Quadratic Functions and Equations Lesson #6:
Solving Quadratic Equations - The Quadratic Formula

In previous lessons we have determined the roots of quadratic equations by graphing, and by factoring using inspection or decomposition.

In this lesson we will use

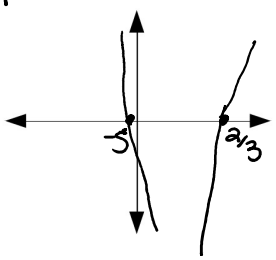
- **the square root method**
- **the completing the square method**
- **the quadratic formula**

to solve quadratic equations.

Review

Solve the equation $3x^2 + 13x - 10 = 0$ by graphing. ← calculator
Use the sketch to illustrate your solution.

$x_{int} = -5, \frac{2}{3}$



Solve the following equations by factoring.

a) $x^2 + 7x - 18 = 0$ $\begin{array}{c|c} x & t \\ \hline -10 & 7 \end{array}$

$(x+9)(x-2)=0$

$x = -9, 2$

b) $3x^2 + 13x - 10 = 0$ $\begin{array}{c|c} x & t \\ \hline -30 & 13 \end{array}$

$3x^2+15x-2x-10=0$

$3x(x+5)-2(x+5)=0$

$(x+5)(3x-2)=0$

$x = -5, \frac{2}{3}$

The Square Root Method

The solution to the equation $x^2 = k$ is found by taking the square root of each side to get $x = \pm\sqrt{k}$.

Use the square root method to solve the following quadratic equations.

a) $3x^2 = 27$

$\frac{3}{3} \quad \frac{3}{3}$

$\sqrt{x^2} = \pm\sqrt{9}$

$x = \pm 3$

$x = 3, -3$

b) $(2x-1)^2 = 64$ $\pm\sqrt{\ }$

$2x - 1 = \pm 8$

$2x-1=8$ or $2x-1=-8$

$2x = 9$ $\quad 2x = -7$

$x = 9/2$ $\quad x = -7/2$

$x = 9/2, -7/2$

c) $(x+3)^2 - 8 = 0$

Completing the Square Method

This is an extension of the method used in Class Ex. #3c).

a) Explain why the quadratic equation $3x^2 - 12x - 8 = 0$ cannot be solved by factoring.

can't find #'s that work

$\begin{array}{c|c} x & + \\ \hline -24 & -12 \end{array}$

$1 \cdot 24$
$2 \cdot 12$
$3 \cdot 8$
$4 \cdot 6$

b) Solve the equation $3x^2 - 12x - 8 = 0$ by expressing the left side in completed square form and then using the square root method to complete solving the equation.

$3x^2 - 12x - 8$

$3(x^2 - 4x + 4 - 4) - 8$

$= 3(x-2)^2 - 12 - 8$

$= 3(x-2)^2 - 20$

$\frac{-4}{2} = (-2)^2 = 4$

solve for x

$3(x-2)^2 - 20 = 0$

$3(x-2)^2 = 20$

$\sqrt{(x-2)^2} = \pm\sqrt{\frac{20}{3}}$

$x - 2 = \pm\sqrt{\frac{20}{3}}$

$x = 2 \pm \sqrt{\frac{20}{3}}$

$x = 2 + \sqrt{\frac{20}{3}}, \; 2 - \sqrt{\frac{20}{3}}$

Complete Assignment Questions #1 - #4

Complete Assignment Questions #1 - #4

Developing the Quadratic Formula

We will use the completing the square method to develop a formula that can be used to solve any quadratic equation of the form $ax^2 + bx + c = 0$.

Solve the following equations by completing the square.

a) $2x^2 - 8x + 5 = 0$ **b)** $ax^2 + bx + c = 0$

The solution to Class Ex. #1b) is a formula which can be used to solve any quadratic equation of the form $ax^2 + bx + c = 0$. The formula is known as the **quadratic formula**.

Solving a quadratic equation by completing the square is rarely used as the quadratic formula is usually a more efficient method.

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The Quadratic Formula

The quadratic equation $ax^2 + bx + c = 0, a \neq 0$ has the roots

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Find the roots of the following equations using the quadratic formula.
Give answers as exact values in simplest form and to the nearest tenth.

a) $x^2 + 2x - 1 = 0$

$a=1 \quad b=2 \quad c=-1$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-2 \pm \sqrt{2^2-4(1)(-1)}}{2(1)}$

$= \frac{-2 \pm \sqrt{4+4}}{2}$

$= \frac{-2 \pm \sqrt{8}}{2}$ → $\sqrt{4 \cdot 2} = 2\sqrt{2}$

$= \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$

$x = -1 \pm \sqrt{2}$ exact value

$x = -1 + \sqrt{2} = 0.4$

$x = -1 - \sqrt{2} = -2.4$

$x = 0.4, -2.4$ nearest tenth

b) $4x^2 - 12x + 3 = 0$

$a=4 \quad b=-12 \quad c=3$

$x = \frac{-(-12) \pm \sqrt{(-12)^2-4(4)(3)}}{2(4)}$

$x = \frac{12 \pm \sqrt{144-48}}{8}$

$x = \frac{12 \pm \sqrt{96}}{8}$ → $\sqrt{16 \cdot 6}$, $4\sqrt{6}$

$\sqrt{96}$: $2 \cdot 48$, $2 \cdot 24$, $2 \cdot 12$, $2 \cdot 6$, $2 \cdot 3$

$x = \frac{12 \pm 4\sqrt{6}}{8}$

$x = \frac{3 \pm \sqrt{6}}{2}$

$x = \frac{3+\sqrt{6}}{2} = 2.7$

$x = \frac{3-\sqrt{6}}{2} = 0.3$

$x = 0.3, 2.7$

c) $4x^2 = 3(4x + 5)$

$4x^2 = 12x + 15$

$4x^2 - 12x - 15 = 0$

$a=4 \quad b=-12 \quad c=-15$

$x = \frac{-(-12) \pm \sqrt{(-12)^2-4(4)(-15)}}{2(4)}$

$x = \frac{12 \pm \sqrt{144+240}}{8}$ ← if under radical is negative, then x has no solutions

$x = \frac{12 \pm \sqrt{384}}{8}$ → $\sqrt{64 \cdot 6}$

$x = \frac{12 \pm 8\sqrt{6}}{8}$

$x = \frac{3 \pm 2\sqrt{6}}{2}$

$x = (3 + 2\sqrt{6}) \div 2 = 3.9$

$x = (3 - 2\sqrt{6}) \div 2 = -0.9$

#1, 5, 6, 7, 8ac, 9ab, 10

tomorrow: L7/8
mon: L9
Tues: Quiz L5-9
Wed: Review *
Thurs: Unit test

Find the zeros of the quadratic function $f(x) = -3x^2 + 4x + 1$.
Give answers as exact values in simplest form and to the nearest hundredth.

Complete Assignment Questions #5 - #13

Assignment

1. Solve the equation $2x^2 - 32 = 0$ by

a) graphing **b)** factoring **c)** the square root method

2. Solve the following equations by the square root method. Answer using exact values in simplified form.

a) $4x^2 = 9$ **b)** $3x^2 - 12 = 0$ **c)** $\frac{1}{2}x^2 - 12 = 0$

d) $(x - 5)^2 = 36$ **e)** $2(1 - x)^2 = 32$ **f)** $3(2x + 1)^2 = 24$

3. Use the completing the square method to solve the following quadratic equations. Answer using exact values.

a) $x^2 + 6x + 3 = 0$ **b)** $4x^2 - 8x - 5 = 0$ **c)** $-4x^2 + 16x - 15 = 0$

Answer using exact values.

a) $x^2 + 6x + 3 = 0$ **b)** $4x^2 - 8x - 5 = 0$ **c)** $-4x^2 + 16x - 15 = 0$

4. Determine, to the nearest tenth, the x–intercepts of the graph of the function $f(x) = x^2 - 10x + 1$

5. Solve the equation $x^2 - 3x - 10 = 0$ by using

a) inspection **b)** the quadratic formula

6. Solve the equation $4x^2 - 11x - 3 = 0$ by using

a) decomposition **b)** the quadratic formula

7. Find the exact roots of the equation $6x^2 + 5x + 1 = 0$ by using

a) graphing **b)** the quadratic formula

8. Find the roots of the following quadratic equations (to the nearest tenth) using the quadratic formula.

a) $2x^2 + x - 4 = 0$ **b)** $2x^2 - 3x - 4 = 0$ **c)** $10t^2 = 7t + 1$

9. Solve the following quadratic equations (as exact values) using the quadratic formula.

a) $x^2 - 10x - 15 = 0$ **b)** $x^2 + 6x + 17 = 0$ **c)** $3x^2 - 12x + 11 = 0$

10. Find the zeros of the following quadratic functions.
Give answers as exact values in simplest form and to the nearest hundredth.

a) $f(x) = x^2 + 20x + 15$ **b)** $f(x) = 5x^2 + 12x - 5$

Multiple Choice **11.** The roots of the quadratic equation $dx^2 + ex + f = 0$ are

A. $x = \dfrac{e \pm \sqrt{e^2 - 4df}}{2d}$ **B.** $x = \dfrac{-e \pm \sqrt{e^2 - 4df}}{2d}$

C. $x = \dfrac{e \pm \sqrt{e^2 + 4df}}{2d}$ **D.** $x = \dfrac{-e \pm \sqrt{e^2 + 4df}}{2d}$

12. The zeros of the quadratic function $f(x) = 6x^2 + 2x - 1$ are

A. $\dfrac{-1 \pm \sqrt{14}}{6}$

_ $-1 \pm 2\sqrt{7}$

A. $\frac{6}{6}$

B. $\frac{-1 \pm 2\sqrt{7}}{6}$

C. $\frac{-1 \pm \sqrt{7}}{6}$

D. $\frac{-2 \pm \sqrt{7}}{6}$

Numerical Response

13. The quadratic equation $2x^2 + 15x + p = 0$ has a root of $-\frac{1}{2}$ when p has the whole number value of _____ .

(Record your answer in the numerical response box from left to right.)

Answer Key

1. ± 4 **2.** **a)** $x = \pm \frac{3}{2}$ **b)** $x = \pm 2$ **c)** $x = \pm 2\sqrt{6}$ **d)** $x = -1, 11$ **e)** $x = -3, 5$ **f)** $x = -\frac{1}{2} \pm \sqrt{2}$

3. **a)** $-3 \pm \sqrt{6}$ **b)** $x = -\frac{1}{2}, \frac{5}{2}$ **c)** $\frac{3}{2}, \frac{5}{2}$ **4.** 0.1, 9.9

5. **a)** –2, 5 **b)** –2, 5 **6.** **a)** $-\frac{1}{4}, 3$ **b)** $-\frac{1}{4}, 3$

7. **a)** $-\frac{1}{3}, -\frac{1}{2}$ **b)** $-\frac{1}{3}, -\frac{1}{2}$

8. **a)** –1.7, 1.2 **b)** –0.9, 2.4 **c)** –0.1, 0.8 **9.** **a)** $5 \pm 2\sqrt{10}$ **b)** no solution **c)** $\frac{6 \pm \sqrt{3}}{3}$

10. **a)** $-10 \pm \sqrt{85}$ –0.78, –19.22 **b)** $\frac{-6 \pm \sqrt{61}}{5}$ –2.76, 0.36

11. B **12.** C **13.**

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