Quadratic Functions and Equations Lesson #1:
Connecting Zeros, Roots, and x-intercepts

Function and Function Notation

Recall the following information from previous math courses:

Function
A functional relation, or **function**, is a special type of relation in which each element of the domain is related to exactly one element of the range. If any element of the domain is related to more than one element of the range, then the relation is not a function.

Function Notation
Under a function, f, the image of an element, x, in the domain is denoted by $f(x)$, which is read "f of x".

$$x \xrightarrow{\text{“}f\text{”}} f(x)$$

y

"f of x" function of x

Consider a function f defined by the formula $f(x) = 4x - 8$.
The notation $f(x) = 4x - 8$ is called **function notation**.

We can show that, under the function f, the image of 5 is 12. We write $f(5) = 12$.

x y

function notation	equation of graph of function
$f(x) = 4x - 8$	$y = 4x - 8$
$f(5) = 4(5) - 8$	$y = 4(5) - 8$
$f(5) = 12$	$y = 12$

input (x)

We can also show that, under the function f, the image of 2 is 0. We write $f(2) = 0$.

output y

function notation	equation of graph of function
$f(x) = 4x - 8$	$y = 4x - 8$
$f(2) = 4(2) - 8$	$y = 4(2) - 8$
$f(2) = 0$	$y = 0$

We can say:
"The zero of the function $f(x) = 4x - 8$ is 2." "The root of the equation $y = 4x - 8$ is 2."

Zero(s) of a Function

x

A **zero of a function** is a value of the independent variable which makes the value of the function equal to zero. Zero(s) of a function can be found by solving the equation $f(x) = 0$.

Find the zero of the function f where $f(x) = 7x - 21$.

$7x - 21 = 0$ solve for x
$+21 \quad +21$
$\frac{7x}{7} = \frac{21}{7}$
$\boxed{x = 3}$

the zero of the function f is 3

Investigation #1 *Connecting Roots, x-intercepts, and Zeros in a Linear Relation*

y=0

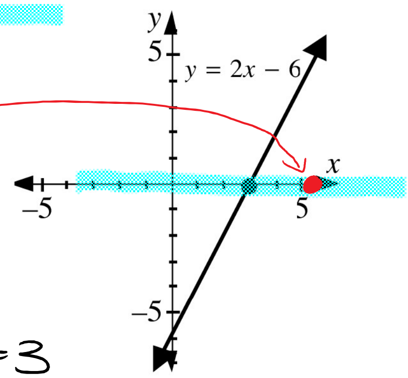
a) The graph of $y = 2x - 6$ is shown. Determine the x-intercept of the graph algebraically and graphically.

y
5

a) The graph of $y = 2x - 6$ is shown. Determine the x-intercept of the graph algebraically and graphically.

$2x - 6 = 0$
$2x = 6$
$x = 3$

$X_{int} = 3$



b) Determine the root of the equation $2x - 6 = 0$.

$2x = 6$
$x = 3$
root is $x = 3$

c) State the connection between the x-intercepts of the graph of $y = 2x - 6$ and the roots of the equation $2x - 6 = 0$.

same value

d) Consider the function $f(x) = 2x - 6$. What is the zero of the function?

$0 = 2x - 6$
$6 = 2x$
$3 = x$

e) What is the connection between the x-intercepts of the graph of $y = 2x - 6$, the roots of the equation $2x - 6 = 0$, and the zero of the function $f(x) = 2x - 6$?

all the same value

Investigation #2 *Connecting Roots, x-intercepts, and Zeros in a Quadratic Relation*

Investigation #2 | *Connecting Roots, x-intercepts, and Zeros in a Quadratic Relation*

a) Jacques is determining the roots of the equation $x^2 - x - 6 = 0$.
He wrote the equation in factored form and used the Zero Product Law to determine the roots of the equation.

Complete his work to solve for x.

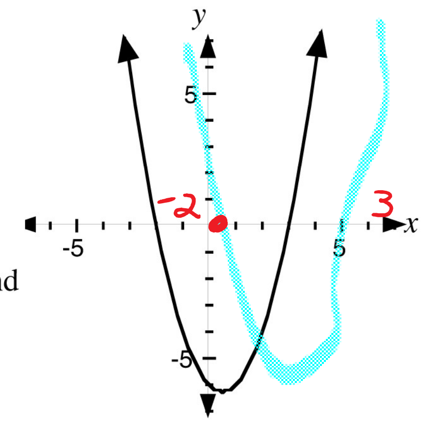
$x^2 - x - 6 = 0$ $\quad$ $\times$ | $+$; -6 | -1

$(x-3)(x+2) = 0$

$x = -2, 3$

b) The graph of $y = x^2 - x - 6$ is shown.
State the x-intercepts of the graph and mark them on the grid.

Xint = $-2, 3$



c) **i)** State the connection between
the ***x*-intercepts** of the graph of $y = x^2 - x - 6$ and
the **roots** of the equation $x^2 - x - 6 = 0$.

same values

ii) Explain the connection between
the **factors** of $x^2 - x - 6$ and
the **roots** of the equation $x^2 - x - 6 = 0$.

d) Consider the function $g(x) = x^2 - x - 6$. Determine the zeros of the function.

$(x-3)(x+2) = 0$ $\quad$ the zeros are 3 & -2

$x = 3, -2$

e) State the connection between
- the ***x*-intercepts** of the graph of $y = x^2 - x - 6$
- the **roots** of the equation $x^2 - x - 6 = 0$, and
- the **zeros** of the function $g(x) = x^2 - x - 6$

all the same values

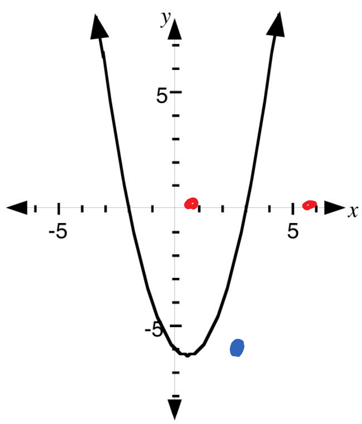
Class Ex. #2 **a)** Fill in the blanks in the following statement regarding the function with equation $y = f(x)$.

Class Ex. #2

a) Fill in the blanks in the following statement regarding the function with equation $y = f(x)$.

"The zeros of the function, the x-ints of the graph of the function, and the roots of the corresponding equation $y = 0$, are the same numbers."

b) The graph of $f(x) = x^2 - x - 6$ is shown. Fill in the blanks.



The ***graph*** of	The ***function***	The ***equation***
$f(x)=x^2-x-6$	$f(x)=x^2-x-6$ $= (x+2)(x-3)$	$x^2-x-6=0$
has ***x-intercepts***	has ***zeros***	has ***roots***
$x =$ -2 and $x =$ 3	-2 and 3	$x =$ -2 and $x =$ 3
with ***y-intercept*** $y =$ -6		

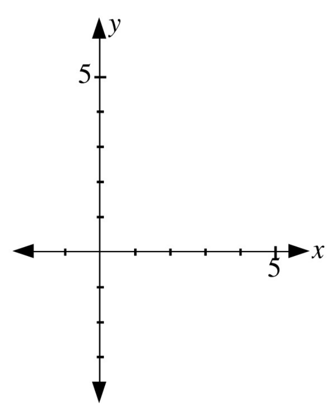
$x=0$, y-int
$0^2-0-6=y$
$-6=y$

Class Ex. #3

Consider the equation $2x^2 - 7x + 3 = 0$.

a) Describe how the **zero feature** of a graphing calculator can be used to determine the roots of the equation.

b) Use a graphing calculator to determine the roots of the equation and sketch the graph on the grid provided.



c) Use the x-intercepts of the graph of $y = 2x^2 - 7x + 3$ to factor the expression $2x^2 - 7x + 3$.

d) What are the zeros of the function $f(x) = 2x^2 - 7x + 3$?

Note

Finding Zeros of a Function

To find the zeros of a function, $f(x)$, either

- substitute zero for $f(x)$ and find the roots of the resulting equation [redacted] To find the zeros of a function, $f(x)$, either

- substitute zero for $f(x)$ and find the roots of the resulting equation

  or
- graph the function and determine the x-intercepts of the graph

Finding the Roots of an Equation by Factoring

Finding the roots of a single variable equation may involve factoring. Except in the case of a linear equation, set the equation to zero before factoring.

Recall the following techniques for factoring common factors, difference of two squares, trinomials of the form $x^2 + bx + c = 0$, and trinomials of the form $ax^2 + bx + c = 0$.

Class Ex. #4

Find the roots of the following equations. Set one side to zero, FACTOR, solve

a) $x^2 + 8x = 33$

$x^2+8x-33=0$ $\quad \frac{\times \mid +}{-33 \mid 8}$

$(x+11)(x-3)=0$

$x=-11, 3$

roots are -11 & 3

b) $6(4x + 5)(x - 3) = 0$

$x=\frac{-5}{4}, 3$

roots are $\frac{-5}{4}$ & 3

Reminder

$4x+5=0$

$4x=-5$

$x=\frac{-5}{4}$

c) $2x^2 - 8 = 0$

$2(x^2-4)=0$

$2(x-2)(x+2)=0$

$x=\pm 2$

roots are -2 & 2

Class Ex. #5

For the following functions

i) find the zeros **ii)** find the y-intercept of the graph of the function

a) $f(x) = 5x^2 + 15x - 20$

i) $5x^2+15x-20=0$

$5(x^2+3x-4)=0$ $\quad \frac{\times \mid +}{-4 \mid 3}$

$5(x+4)(x-1)=0$

$x=-4, 1$

zeros are -4 & 1

ii) y-int, $x=0$

$f(0)=5(0)^2+15(0)-20$

when $x=0$, $f(x)=-20$

$y$int $=-20$

b) $f(x) = 3x^2 - 11x + 10$

i) $3x^2-11x+10$ $\quad \frac{\times \mid +}{30 \mid -11}$

$3x^2-6x-5x+10$

$3x(x-2)-5(x-2)$

$(3x-5)(x-2)=0$

$x=\frac{5}{3}, 2$

zeros are $2, \frac{5}{3}$

ii) when $x=0$, $f(x)=10$

y-int $=10$

c) $g(x) = 2x(2x + 1)$

i) $2x(2x+1)=0$

$x=0, -\frac{1}{2}$

zeros are 0 & $-\frac{1}{2}$

ii) y-int $x=0$

$g(0)=2(0)(2(0)+1)$

$=0(1)=0$

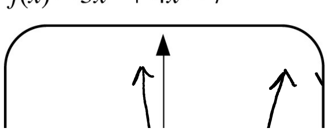
y-int $=0$

Class Ex. #6

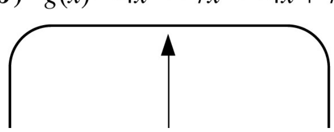
For each of the following functions, use a graphing calculator to

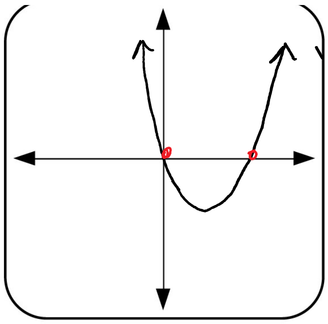
- sketch the graph of the function
- find the zeros of the function as exact values
- write the function in factored form

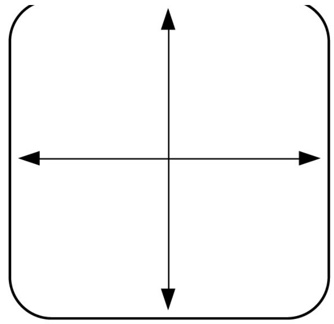
a) $f(x) = 3x^2 + 4x - 7$



b) $g(x) = 4x^3 - 7x^2 - 4x + 7$



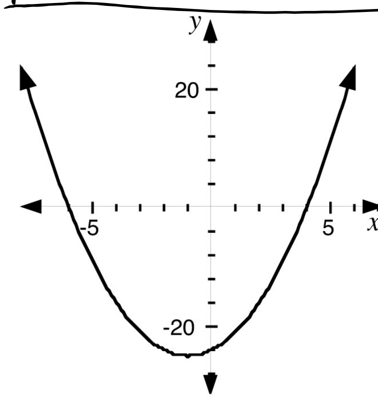




Complete Assignment Questions #1 - #11

$\boxed{\#1, 2, 3, 4, 6}$

Assignment



1. The graph of a function, f, is shown.
The x and y-intercepts of the graph are integers.

a) State the x and y-intercepts of the graph.

b) State the zeros of the function f.

2. Find the roots of the following equations.

a) $2x(x + 3) = 0$

b) $2x^2 - 10x + 12 = 0$

$2(x^2 - 5x + 6) = 2(\quad x \quad) = 0$

c) $x^3 + 8x^2 = 20x$

d) $4x^2 + 4x - 3 = 0$

3. Find the zeros of the following functions.

a) $f(x) = \frac{x}{3} + 5$

b) $g(x) = 25x^2 - 64$

c) $P(x) = 3(2x - 5)(x + 1)$

d) $P(x) = x(x - 3)(2x + 1)$

c) $P(x) = 3(2x - 5)(x + 1)$ **d)** $P(x) = x(x - 3)(2x + 1)$

4. In each of the following

i) determine the zeros of the function
ii) determine the y-intercept of the graph of the function

a) $f(x) = 5x^2 - 35x$ **b)** $f(x) = 3x(x^2 - 49)$

c) $f(x) = 2x^2 - x - 15$ **d)** $P(x) = 8x^2 + 14x - 15$

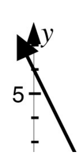
5. Use a graphing calculator to find the zeros (as exact values) of the following functions.

a) $f(x) = 18x^2 - 5x - 7$ **b)** $g(x) = 3x^3 - 11x^2 + 6x$

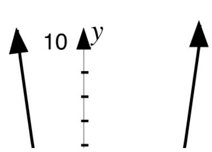
6. In each case, the graph of a function with $y = f(x)$ is shown. The x and y-intercepts of the graph are integers. Determine

- the zeros of the function
- the y-intercept of the graph of the function
- the equation of the function in factored form

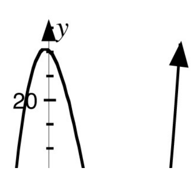
a)

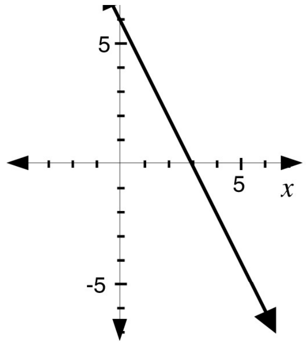


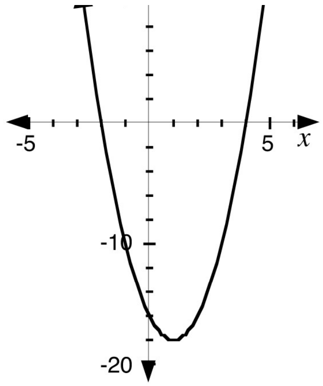
b)

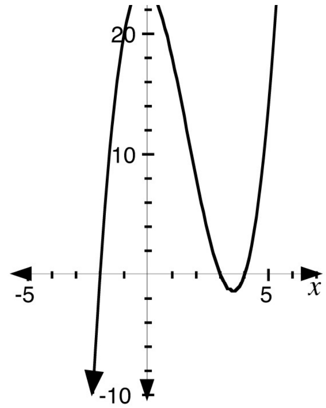


c)









7. Use a graphing calculator to write the equation in factored form.

a) $y = 2x^2 - 3x - 9$ **b)** $y = 5x^3 - 7x^2 - 21x - 9$

Multiple Choice

8. The zeros of the function $f(x) = 2(x - 3)(4x + 7)$ are

A. $3, -\frac{7}{4}$ **B.** $-3, \frac{7}{4}$

C. $0, 3, -\frac{7}{4}$ **D.** $2, 3, -\frac{7}{4}$

9. The roots of the equation $3x(x + 1) = 6$ are

A. $0, -1$
B. $2, 5$
C. $2, -1$
D. $-2, 1$

10. The least possible zero of the function $f(x) = 2x^3 - 7x^2 + 3x$ is

A. 0
B. 1

A. 0
B. $\frac{1}{2}$
C. 3
D. –3

Numerical Response **11.** The y-intercept of the graph of the function $f(x) = (x + 4)(3 - 2x)(x + 1)$, to the nearest whole number, is _____ .

(Record your answer in the numerical response box from left to right.)

Answer Key

1. a) x-intercepts are –6, 4 and y-intercept is –24. **b)** -6, 4

2. a) –3, 0 **b)** 2, 3 **c)** –10, 0, 2 **d)** $-\frac{3}{2}, \frac{1}{2}$

3. a) –15 **b)** $-\frac{8}{5}, \frac{8}{5}$ **c)** $-1, \frac{5}{2}$ **d)** $-\frac{1}{2}, 0, 3$

4. a) i) 0, 7 **ii)** 0 **b) i)** –7, 0, 7 **ii)** 0
c) i) $-\frac{5}{2}, 3$ **ii)** –15 **d) i)** $-\frac{5}{2}, \frac{3}{4}$ **ii)** –15

5. a) $-\frac{1}{2}, \frac{7}{9}$ **b)** $0, \frac{2}{3}, 3$

6. a) zero: 3
y-intercept: 6
$f(x) = -2(x - 3)$

b) zeros: –2, 4
y-intercept: –16
$f(x) = 2(x + 2)(x - 4)$

c) zeros: –2, 3, 4
y-intercept: 24
$f(x) = (x + 2)(x - 3)(x - 4)$

7. a) $y = (2x + 3)(x - 3)$ **b)** $y = (5x + 3)(x - 3)(x + 1)$

8. A **9.** D **10.** A **11.**

1	2		