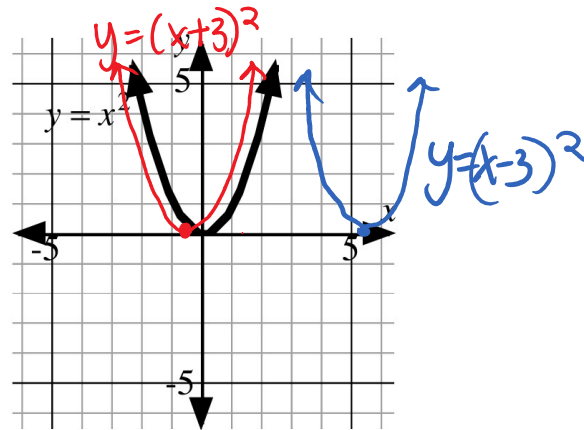
The graph of $y = f(x) = x^2$ is shown.

a) Write an equation which represents each of the following:

- $y = f(x + 3)$
- $y = f(x - 3)$

$y = (x + 3)^2$

$y = (x - 3)^2$



b) Use a graphing calculator to sketch $y = f(x + 3)$ and $y = f(x - 3)$ on the grid.

c) Complete the following chart.

horizontal \longleftrightarrow

Function	Equation Representing Function	Vertex	Max/Min Value	Equation of Axis of Symmetry	Description of Transformation
$y = f(x)$	$y = x^2$	$(0, 0)$	min, 0	$x = 0$	no transformation
$y = f(x + 3)$	$y = (x + 3)^2$	$(-3, 0)$	min, 0	$x = -3$	horizontal translation 3 units left
$y = f(x - 3)$	$y = (x - 3)^2$	$(3, 0)$	min, 0	$x = 3$	horizontal translation 3 units right
$y = f(x - p)$	$y = (x - p)^2$	$(p, 0)$	min, 0	$x = p$	horizontal translation p units right

d) What is the effect of the parameter, p , on the graph of $y = (x - p)^2$?

e) Compared to the graph of $y = x^2$, the graph of $y = (x - p)^2$ results in a horizontal translation (shift) of p units.

If $p > 0$, the parabola moves right $p < 0$, the parabola moves left

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Investigation #3

Analyzing the Graph of $y = (x - p)^2 + q$

Consider the function $f(x) = x^2$.

a) Write an equation which represents $f(x + 2) - 4$.

$y_1 = x^2$
 $y_2 = (x+2)^2 - 4$

b) Predict the transformations on $y = x^2$ in a). Use a graphing calculator to verify the results.

2 units left
 4 units down



c) Complete the following chart.

Function	Equation Representing Function	Vertex	Max/Min Value	Equation of Axis of Symmetry	Description of Transformation
$y = f(x)$	$y = x^2$	$(0, 0)$	min, 0	$x = 0$	no transformation
$y = f(x + 2) - 4$	$y = (x+2)^2 - 4$	$(-2, -4)$	min -4	$x = -2$	translation 2 left 4 down
$y = f(x - p) + q$	$y = (x-p)^2 + q$	(p, q)	min q	$x = p$	translation p right q up



Describe how the graphs of the following functions relate to the graph of $y = x^2$.

a) $y = (x + 10)^2$

$p = -10$
 left 10 units

b) $y = x^2 + 4$

vertical shift
 $q = 4$

c) $y + 8 = (x - 5)^2$

$y = (x - 5)^2 - 8$
 $p = 5$ $q = -8$
 right 5 $q = -8$

left 10 units
horizontal

$q = 4$
up 4 units

$p = 5$ $q = -8$
right 5
down 8



Class Ex. #2

The following transformations are applied to the graph of $y = x^2$. Write the equation of the image function for each.

a) a horizontal translation of 5 units right

$y = (x - 5)^2$

b) a translation of 6 units down and 4 units left

$y = (x + 4)^2 - 6$



Class Ex. #3

Write the coordinates of the image of the point $(3, 9)$ on the graph $y = x^2$ when a translation of two units up and seven units right is applied.

$(3, 9) \rightarrow (3, 11) \rightarrow (10, 11)$

#1-6

Complete Assignment Questions #1 - #10

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282 Quadratic Functions and Equations Lesson #2: Analyzing Quadratic Functions - Part One

Assignment

1. Describe how the graphs of the following functions relate to the graph of $y = x^2$.

a) $y = (x + 5)^2$

b) $y = x^2 - 7$

c) $y - 8 = x^2$

d) $y = 5 + (x - 2)^2$

e) $y + 7 = (x + 1)^2 - 10$

f) $y = (x - a)^2 - b$

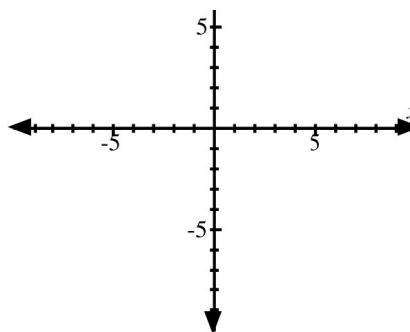
2. Consider the graph of the function $f(x) = (x - 2)^2 + 3$.

a) Without using a graphing calculator, sketch the graph on the grid.

b) State the coordinate of the vertex.



Graph on the grid.



- b) State the coordinate of the vertex.
 - c) State the maximum or minimum value of the function.
 - d) State the domain and range of the function.
3. The following transformations are applied to the graph of $y = x^2$. Write the equation of the image function for each.
- a) a horizontal translation of 7 units right b) a vertical translation of 2 units down
 - c) a translation 3 units left and 8 units up d) a translation c units down and d units right
4. Write the coordinates of the image of the point $(-2, 4)$ on the graph $y = x^2$ when each of the following transformations is applied.
- a) A horizontal translation of 2 units to the left.
 - b) A translation of 3 units up and 11 units right.

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Quadratic Functions and Equations Lesson #2: *Analyzing Quadratic Functions - Part One* **283**

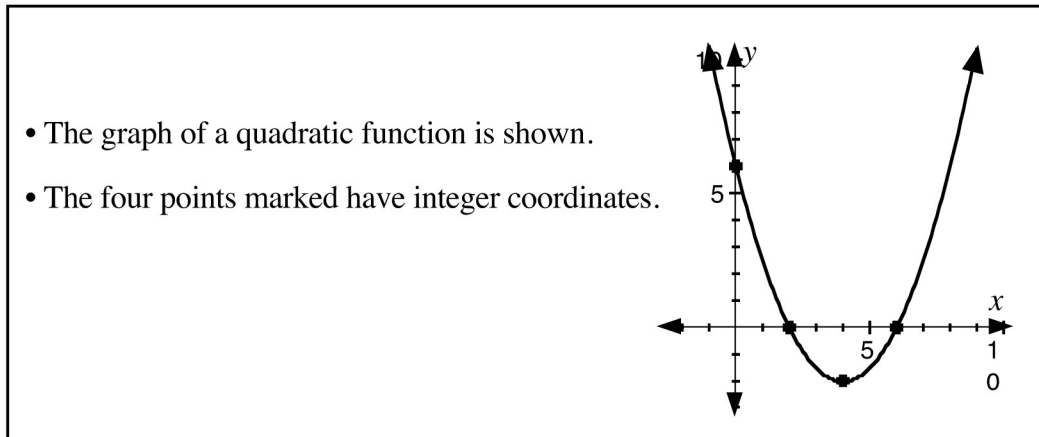
5. Complete the following table.

Function	$y = x^2 + 5$	$y = (x + 3)^2 - 4$	$y + 9 = (x - 6)^2 + 1$	$y - w = (x + r)^2$
Coordinates of Vertex				
Max/Min Value				
Eqn. of Axis of Symmetry				
Domain				
Range				

6. After a combination of a horizontal and a vertical translation, the graph of $y = x^2$ has an image graph with a vertex at $(2, -6)$. Describe the translations.

image graph with a vertex at $(2, -6)$. Describe the translations.

Use the following information to answer questions #7 and #8.



Multiple Choice

7. The domain and range, respectively, of the function are

- A. $x \in R$ and $y \in R$
- B. $x \geq -2$ and $y \in R$
- C. $x \in R$ and $y \geq -2$
- D. $2 \leq x \leq 6$ and $y \geq -2$

Numerical Response

8. The sum of the x and y -intercepts is _____ .

(Record your answer in the numerical response box from left to right.)

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284 Quadratic Functions and Equations Lesson #2: *Analyzing Quadratic Functions - Part One*

Multiple Choice

9. Which of the following transformations shifts the graph of $y = x^2$ to the graph of $y + a = (x - b)^2$?

- A. a units right and b units down
- B. b units right and a units down
- C. b units up and a units right.
- D. a units down and b units left

10. The function defined by the equation $y = x^2$ is transformed to $y = (x + 2)^2 + 4$. If the point $(2, 4)$ lies on the graph of $y = x^2$, which of the following points must lie on the graph of $y = (x + 2)^2 + 4$?

- A. $(0, 0)$
- B. $(4, 0)$
- C. $(4, 8)$
- D. $(0, 8)$

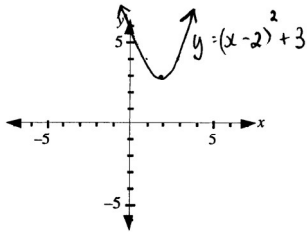
C. (4, 8)

D. (0, 8)

Answer Key

1. a) horizontal translation 5 units left b) vertical translation 7 units down
 c) vertical translation 8 units up d) translation 2 units right and 5 units up
 e) translation 1 unit left and 17 units down f) translation a units right and b units down

2. a) See Graph below



- b) (2, 3)
 c) minimum value of 3
 d) Domain: $\{x \mid x \in \mathbb{R}\}$ Range: $\{y \mid y \geq 3, y \in \mathbb{R}\}$

3. a) $y = (x - 7)^2$ b) $y = x^2 - 2$
 c) $y = (x + 3)^2 + 8$ d) $y = (x - d)^2 - c$

4. a) (-4, 4) b) (9, 7)

5. See table below

Function	$y = x^2 + 5$	$y = (x + 3)^2 - 4$	$y + 9 = (x - 6)^2 + 1$	$y - w = (x + r)^2$
Coordinates of Vertex	(0, 5)	(-3, -4)	(6, -8)	(-r, w)
Max/Min Value	min, 5	min, -4	min, -8	min, w
Eqn. of Axis of Symmetry	$x = 0$	$x = -3$	$x = 6$	$x = -r$
Domain	$\{x \mid x \in \mathbb{R}\}$	$\{x \mid x \in \mathbb{R}\}$	$\{x \mid x \in \mathbb{R}\}$	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \geq 5, y \in \mathbb{R}\}$	$\{y \mid y \geq -4, y \in \mathbb{R}\}$	$\{y \mid y \geq -8, y \in \mathbb{R}\}$	$\{y \mid y \geq w, y \in \mathbb{R}\}$

6. horizontal translation 2 units right, vertical translation 6 units down.

7. C

8.

1	4		
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9. B

10. D

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