

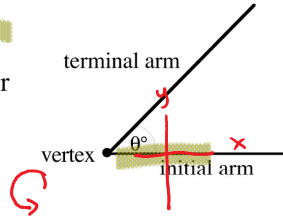
Trigonometry - Angles and Ratios Lesson #1: Rotation Angles and Reference Angles

Angles in Standard Position

Angles can be measured in degrees where 360° is one complete rotation.

A **rotation angle** is formed by rotating an **initial arm** (or initial side) through an angle θ° about a fixed **point** (the vertex).

The angle formed between the initial arm and the terminal arm (or terminal side) is the rotation angle.



A **positive angle** results from a counter clockwise rotation.



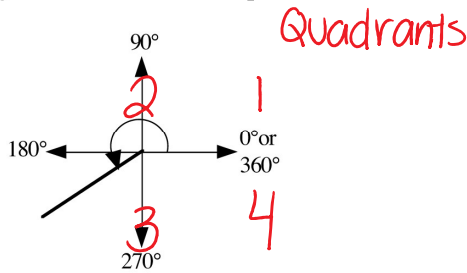
A **negative angle** results from a clockwise rotation.



The angle shown in the above diagram is said to be in **standard position**.

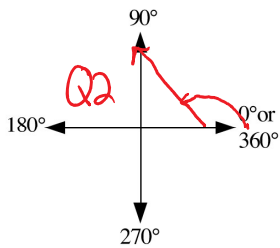
On a coordinate grid, standard position means the initial arm is along the positive x -axis and the rotation is about the origin.

The diagram below shows an angle of 220° in standard position.

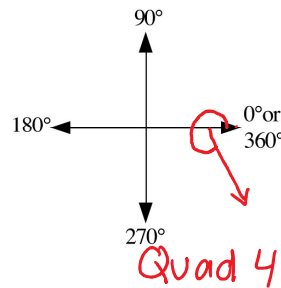


Sketch the rotation angle in standard position and state the quadrant in which the angle terminates.

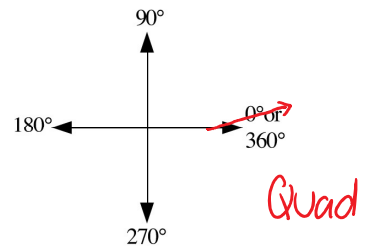
a) 120°



b) 309°



c) 17°



Copyright © by Absolute Value Publications. This book is **NOT** covered by the Copyright agreement.

160 Trigonometry - Angles and Ratios Lesson #1: *Rotation Angles and Reference Angles*



Draw the rotation angle in standard position.

a)

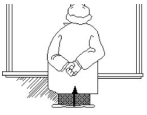


b)

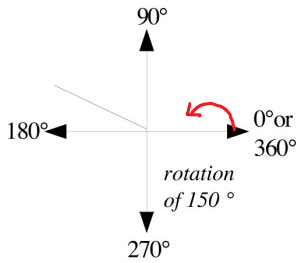


c)

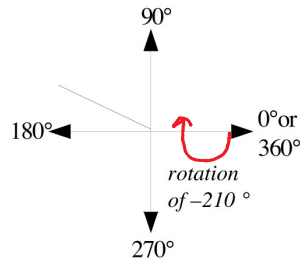




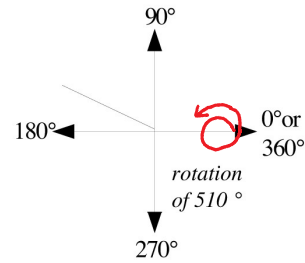
a)



b)



c)



Angles with the same terminal arm are called **coterminal angles**.

Since 150° is the measure of the smallest positive rotation angle coterminal with the angles in Class Example #2, it is called the **principal angle**.

The principal angle will always have a measure between 0° and 360° .

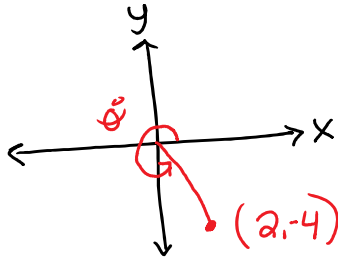
There are infinitely many angles that are coterminal with a given angle.



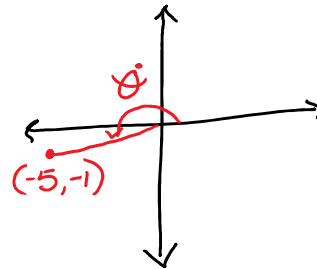
Class Ex. #3

The point P lies on the terminal arm of the angle θ° . Draw the angle θ° in standard position.

a) $P(2, -4)$



b) $P(-5, -1)$



Complete Assignment Question #1 - #2

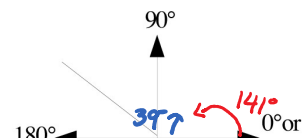
Copyright © by Absolute Value Publications. This book is **NOT** covered by the Cancopy agreement.

Trigonometry - Angles and Ratios Lesson #1: *Rotation Angles and Reference Angles* **161**

Reference Angles

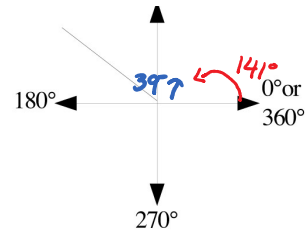
A **reference angle** is the acute angle formed between the terminal arm of the rotation angle and the x -axis.

The diagram shows the terminal arm of a rotation angle of 141°



A **reference angle** is the acute angle formed between the terminal arm of the rotation angle and the x -axis.

The diagram shows the terminal arm of a rotation angle of 141° with a reference angle of 39° .



Mark 141° and 39° on the diagram.

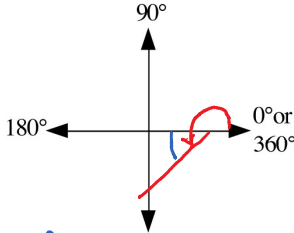
↑ rotation angle
↑ reference angle



Class Ex. #4

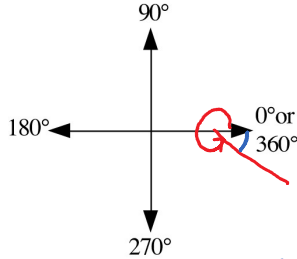
In each case, sketch the rotation angle and state the reference angle.

a) 243°



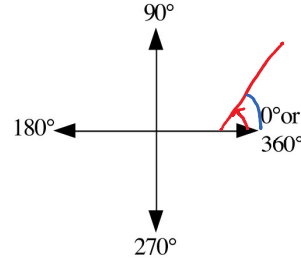
reference angle
 $243 - 180 = 63^\circ$

b) 337°



reference angle
 $360 - 337 = 23^\circ$

c) 70°



reference angle 70°

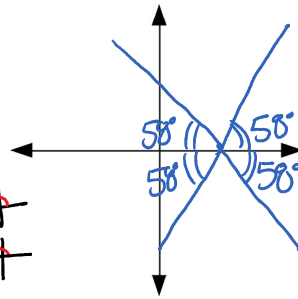


Class Ex. #5

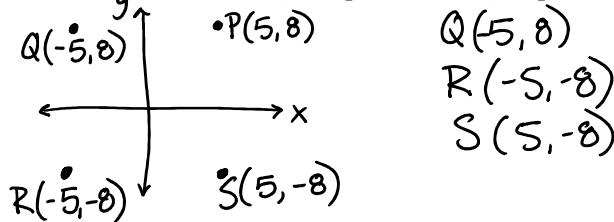
a) On the grid, draw a reference angle of 58° in each of quadrants one to four.

b) State the measure of the rotation angle in each quadrant.

Quadrant 1: 58°
 Quadrant 2: $180 - 58 = 122^\circ$
 Quadrant 3: $180 + 58 = 238^\circ$
 Quadrant 4: $360 - 58 = 302^\circ$



c) Let $P(5, 8)$ be a point on the terminal arm of the rotation angle in quadrant one. State the coordinates of points Q , R , and S which are on the terminal arms of the rotation angles in quadrant two, quadrant three, and quadrant four, respectively.



Copyright © by Absolute Value Publications. This book is **NOT** covered by the Cancopy agreement.

162 Trigonometry - Angles and Ratios Lesson #1: Rotation Angles and Reference Angles



Class Ex. #6

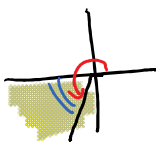
Determine the measure of the rotation angle, x , $0^\circ \leq x < 360^\circ$, given the reference angle and the quadrant.

Reference Angle	Quadrant	Sketch	Rotation Angle
25°	2		$180 - 25 = 155^\circ$
60°	4		$360 - 60 = 300^\circ$

60°	4		$360^\circ - 60^\circ = 300^\circ$
8°	3		$180^\circ + 8^\circ = 188^\circ$
39°	1		39°
90°	between 3 and 4		$180^\circ + 90^\circ = 270^\circ$



Determine three angles between 0° and 360° which have the same reference angle as a rotation angle of 256° .



Ref. angle $256 - 180 = 76^\circ$

Quad 1: 76°

Quad 2: $180 - 76^\circ = 104^\circ$

Quad 4: $360 - 76 = 284^\circ$

Complete Assignment Question #3 - #19

Assignment

#1-9, 13

12 is great practice!

1. Sketch the following rotation angles in standard position, and state the quadrant in which the angle terminates.

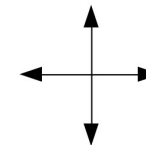
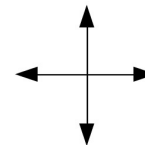
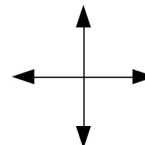
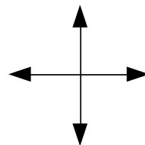
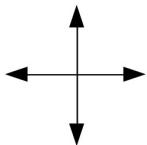
a) 135°

b) 300°

c) 190°

d) 70°

e) 270°



2. In each case, the given point is on the terminal arm of an angle of θ° . Draw the angle θ° in standard position.

a) $P(7, -4)$

b) $Q(-2, 3)$

c) $R(-1, -4)$

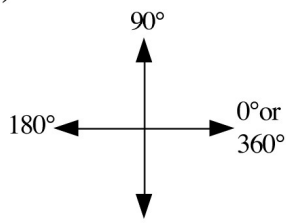
Copyright © by Absolute Value Publications. This book is **NOT** covered by the Cancopy agreement.

Trigonometry - Angles and Ratios Lesson #1: Rotation Angles and Reference Angles

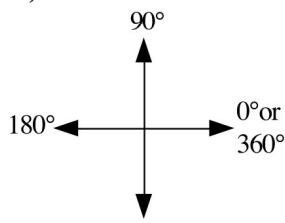
163

3. In each case, sketch the rotation angle and state the reference angle.

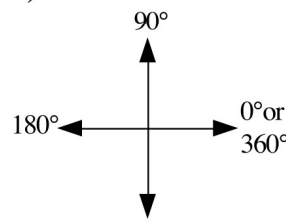
a) 230°

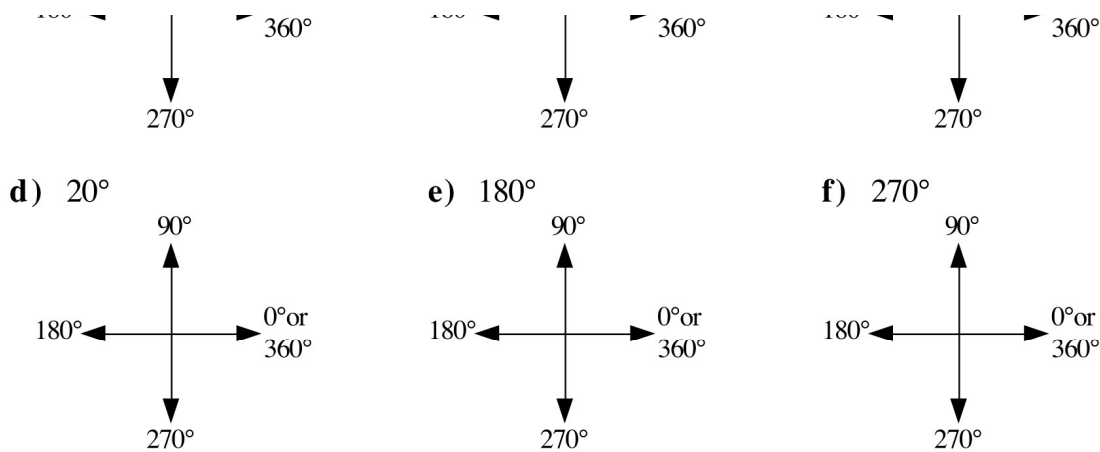


b) 313°



c) 109°





4. Find the reference angle for the following rotation angles.

- a) 135° b) 296° c) 237° d) 90°

5. For each of the following angles, determine
 (i) the quadrant of the terminal arm of the angle

(ii) the reference angle

- a) 355° b) 170° c) 190° d) 51°

6. a) Sketch a diagram to show a reference angle of 30° in each of quadrants one to four.

b) State the measure of the rotation angle in each quadrant.

c) Let $P\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ be a point on the terminal arm of the rotation angle in quadrant one.

State the coordinates of points Q , R and S which are on the terminal arms of the rotation angles in quadrant two, quadrant three, and quadrant four, respectively.

Copyright © by Absolute Value Publications. This book is **NOT** covered by the Cancopy agreement.

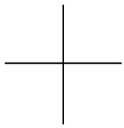
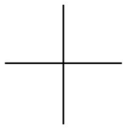
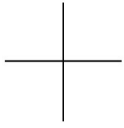
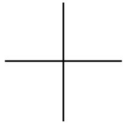
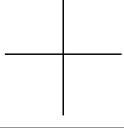
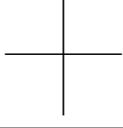
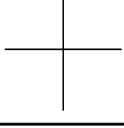
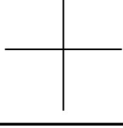
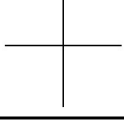
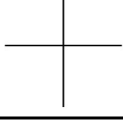
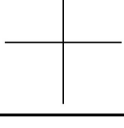
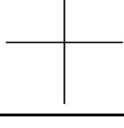
164 Trigonometry - Angles and Ratios Lesson #1: *Rotation Angles and Reference Angles*

7. a) Sketch a diagram to show a reference angle of 77° in each of quadrants one to four.

b) State the measure of the rotation angle in each quadrant.

c) Let $P(a, b)$ be a point on the terminal arm of the rotation angle in quadrant one. State the coordinates of points Q , R , and S which are on the terminal arms of the rotation angles in quadrant two, quadrant three, and quadrant four, respectively.

8. Complete the following tables, given the reference angle and the quadrant.

Reference Angle	Quadrant	Sketch	Rotation Angle	Reference Angle	Quadrant	Sketch	Rotation Angle
30°	2			30°	1		
30°	3			30°	4		
60°	1			4°	3		
55°	2			89°	2		
15°	4			0°	between 2 and 3		
76°	3			90°	between 1 and 2		

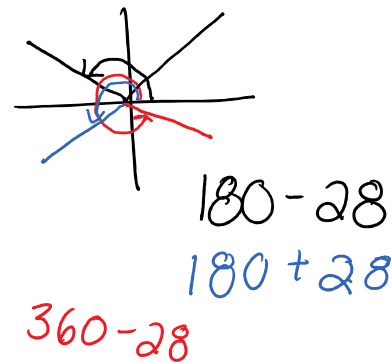
Copyright © by Absolute Value Publications. This book is **NOT** covered by the Cancopy agreement.

9. Students were asked to determine the reference angle for a rotation angle of 214° .

9. Students were asked to determine the reference angle for a rotation angle of 214° .
- Jeff gave an incorrect answer of 56° . Use a diagram to explain how he arrived at his answer.
 - Mandy gave an incorrect answer of 146° . Use a diagram to explain how she arrived at her answer.
 - State the correct answer.
10. Consider a reference angle of 50° in quadrant 1.
- Sketch a diagram to show the reference angle reflected in the y -axis. State the measure of the rotation angle formed.
 - Sketch a diagram to show the original reference angle reflected in the x -axis. State the measure of the rotation angle formed.
 - Sketch a diagram to show the original reference angle reflected in both the x -axis and the y -axis. State the measure of the rotation angle formed.
11. Given that a rotation angle of x° in standard position has a terminal arm in the first quadrant, state expressions for four rotation angles between 0° and 360° which have a reference angle of x° .

12. Complete the following table.

Reference Angle	Rotation Angle in:			
	Quad 1	Quad 2	Quad 3	Quad 4
28°	28°	152°	208°	332°
39°				
a°	a°	$180^\circ - a^\circ$	$180^\circ + a^\circ$	$360^\circ - a^\circ$
	66°	114°	246°	294°
			201°	
				295°



13. Determine three angles between 0° and 360° which have the same reference angle as a rotation angle of 136° .

14. Determine three angles between 0° and 360° which have the same reference angle as a rotation angle of 303° .

Multiple Choice

15. An angle of 134° in standard position has a reference angle of

- A. 44° B. 46°
 C. 134° D. 226°

Use the following information to answer the next question.

Row	Rotation Angle	Quadrant of Terminal Arm	Reference Angle
1	264°	3	96°
2	139°	2	41°
3	357°	4	3°
4	94°	1	86°

16. Which of the following rows contain an error?

- A. row 1 only B. row 4 only
 C. rows 1 and 4 only D. rows 1, 2, 3, and 4

Use the following information to answer the next question.

Meghan makes four statements connecting rotation angles and reference angles.

Statement I: In quadrant 1, the rotation angle is equal to the reference angle.

Statement II: In quadrant 2, the rotation angle is equal to 180° minus the reference angle.

Statement III: In quadrant 3, the rotation angle is equal to 180° plus the reference angle.

Statement IV: In quadrant 4, the rotation angle is equal to 360° minus the reference angle.

17. How many of Meghan's statements are true?

- A. 1 B. 2 C. 3 D. 4

Copyright © by Absolute Value Publications. This book is **NOT** covered by the Cancopy agreement.

18. Which one of the following angles in standard position has the same reference angle as an

18. Which one of the following angles in standard position has the same reference angle as an angle of 165° ?

- A. 25° B. 205°
 C. 255° D. 345°

Numerical Response

19. There are four angles in standard position between 0° and 360° which have a reference angle of 51° . The sum of these angles is _____.

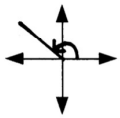
(Record your answer in the numerical response box from left to right.)

--	--	--	--

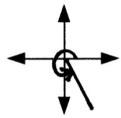
Answer Key

1.

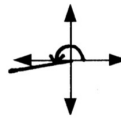
- a) 135° b) 300° c) 190° d) 70° e) 270°



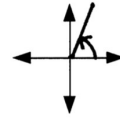
Quadrant 2



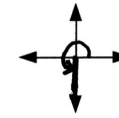
Quadrant 4



Quadrant 3



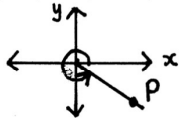
Quadrant 1



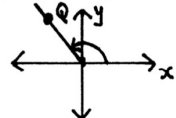
Between Quadrant 3 and Quadrant 4

2.

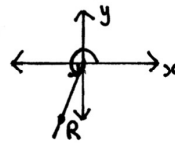
- a) $P(7, -4)$



- b) $Q(-2, 3)$

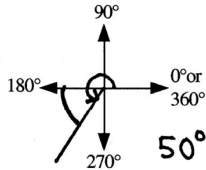


- c) $R(-1, -4)$

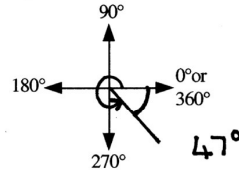


3.

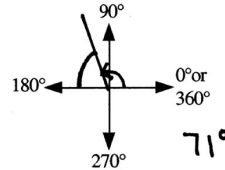
- a) 230°



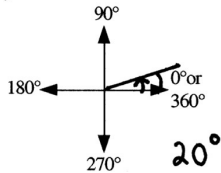
- b) 313°



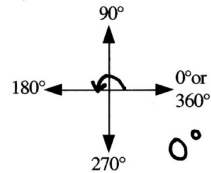
- c) 109°



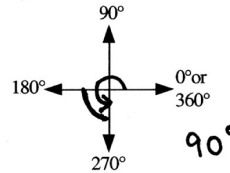
- d) 20°



- e) 180°



- f) 270°

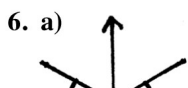


4. a) 45° b) 64° c) 57° d) 90°

5. a) i) 4 ii) 5° b) i) 2 ii) 10° c) i) 3 ii) 10° d) i) 1 ii) 51°

Copyright © by Absolute Value Publications. This book is **NOT** covered by the Cancopy agreement.

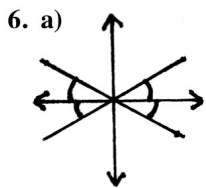
168 Trigonometry - Angles and Ratios Lesson #1: *Rotation Angles and Reference Angles*



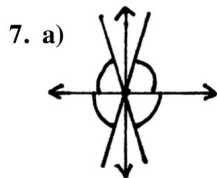
- b) Quadrant 1 $\rightarrow 30^\circ$,
 Quadrant 2 $\rightarrow 150^\circ$,
 Quadrant 3 $\rightarrow 210^\circ$



- b) Quadrant 1 $\rightarrow 77^\circ$,
 Quadrant 2 $\rightarrow 103^\circ$,
 Quadrant 3 $\rightarrow 257^\circ$



- b) Quadrant 1 → 30°
 Quadrant 2 → 150°
 Quadrant 3 → 210°
 Quadrant 4 → 330°

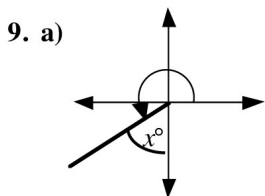


- b) Quadrant 1 → 77°
 Quadrant 2 → 103°
 Quadrant 3 → 257°
 Quadrant 4 → 283°

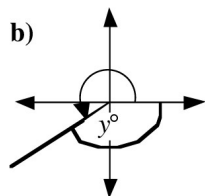
c) $Q\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right) R\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right) S\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

c) $Q(-a, b) R(-a, -b) S(a, -b)$

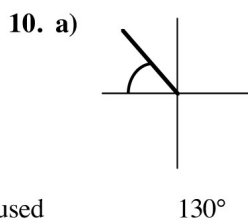
Reference Angle	Quadrant	Sketch	Rotation Angle	Reference Angle	Quadrant	Sketch	Rotation Angle
30°	2		150°	30°	1		30°
30°	3		210°	30°	4		330°
60°	1		60°	4°	3		184°
55°	2		125°	89°	2		91°
15°	4		345°	0°	between 2 and 3		180°
76°	3		256°	90°	between 1 and 2		90°



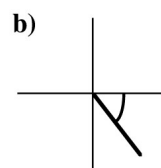
Jeff incorrectly used the angle marked x° as the reference angle



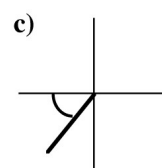
Mandy incorrectly used the angle marked y° as the reference angle



130°



310°



230°

c) 34°

11. $(180 - x)^\circ, (180 + x)^\circ, (360 - x)^\circ$

12.

13. 44°, 224°, 316°

14. 57°, 123°, 237°

15. B 16. C 17. D 18. D

19.

7	2	0	
---	---	---	--

Reference Angle	Rotation Angle in:			
	Quad 1	Quad 2	Quad 3	Quad 4
28°	28°	152°	208°	332°
39°	39°	141°	219°	321°
a°	a°	$(180 - a)^\circ$	$(180 + a)^\circ$	$(360 - a)^\circ$
66°	66°	114°	246°	294°
21°	21°	159°	201°	339°
65°	65°	115°	245°	295°

Copyright © by Absolute Value Publications. This book is **NOT** covered by the Cancopy agreement.