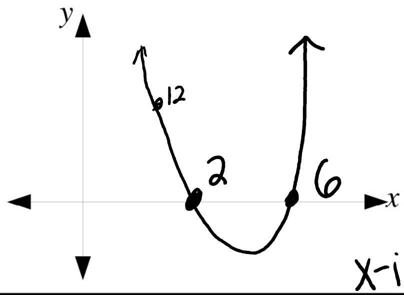
Lesson 7: Roots of Quadratic Equations - The Discriminant

Quadratic Functions and Equations Lesson #7:
Roots of Quadratic Equations - The Discriminant

Review

Find the roots of the quadratic equation $x^2 - 8x + 12 = 0$ by each of the following methods.

i) by graphing (on calculator)



x-ints

ii) by factoring

$x^2 - 8x + 12 = 0$

$(x-6)(x-2) = 0$

$x = 6, 2$

$\times$ | $+$: 12 | −8

zeros

iii) by completing the square

$x^2 - 8x + 16 - 16 + 12 = 0$

$\frac{-8}{2} = -4$

$(-4)^2 = 16$

$(x-4)^2 - 4 = 0$

solve for x

$\sqrt{(x-4)^2} = \pm\sqrt{4}$

$x - 4 = \pm 2$

$x = 4 \pm 2$ < $4+2=6$, $4-2=2$

$x = 6, 2$

iv) by the quadratic formula

$x^2 - 8x + 12 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1 \quad b = -8 \quad c = 12$

$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(12)}}{2(1)}$

$x = \frac{8 \pm \sqrt{64-48}}{2} = \frac{8 \pm \sqrt{16}}{2}$

$x = \frac{8 \pm 4}{2} = \frac{8+4}{2}$ or $\frac{8-4}{2}$ $\quad$ 6, 2

Discuss when each of the following methods might be appropriate or not appropriate for solving a quadratic equation.

- by factoring using inspection or decomposition
  - when in general form and the numbers are factorable $\times$ | $+$
- by quadratic formula
  - when in general form and the #'s are difficult or impossible to factor
- by completing the square
  - when in standard form
- by graphing
  - always, but can't get exact values if the root is irrational ($\sqrt{2}$ for example)

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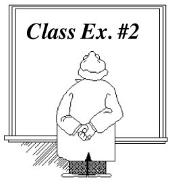


Form a quadratic equation and solve. $\frac{2}{a^2} + \frac{3}{a} = -1, \; a \neq 0$

$\frac{2a^2}{a^2} + \frac{3a^2}{a} = -1a^2$

$2 + 3a = -a^2$

$a^2 + 3a + 2 = 0$

general form ✓

decide how to solve

$(x+1)(x+2)$

$x = -1, -2$

$(0.1x^2 + 0.5x + 1)^{10} = (0)^{10}$

$x^2 + 5x + 10 = 0$

#1-3

Complete Assignment Questions #1 - #3

Extension Enrichment

Investigating the Nature of the Roots of a Quadratic Equation

Insert the missing values.

Equation #1

$x^2 - 6x + 5 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

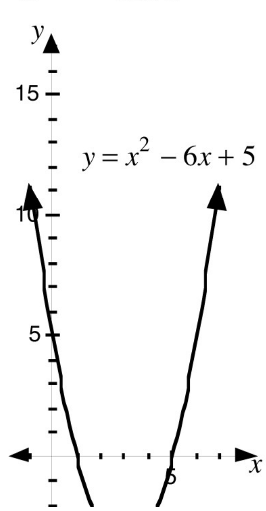
$x = \frac{6 \pm \sqrt{\quad}}{2}$

$= \frac{6 \pm \sqrt{\quad}}{2}$

$= \frac{6 +}{2}$ and $\frac{6 -}{2}$

∴ the roots are

$x =$ and $x =$



Equation #2

$x^2 - 6x + 9 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

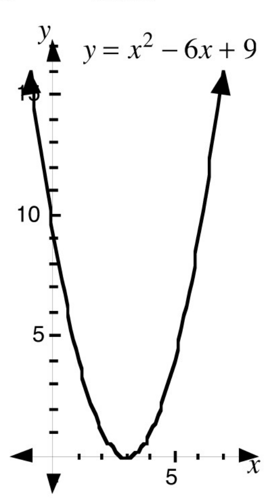
$x = \frac{6 \pm \sqrt{\quad}}{2}$

$= \frac{6 \pm \sqrt{\quad}}{2}$

$= \frac{6 +}{2}$ and $\frac{6 -}{2}$

∴ the roots are

$x =$ and $x =$



Equation #3

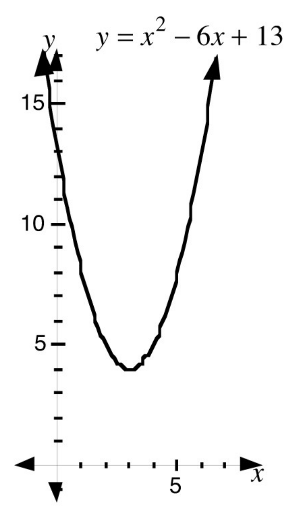
$x^2 - 6x + 13 = 0$

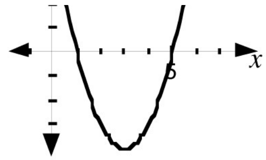
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

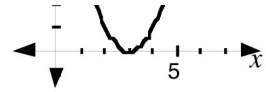
$x = \frac{6 \pm \sqrt{\quad}}{2}$

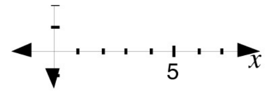
$= \frac{6 \pm \sqrt{\quad}}{2}$

∴ the roots are









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The Nature of the Roots of a Quadratic Equation

The roots of a quadratic equation are represented by the x-intercepts of the graph of the corresponding quadratic function.

The roots of a quadratic equation can be **equal or unequal** and **real or non-real**.

Consider the graphs from the previous page.

- In graph 1 the roots of the equation $x^2 - 6x + 5 = 0$ are real and unequal (distinct).
- In graph 2 the roots of the equation $x^2 - 6x + 9 = 0$ are real and equal.
- In graph 3 the roots of the equation $x^2 - 6x + 13 = 0$ are non-real.

The Discriminant

The nature of the roots of a quadratic equation can be determined without actually solving the equation or drawing its graph.

The number $b^2 - 4ac$, which appears under the radical symbol in the quadratic formula can be used to discriminate between the different types of roots, and is called the **discriminant**.

$$\text{discriminant} = b^2 - 4ac$$

a) Complete the table using the calculations from the investigation on the previous page.

Equation	**Roots**	**Nature of Roots**	$b^2 - 4ac$
$x^2 - 6x + 5 = 0$			
$x^2 - 6x + 9 = 0$			
$x^2 - 6x + 13 = 0$			

b) Complete the following:

b) Complete the following:

- If the discriminant $b^2 - 4ac = 0$, then the roots are ____________ and ____________ .
- If the discriminant $b^2 - 4ac > 0$, then the roots are ____________ and ____________ .
- If the discriminant $b^2 - 4ac < 0$, then the roots are ____________ .

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Determine the nature of the roots of the following equations without solving or graphing.

a) $6x^2 - x - 1 = 0$ **b)** $x^2 + 16 = 8x$ **c)** $5x^2 + 2x + 1 = 0$.

Determine for what value(s) of m the quadratic equation $x^2 - 8x + m$ has

a) real and distinct roots **b)** real and equal roots **c)** non-real roots

a) State a condition for $b^2 - 4ac$ so that the equation $ax^2 + bx + c = 0$ has real roots.

b) Given that the equation $ax^2 + bx + c = 0$ has real roots, state a condition for $b^2 - 4ac$ so that the roots are: **i)** rational, **ii)** irrational.

c) Show that the roots of the equation $(m - 2)x^2 - (3m - 2)x + 2m = 0$ are always real and

c) Show that the roots of the equation $(m-2)x^2-(3m-2)x+2m=0$ are always real and rational.

Complete Assignment Questions #4 - #12

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Assignment

1. Form a quadratic equation and solve. Answer to the nearest tenth.

a) $x+\frac{1}{x}=3,\ x\neq 0$

b) $(2x-1)(3x+2)=(x+3)(2x+1)$

2. Form a quadratic equation and solve. Give answers as exact values in simplest form.

a) $\frac{4}{x^2}+\frac{2}{x}=3$

b) $3x(x-4)=8$

c) $3(x-1)(x+2)-(x^2+3)=0$

3. Find a quadratic equation in simplest form which is equivalent to the given equation, but has integral coefficients. Hence find the roots of the given equation to the nearest tenth.

a) $1.4x^2 - 2.8x = 1.8$ ×10   **b)** $\frac{x^2}{2} - x - \frac{5}{4} = 0$

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4. Find the value of the discriminant in each of the following equations.

a) $x^2 + x + 9 = 0$ **b)** $3x^2 - 18x + 27 = 0$

5. Determine the nature of the roots of the following equations without solving or graphing.

a) $2x^2 + 4x + 8 = 0$ **b)** $9x^2 - 24x + 16 = 0$ **c)** $-2x^2 - x + 3 = 0$

d) $-2(x+3)^2+40=0$ **e)** $x^2+10+3x=0$ **f)** $4x^2+4x+1=0$

6. **a)** Determine for what value(s) of d the quadratic equation $5x^2-10x+d=0$ has
i) real and distinct roots **ii)** real and equal roots **iii)** non-real roots

b) Determine an integer value for d such that the equation has rational, non-zero roots.

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7. For what values of n does each equation have real roots?
a) $nx^2-2x+1=0$ **b)** $2x^2+20x+n=0$

8. For what values of a does the equation $ax^2 + (2a - 3)x + a = 0$ have non-real roots?

9. Show that the roots of the equation $x(x - 3) = k^2 - 2, k \in R$, are always real.

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Use the following information for questions #10 and #11.

Rosa was analyzing the following four functions:

I. $f(x) = x^2 - x - 11$ **II.** $f(x) = 2x^2 - x + 3$

III. $f(x) = (3x - 1)(x - 2)(x + 3)$ **IV.** $f(x) = 4x^2 - 12x + 9$

Multiple Choice

10. Which of these functions is a quadratic function with real and equal zeros?
A. I
B. II
C. III
D. IV

11. Which of these functions is a quadratic function with no real zeros?
A. I
B. II
C. III
D. IV

Numerical Response

12. The discriminant for the quadratic equation $3x^2 - 8 - 7x = 0$ is _____ .

(Record your answer in the numerical response box from left to right.)

Answer Key

1. a) 0.4, 2.6 **b)** –0.6, 2.1

2. a) $\frac{1 \pm \sqrt{13}}{3}$ **b)** $\frac{6 \pm 2\sqrt{15}}{3}$ **c)** $-3, \frac{3}{2}$

3. a) $7x^2 - 14x - 9 = 0$, –0.5, 2.5 **b)** $2x^2 - 4x - 5 = 0$, –0.9, 2.9

4. a) –35 **b)** 0

5. a) non-real **b)** real and equal **c)** real and unequal
d) real and unequal **e)** non-real **f)** real and equal

5. **a)** non-real **b)** real and equal **c)** real and unequal
d) real and unequal **e)** non-real **f)** real and equal

6. **a)** **i)** $d < 5$ **ii)** $d = 5$ **iii)** $d > 5$ **b)** $d = -15$ or -40 or -75, etc.

7. **a)** $n \leq 1$ **b)** $n \leq 50$

8. $a > \frac{3}{4}$

9. $b^2 - 4ac = 1 + 4k^2$ which is always positive.

10. D

11. B

12.

1	4	5	

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