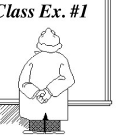
Quadratic Functions and Equations Lesson #8:
Applications of Quadratic Functions - A Graphical Approach

Using A Graphing Calculator to Find Maximum or Minimum Values

1. Enter the equation of the function into [Y1] and press [GRAPH].
2. Access the CALC feature by entering [2nd] then [TRACE].
3. Select "minimum" or "maximum".
4. On the bottom left hand side of the screen the calculator will ask for a left bound. Select a value on the left side of the max/min point and press [ENTER].
5. On the bottom left hand side of the screen the calculator will ask for a right bound. Select a value on the right side of the max/min point and press [ENTER].
6. On the bottom left hand side of the screen the calculator will ask for a guess. Press [ENTER]. The ***y* value** will be the max/min answer.

[2nd] [TRACE]
[1:] value
X = ______
↑
Solve for any y value this way.
X = 0 [Enter] for y-int

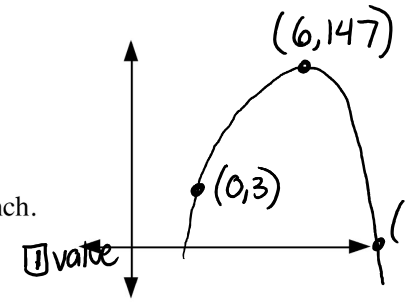
Class Ex. #1



The height, h, in metres above the ground of a projectile at any time, t, in seconds after the launch, is defined by the function $h(t) = -4t^2 + 48t + 3$.

Use a graphing calculator to answer the following.

a) Sketch the relevant part of the parabola on the grid.



Xmin -1
Xmax 15
ymin -1
ymax 160

b) Find the height of the projectile 3 seconds after the launch.

h(3) = 111   height = 111m

c) Find the maximum height reached by the projectile.

147m

d) How many seconds after the launch is the maximum height reached?

6 seconds

e) What was the height of the projectile at the launch?

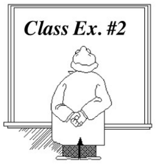
3m

f) Determine when the projectile hit the ground to the nearest tenth of a second.

h(t) = 0   t = 12.1 seconds
x-int

#1, 4

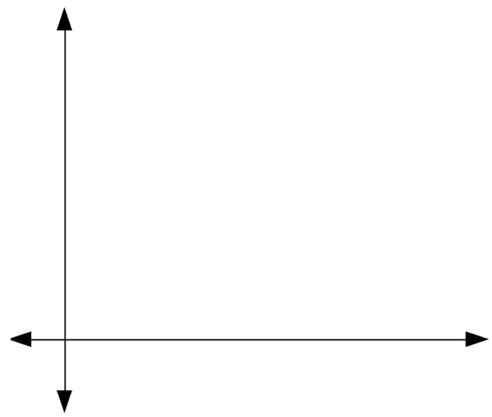
Class Ex. #2 Last season, a struggling hockey club had only 7 200 season ticket holders. The owner of the



Last season, a struggling hockey club had only 7 200 season ticket holders. The owner of the hockey club has decided to raise the price of a package of season tickets for the new season to generate more revenue. The existing cost of a package of season tickets is $1 400. Before raising the price of a package of season tickets, he hired a market research company to gather data on the proposed increase. The research company reported that for every $25 increase in price, approximately 100 season ticket holders would not renew their season tickets.

If the price increase is to be a multiple of $25, use the following procedure to determine what price would maximize the revenue from season tickets.

a) Let x be the number of $25 increases from the current price of a season ticket. Write expressions in x for the cost of a package of season tickets and the potential number of season ticket holders.



b) Use the results of a) to generate an expression which represents the revenue obtained.

c) Determine the price of a package of season tickets which would generate maximum revenue.

d) How many season ticket holders would there be if this plan was implemented?

e) How much more revenue would be generated if the plan in c) was implemented?

Complete Assignment Questions #1 - #7

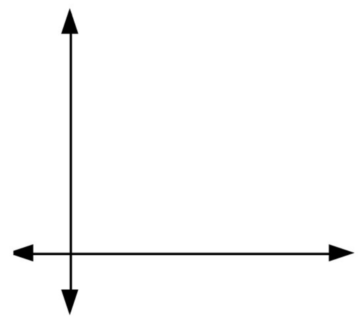
Assignment

1. A football punted during a high school football game followed the path of a parabola. The path can be modelled by the function

$$d(t) = -5t^2 + 15t + 1, \ t \geq 0$$

where t is the number of seconds which have elapsed since the football was punted, and $d(t)$ is the number of metres above the ground after t seconds.

a) Sketch the graph on the grid.



In the following questions, answer to the nearest hundredth of a unit where necessary.

b) What was the height of the football above the ground as the punter made contact with the football?

c) What was the height of the football above the ground 1 second after contact?

d) What is the maximum height reached by the football? What relation does this have to the vertex of the parabola?

e) How many seconds had elapsed when the football reached its maximum height? What relation does this have to the vertex?

f) The punt was not fielded by the opposition and the football hit the ground. How many seconds did it take for the football to hit the ground?

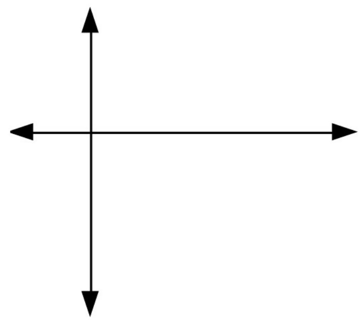
g) The original domain was given as $t \geq 0$. Write a more accurate domain for the function which describes the path of the football.

2. The cross section of a river, from one bank to the other, can be represented by the function

$$d(w) = \frac{1}{14}w^2 - \frac{5}{7}w$$

where $d(w)$ is the depth, in metres, of the river w metres from the left edge of the river bank.

a) Sketch the graph of the cross section of the river using a graphing calculator.



b) Determine the depth of the river 3 metres from the left edge.

c) What is the maximum depth of the river, to the nearest hundredth of a metre?

d) How far from the left edge of the river, to the nearest tenth of a metre, is the deepest part of the river?

e) What is the width of the river to the nearest tenth of a metre?

3. Recall the following information from Class Ex. #2 on page 324.

The hockey club had 7 200 season ticket holders who each paid $1 400 for a package of season tickets. The owner had suggested raising the price to generate more revenue, but knew that the number of season ticket holders would be reduced.

The general manager suggested that more revenue might be obtained by decreasing the price and thus attracting more fans to buy a package of season tickets. The research company, that the owner hired to explore the general manager's suggestion, reported that for every $50 decrease in price, approximately 400 new season ticket holders would be generated.

a) If the price decrease is to be a multiple of $50, determine the following.

i) The price of a package of season tickets which would generate maximum revenue.

ii) The number of season ticket holders which would be generated.

iii) The revenue which would be generated if the plan in a) was implemented.

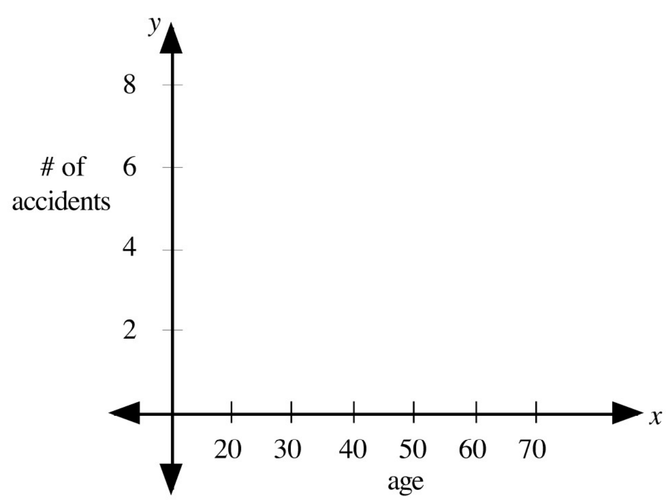
iii) The revenue which would be generated if the plan in a) was implemented.

b) What advice would you give the owner in regards to the direction he should take to obtain maximum revenue?

4. The cost of car insurance depends on many factors, one of which is the age of the driver. Insurance companies know that younger drivers under the age of 25 and older drivers over the age of 70 are statistically more likely to have accidents than drivers between the ages of 25 and 70. The following data shows the number of accidents, per million kilometres driven, by drivers of a particular age.

Age (x)	18	30	45	60	75
Number of Accidents (y)	5.2	3.1	2.2	2.8	4.7

a) If x represents the age of drivers and y represents the number of accidents per million kilometres driven, plot the data on a Cartesian plane, and join the points with a smooth curve.



b) The data looks like it could be modelled by a quadratic function with equation $y = ax^2 + bx + c$. Using the technique of quadratic regression (which is taught in a higher level math course), a teacher determines that the equation which best models the data is

$$y = 0.0034x^2 - 0.3232x + 9.8505$$

Use the above model to determine what age, to the nearest year, results in the lowest number of accidents per million kilometres?

c) Determine the lowest number of accidents per million kilometres. Answer to the nearest tenth.

c) Determine the lowest number of accidents per million kilometres.
Answer to the nearest tenth.

d) Based on this model, who is more likely to have an accident - a 17 year old student or a 78 year old senior?

5. Luigi owns a potato farm in south-eastern B.C. Each year he faces a dilemma as to when to harvest his crop. He knows that if he harvests early the price will be high but his yield will be low, and if he harvests late, the price will be low but the yield will be high.

From past experience, he knows that if he harvests on July 15, he can expect approximately 2000 kg of potatoes which he could sell at \$0.60 per kg.

For each week he waits after July 15, he can expect an extra 400 kg of potatoes, but the price will reduce by \$0.05 per kg.

When should he harvest his crop for maximum revenue?

Use the following information to answer questions #6 and #7.

Researchers predict that the world population will peak sometime during the 21st century before starting to decline. In the year 2000, the world population was approximately 6 100 000 000 (or 6.1 billion).

The following model has been suggested as an approximate relationship between the number of years, x, since the year 2000 and the world population, y.

The equation of the relationship is $y = -595\,000x^2 + 83\,000\,000x + 6\,100\,000\,000$

between the number of years, x, since the year 2000 and the world population, y.

The equation of the relationship is $y = -595\ 000x^2 + 83\ 000\ 000x + 6\ 100\ 000\ 000$

Numerical Response **6.** The world population is expected to peak in the year _____ .

(Record your answer in the numerical response box from left to right.)

7. The maximum population, to the nearest tenth of a billion, is expected to be _____ .

(Record your answer in the numerical response box from left to right.)

Answer Key

1. b) 1 metre **c)** 11 metres **d)** 12.25 metres. It is the y-coordinate of the vertex.
e) 1.50 seconds. It is the x-coordinate of the vertex. **f)** 3.07 seconds **g)** $0 \leq t \leq 3.07$

2. b) 1.5 metres **c)** 1.79 metres **d)** 5.0 metres **e)** 10.0 metres

3. a) i) \$1,150.00 **ii)** 9 200 **iii)** \$10,580 000
b) It would be better to reduce the price to \$1,150 than to increase the price to \$1,600.

4. b) 48 years **c)** 2.2 accidents per million km **d)** both are about equally likely (17 year old is slightly more likely).

5. $3\frac{1}{2}$ weeks after July 15.

6.

2	0	6	9

7.

9	.	0	

Quadratic Functions and Equations Lesson #9:
Applications of Quadratic Functions - An Algebraic Approach

Review

The standard form of quadratic functions is useful to solve, analyze, and interpret problems whose graphical model is parabolic in shape. Complete the following statements for the standard form equation of a parabola $y = a(x - p)^2 + q$.

a) The coordinates of the vertex are (p, q)

b) When a < 0, the maximum value is q. When a > 0, the minimum value is q.
negative

c) The equation of the axis of symmetry is $x = p$

Maximum/Minimum Applications

In this lesson, all the questions are intended to be completed algebraically.

Class Ex. #1

Consider the following information taken from Lesson 8, page 323, Class Ex. #1.

"The height, h, in metres above the ground, of a projectile at any time, t, in seconds after the launch is defined by the function $h(t) = -4t^2 + 48t + 3$."

a) Complete the square to write h in standard form. ✓

$-4(t^2 - 12t + 36 - 36) + 3$ $\quad \frac{-12}{2} = (-6)^2 = 36$

$-4(t-6)^2 + 144 + 3$

$= -4(t-6)^2 + 147$ $\quad$ vertex $(6, 147)$ ← max

b) Find the height of the projectile 3 seconds after the launch.

$-4(3-6)^2 + 147 = -4(-3)^2 + 147 = -4(9) + 147$
$= -36 + 147 = 111\text{ m}$

c) Find the maximum height reached by the projectile.

147m (from vertex)

d) How many seconds after the launch is the maximum height reached?

6 seconds (from vertex)

e) What was the height of the projectile at the launch? $t = 0$

$-4(0-6)^2 + 147 = -144 + 147 = 3\text{m}$

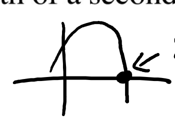
f) Determine when the projectile hits the ground to the nearest tenth of a second.

$0 = -4(t-6)^2 + 147$
$-147 \quad\quad -147$

$-147 = -4(t-6)^2$
$\div -4 \quad \div -4$

$\sqrt{\frac{147}{4}} = \sqrt{(t-6)^2}$

$\pm\sqrt{\frac{147}{4}} = t - 6$



$6 \pm \sqrt{\frac{147}{4}} = t$

$-147 = -4(t-6)^2$

$\frac{-147}{-4} = \frac{-4(t-6)^2}{-4}$

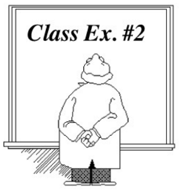
$\pm\sqrt{\frac{147}{4}} = t-6$

$6 \pm \sqrt{\frac{147}{4}} = t$

$t = 12.1$

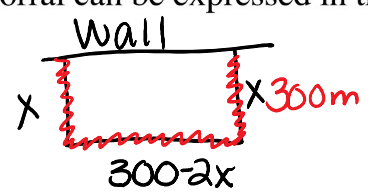
~~$t = -0.1$~~

g) Compare the answers from b) - f) with those on page 323.



A rancher has 300 m of fencing with which to form a rectangular corral (an enclosure for confining livestock), one of whose sides is an existing wall which does not require fencing.

a) If two of the sides of the rectangle are each x metres in length, show that the area of the corral can be expressed in the form $A(x) = 300x - 2x^2$. ✓



3rd side: $300 - x - x$ $= 300 - 2x$

Area: $x(300-2x) = 300x - 2x^2$

b) Use the method of completing the square to determine the maximum area possible.

$-2x^2 + 300x$

$-2(x^2 - 150x + 5625 - 5625)$

$-2(x-75)^2 + 11250$

$\frac{-150}{2} = (-75)^2 = 5625$

vertex $(75, 11250)$

c) State the dimensions of the rectangle which gives the maximum area.

$x = 75$

3rd side $300 - 2x = 300 - 150 = 150$

75 by 150

Ashley was asked by her Math teacher to find two numbers which differ by 8 and whose product is a minimum.

a) If x represents the smaller number, write a quadratic expression in x for the product of the two numbers.

small #: x

big #: $x + 8$

$x(x+8) = x^2 + 8x$

b) Write the product in completed square form.

$x^2 + 8x + 16 - 16$

$(x+4)^2 - 16$

$\frac{8}{2} = (4)^2 = 16$

vertex $(-4, -16)$

$(x+4)^2-16$ vertex $(-4,-16)$

c) Determine the numbers and the minimum product.

$x=-4$

large # $= -4+8 = 4$

-4 & 4

#1,3,5

Complete Assignment Questions #1 - #9

Extension: The Vertex Formula

The coordinates of the vertex of the graph of a quadratic function $f(x) = ax^2 + bx + c$ can be found by completing the square as follows:

$$f(x) = ax^2 + bx + c$$

$$= a\left(x^2 + \frac{b}{a}x \qquad\right) + c$$

$$= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}\right) + c$$

$$= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - \frac{b^2}{4a}$$

$$= a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$$

Vertex $= \left(\frac{-b}{2a}, \frac{4ac - b^2}{4a}\right)$

Maximum / Minimum value $= \frac{4ac - b^2}{4a}$

Use an appropriate procedure to determine the coordinates of the vertex of the graph of each of the following functions. State the maximum or minimum value of each function.

a) $f(x) = 2(x + 5)^2 + 8$

b) $P(x) = -2x^2 + 12x - 13$

Complete Assignment Question #10

Assignment

In this assignment, all the questions are intended to be completed algebraically.

1. At a local golf course, on the par 3 hole, Linda used a seven iron to reach the green. Her golf ball followed the path of a parabola, approximated by the function

$$h(t) = -5t^2 + 25t + 0.05$$

where t is the number of seconds which have elapsed since Linda hit the ball,
and $h(t)$ is the height, in metres, of the ball above the ground after t seconds.

a) Write the function in standard form.

b) Find the height of the golf ball 2 seconds after the ball is hit.

c) Find the maximum height reached by the golf ball.

c) Find the maximum height reached by the golf ball.

d) How many seconds did it take for the golf ball to reach its maximum height?

e) How high, in centimetres, did Linda tee up her golf ball before she hit it?

f) How long, to the nearest tenth of a second, did it take for the golf ball to hit the ground?

2. The sum of a number, x, and its reciprocal is $\frac{29}{10}$.
Form an equation and find the original number.

3. The perimeter of a rectangular plot of land is 84 metres and its area is 320 metres^2. If the length of the plot is represented by x metres, form a quadratic equation in x, and solve it to find the length and width of the plot.

4. The paved walkway from the main school building to the Physical Education block at a school is "L" shaped, with the total distance being 180 metres. A student, taking a short cut diagonally across the grass, shortens the distance to 130 m.

a) Draw a sketch to illustrate this information.

b) If one of the "L" shaped sides has a length of x metres, state the length of the other "L" shaped side in terms of x.

c) Use the Pythagorean Theorem to write a quadratic equation in x. Solve the equation to determine the length of the two legs of the paved walkway. Answer to the nearest tenth of a metre.

5. A stone is thrown vertically upward at a speed of 22 m/s. Its height, h metres, after t seconds, is given approximately by the function $h(t) = 22t - 5t^2$.
Use this formula to find, to the nearest tenth of a second, when the stone is 15 metres up and explain the double answer.

Multiple Choice

6. Two numbers have a difference of 20. When the squares of the numbers are added together, the result is a minimum. The larger of the two numbers is

A. 0

B. 10

C. 20

D. 30

Numerical Response

7. A springboard diver's height, in metres, above the water, is given by the formula

$$h(t) = -5t^2 + 8t + 4$$

where t is the number of seconds which have elapsed since the start of the dive,
and $h(t)$ is the height, in metres of the diver above the water after t seconds.

The time taken, to the nearest tenth of a second, for the diver to enter the water is _____ .

(Record your answer in the numerical response box from left to right.)

8. One positive integer is 3 greater than 4 times another positive integer. If the product of the two integers is 76, then the sum of the two integers is _____ .

(Record your answer in the numerical response box from left to right.)

9. A whole number is multiplied by 5 and added to 3 times its reciprocal to give a sum of 16. The number is _____ .

(Record your answer in the numerical response box from left to right.)

10. Use the vertex formula to determine the coordinates of the vertex of the graph of each of the following functions. State the maximum or minimum value of each function.

a) $f(x) = 5x^2 + 3x - 2$ **b)** $f(x) = -3x^2 - 7x - 1$ **c)** $f(x) = x^2 + 9x + 4$

Answer Key

1. **a)** $h(t) = -5(t - 2.5)^2 + 31.3$ **b)** 30.05 metres **c)** 31.3 metres
d) 2.5 seconds **e)** 5 cm **f)** 5.0 seconds

2. $\frac{2}{5}$ or $\frac{5}{2}$

3. length = 32 metres, width = 10 metres

4. **b)** $(180 - x)$ metres **c)** $2x^2 - 360x + 15500 = 0$, 108.7m and 71.3m

5. 0.8 seconds and 3.6 seconds. There are two answers as the stone goes up and then comes down.

6. B

7.

2	.	0	

8.

2	3		

9.

3			

10. **a)** vertex $\left(-\frac{3}{10}, -\frac{49}{20}\right)$ minimum value is $-\frac{49}{20}$ **b)** vertex $\left(-\frac{7}{6}, \frac{37}{12}\right)$ maximum value is $\frac{37}{12}$

c) vertex $\left(-\frac{9}{2}, -\frac{65}{4}\right)$ minimum value is $-\frac{65}{4}$

c) vertex $\left(-\frac{9}{2}, -\frac{65}{4}\right)$ minimum value is $-\frac{65}{4}$