Operations on Radicals Lesson #3:
Dividing Radicals - Part One

Dividing Radicals

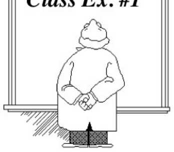
In previous work, we discovered that $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, $a \ge 0$, $b > 0$, and $a, b \in R$.

We can use this rule to divide radicals of the form $\frac{m\sqrt{a}}{n\sqrt{b}}$.

To divide radicals, the index must be the same in each radical.

- Divide numerical coefficients by numerical coefficients.
- Divide radicand by radicand.
- Simplify into mixed radical form if possible.

Class Ex. #1



Divide and simplify where possible.

a) $\frac{\sqrt{30}}{\sqrt{6}}$ $= \sqrt{\frac{30}{6}}$ $= \sqrt{5}$

b) $\frac{8\sqrt[3]{21}}{2\sqrt[3]{3}}$ $= 4\sqrt[3]{7}$

c) $\frac{15\sqrt{48}}{10\sqrt{26}}$ $\frac{3}{2}\sqrt{8}$ or $\frac{3\sqrt{8}}{2}$

d) $\frac{4\sqrt{ab}}{12\sqrt{3a}}$ $\frac{1}{3}\sqrt{b}$ or $\frac{\sqrt{b}}{3}$

In some cases, converting a radical into its simplest mixed radical form before dividing will make the calculation easier.

Class Ex. #2

Simplify numerator and denominator, then divide.

a) $\frac{4\sqrt{54}}{3\sqrt{8}}$ $= \frac{4(3)\sqrt{6}}{3(2)\sqrt{2}}$ $= \frac{12\sqrt{6}}{6\sqrt{2}} = 2\sqrt{3}$

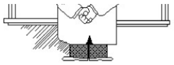
b) $\frac{8\sqrt{126}}{\sqrt{112}}$

c) $\frac{10\sqrt[3]{162}}{20\sqrt[3]{128}}$

Class Ex. #3

Divide each term in the numerator by the denominator, and simplify.

$\frac{\sqrt{24} + \sqrt{48} - \sqrt{108}}{\sqrt{6}}$ $\frac{\sqrt{6}}{\sqrt{6}} = \frac{\quad}{6}$



$\sqrt{6}$

Complete Assignment Questions #1 - #4

Rationalizing the Denominator

Usually answers are written in **simplest form**, e.g. $\frac{1}{6}+\frac{1}{3}=\frac{3}{6}$ which simplifies to $\frac{1}{2}$.

In the division of radicals in this unit, regard simplest form as the form in which

i) the denominator of the fraction is a rational number, i.e. it does not contain a radical

ii) the radicand cannot contain a fraction and is expressed in simplest mixed form

The process of eliminating the radical from the denominator (i.e. converting the denominator from an irrational number to a rational number) is called **rationalizing the denominator**. The denominators in this lesson are all of monomial form. Denominators in binomial form will be discussed in the next lesson.

Simplify by rationalizing the denominator.

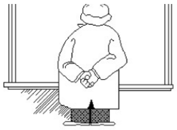
a) $\frac{1}{\sqrt{13}}$ **b)** $\frac{\sqrt{5}}{\sqrt{2}}$ **c)** $\frac{\sqrt{2}}{-\sqrt{6}}$ **d)** $\frac{\sqrt{20}}{\sqrt{3}}$

Simplify.

a) $\frac{7}{3\sqrt{7}}$ **b)** $\sqrt{\frac{18}{5}}$ **c)** $\frac{3\sqrt{12}}{\sqrt{72}}$

Simplify the radical expression $\frac{3\sqrt{18}-\sqrt{12}}{\sqrt{2}}$ by

a) rationalizing the denominator **b)** dividing numerator and denominator by $\sqrt{2}$



a) rationalizing the denominator

$= \frac{3\sqrt{18} - \sqrt{12}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{3\sqrt{36} - \sqrt{24}}{2}$

$= \frac{3(6) - 2\sqrt{6}}{2} = \frac{18 - 2\sqrt{6}}{2} = 9 - \sqrt{6}$

b) dividing numerator and denominator by $\sqrt{2}$

$= 3\sqrt{9} - \sqrt{6}$

$= 3(3) - \sqrt{6}$

$= 9 - \sqrt{6}$

Complete Assignment Questions #5 - #16

Quiz Tuesday

Assignment

1. Simplify.

a) $\frac{\sqrt{50}}{\sqrt{5}}$ **b)** $\frac{\sqrt{35}}{\sqrt{7}}$ $= \sqrt{\frac{35}{7}} = \sqrt{5}$ **c)** $\frac{\sqrt[3]{39}}{\sqrt[3]{3}}$ $= \sqrt[3]{13}$ **d)** $\frac{\sqrt{28}}{\sqrt{7}}$ **e)** $\frac{\sqrt{ab}}{\sqrt{b}}$

f) $\frac{8\sqrt{42}}{2\sqrt{6}}$ **g)** $\frac{25\sqrt{88}}{5\sqrt{8}}$ $= 5\sqrt{11}$ **h)** $\frac{12\sqrt[4]{51}}{-6\sqrt[4]{17}}$ **i)** $\frac{4\sqrt{50}}{8\sqrt{10}}$ **j)** $\frac{6\sqrt{xy^2}}{15\sqrt{xy}}$

2. Simplify.

a) $\frac{\sqrt{270}}{\sqrt{10}}$ **b)** $\frac{\sqrt{90}}{\sqrt{5}}$ **c)** $\frac{\sqrt{96}}{4\sqrt{3}}$ **d)** $\frac{3\sqrt{200}}{2\sqrt{5}}$ $\frac{3}{2}\sqrt{40}$ (4, 10) **e)** $\frac{4\sqrt[3]{144}}{\sqrt[3]{9}}$

$\frac{3}{2}\sqrt{40}$

$= \frac{3}{2} 2\sqrt{10} = \frac{6}{2}\sqrt{10}$

$= 3\sqrt{10}$

3. Simplify.

a) $\dfrac{2\sqrt{150}}{\sqrt{8}}$ **b)** $\dfrac{4\sqrt{90}}{\sqrt{72}}$ **c)** $\dfrac{3\sqrt{240}}{\sqrt{108}}$ **d)** $\dfrac{18\sqrt{24}}{\sqrt{162}}$ **e)** $\dfrac{3\sqrt[3]{32}}{2\sqrt[3]{216}}$

4. Simplify.

a) $\dfrac{\sqrt{35} - \sqrt{21}}{\sqrt{7}}$ **b)** $\dfrac{9\sqrt{20} - 3\sqrt{10}}{3\sqrt{2}}$ **c)** $\dfrac{8\sqrt{42} + 12\sqrt{75}}{4\sqrt{3}}$

$= 3\sqrt{10} - \sqrt{5}$

d) $\dfrac{8\sqrt{20} + 10\sqrt{125}}{2\sqrt{5}}$ **e)** $\dfrac{\sqrt{75} + \sqrt{48} - \sqrt{27}}{\sqrt{3}}$ **f)** $\dfrac{\sqrt{90} + 2\sqrt{40} - \sqrt{160}}{\sqrt{5}}$

5. Simplify by rationalizing the denominator.

a) $\dfrac{1}{\sqrt{2}}$ **b)** $\dfrac{6}{\sqrt{6}}$ **c)** $\dfrac{\sqrt{5}}{\sqrt{3}}$ **d)** $\dfrac{\sqrt{3}}{-\sqrt{2}}$

e) $\dfrac{\sqrt{10}}{\sqrt{7}}$ **f)** $\dfrac{\sqrt{12}}{\sqrt{5}}$ **g)** $\dfrac{2}{5\sqrt{6}} \cdot \dfrac{\sqrt{6}}{\sqrt{6}}$ **h)** $\dfrac{\sqrt{32}}{\sqrt{18}}$

$= \dfrac{2\sqrt{6}}{5(6)} = \dfrac{2\sqrt{6}}{30}$

$= \dfrac{\sqrt{6}}{15}$ or $\dfrac{1}{15}\sqrt{6}$

i) $\dfrac{5}{\sqrt{50}}$ **j)** $\dfrac{14}{\sqrt{98}}$ **k)** $\dfrac{-2}{\sqrt{88}}$ **l)** $\dfrac{3\sqrt{500}}{-\sqrt{27}}$

6. Simplify

6. Simplify.

a) $\sqrt{\frac{27}{10}}$ **b)** $\frac{5\sqrt{14}}{\sqrt{70}}$ **c)** $\sqrt{\frac{243}{2}}$ **d)** $\frac{20\sqrt{12}}{12\sqrt{20}}$

7. Express the following with rational denominators.

a) $\frac{\sqrt{7}-\sqrt{2}}{\sqrt{2}}$ **b)** $\frac{\sqrt{3}+2\sqrt{2}}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ **c)** $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{6}}$

$= \frac{3+2\sqrt{6}}{2(3)} = \frac{3+2\sqrt{6}}{6}$

$= \frac{1}{2} + \frac{\sqrt{6}}{3}$ or $\frac{1}{2} + \frac{1}{3}\sqrt{6}$

8. a) Students are asked to simplify the radical expression $\frac{6\sqrt{40}-8\sqrt{20}}{2\sqrt{5}}$.

Erica decides to simplify the expression by rationalizing the denominator, whereas Jaclyn divides each term in the numerator by the denominator. Determine the simplification by each method, and state which method you prefer.

b) Without doing the simplification, **explain** why Jaclyn's method would be more difficult if the radical expression was $\dfrac{6\sqrt{40} - 8\sqrt{20}}{2\sqrt{7}}$.

9. Simplify and express in lowest terms.

a) $\dfrac{10\sqrt{18} - 5\sqrt{24}}{\sqrt{5}}$

b) $\dfrac{15\sqrt{18} - 3\sqrt{242}}{-3\sqrt{8}}$

10. A rectangular garden has length $3\sqrt{6}$ metres and area $\left(9\sqrt{2} - 6\sqrt{3}\right)$ square meters.

a) Write and simplify an expression for the width of the garden.

b) Determine the perimeter of the garden to the nearest tenth of a metre.

11. [illegible]

11. A triangle has an area of $\left(3\sqrt{288} - 2\sqrt{12}\right)$ square metres with a base of $3\sqrt{2}$ metres. Express the height of the triangle

a) as an exact value in simplest form **b)** as a decimal to the nearest 0.01 m

Multiple Choice

12. Without using technology, determine which of the following expressions is **not** equivalent to the others.

A. $\dfrac{36}{\sqrt{48}}$

B. $\left(\sqrt{3}\right)^3$

C. $\sqrt{192} - \sqrt{75}$

D. $\dfrac{\sqrt{54}}{\sqrt{3}}$

13. $\dfrac{2+\sqrt{8}}{2}$ can be simplified to

A. $1+\sqrt{8}$ **B.** $1+\sqrt{6}$

C. $1+\sqrt{4}$ **D.** $1+\sqrt{2}$

14. If $\sqrt{10} \times \sqrt{12}$ ... $= 2\sqrt{t}$, then t is equal to

14. If $\dfrac{\sqrt{10} \times \sqrt{12}}{\sqrt{6}} = 2\sqrt{t}$, then t is equal to

A. $\sqrt{5}$
B. $\sqrt{10}$
C. 5
D. 10

Numerical Response **15.** The expression $\dfrac{1}{\sqrt{27}} - \dfrac{5\sqrt{3}}{4\sqrt{24}}$ can be written in the form $a\sqrt{3} - b\sqrt{2}$, $a, b > 0$.
To the nearest hundredth, the value of b is _____.
(Record your answer in the numerical response box from left to right.)

16. When the equation $\sqrt{2} + a\sqrt{5} = \sqrt{72}$ is solved for a, the solution is $a = \sqrt{t}$, where $t \in W$. The value of t is _____.
(Record your answer in the numerical response box from left to right.)

Answer Key

1. a) $\sqrt{10}$ **b)** $\sqrt{5}$ **c)** $\sqrt[3]{13}$ **d)** 2 **e)** $\sqrt{a}$
f) $4\sqrt{7}$ **g)** $5\sqrt{11}$ **h)** $-2\sqrt[4]{3}$ **i)** $\frac{1}{2}\sqrt{5}$ **j)** $\frac{2}{5}\sqrt{y}$

2. a) $3\sqrt{3}$ **b)** $3\sqrt{2}$ **c)** $\sqrt{2}$ **d)** $3\sqrt{10}$ **e)** $8\sqrt[3]{2}$

3. a) $5\sqrt{3}$ **b)** $2\sqrt{5}$ **c)** $2\sqrt{5}$ **d)** $4\sqrt{3}$ **e)** $\frac{1}{2}\sqrt[3]{4}$

4. a) $\sqrt{5} - \sqrt{3}$ **b)** $3\sqrt{10} - \sqrt{5}$ **c)** $2\sqrt{14} + 15$ **d)** 33 **e)** 6 **f)** $3\sqrt{2}$

5. a) $\frac{1}{2}\sqrt{2}$ **b)** $\sqrt{6}$ **c)** $\frac{1}{3}\sqrt{15}$ **d)** $-\frac{1}{2}\sqrt{6}$ **e)** $\frac{1}{7}\sqrt{70}$ **f)** $\frac{2}{5}\sqrt{15}$ **g)** $\frac{1}{15}\sqrt{6}$
h) $\frac{4}{3}$ **i)** $\frac{1}{2}\sqrt{2}$ **j)** $\sqrt{2}$ **k)** $-\frac{1}{22}\sqrt{22}$ **l)** $-\frac{10}{3}\sqrt{15}$

6. a) $\frac{3}{10}\sqrt{30}$ **b)** $\sqrt{5}$ **c)** $\frac{9}{2}\sqrt{6}$ **d)** $\frac{1}{3}\sqrt{15}$

7. a) $\dfrac{\sqrt{14} - 2}{2}$ or $\dfrac{1}{2}\sqrt{14} - 1$ **b)** $\dfrac{3 + 2\sqrt{6}}{6}$ or $\dfrac{1}{2} + \dfrac{1}{3}\sqrt{6}$
c) $\dfrac{\sqrt{30} + 2\sqrt{3}}{6}$ or $\dfrac{1}{6}\sqrt{30} + \dfrac{1}{3}\sqrt{3}$

8. a) $6\sqrt{2} - 8$ probably Jaclyn's method **b)** 40 and 20 do not divide exactly by 7

9. a) $6\sqrt{10} - 2\sqrt{30}$ **b)** -2

10. a) $\sqrt{3} - \sqrt{2}$ metres **b)** 15.3 metres

11. a) $\dfrac{72 - 4\sqrt{6}}{3}$ metres or $24 - \dfrac{4}{3}\sqrt{6}$ metres **b)** 20.73 metres

12. D **13.** D **14.** C

15.

0	.	3	1

16.

1	0		

Operations on Radicals Lesson #4:
Dividing Radicals - Part Two

Rationalizing a Denominator in Binomial Form

When the original denominator of the fraction is of binomial form, the process of rationalizing the denominator involves multiplying both numerator and denominator of the fraction by the **conjugate** of the binomial denominator.

Class Ex. #1

Simplify by rationalizing the denominator.

a) $\dfrac{2}{\sqrt{5} - \sqrt{3}}$ b) $\dfrac{\sqrt{6} - 2}{\sqrt{6} + 2}$ c) $\dfrac{1}{1 - \sqrt{x}}$

a) $\dfrac{2}{\sqrt{5} - \sqrt{3}} \cdot \dfrac{\sqrt{5}+\sqrt{3}}{(\sqrt{5}+\sqrt{3})}$; $\dfrac{2\sqrt{5}+2\sqrt{3}}{5-3} = \dfrac{2\sqrt{5}+2\sqrt{3}}{2}$; $= \sqrt{5}+\sqrt{3}$

b) $\dfrac{(\sqrt{6}-2)}{(\sqrt{6}-2)}$; $= \dfrac{6-2\sqrt{6}-2\sqrt{6}+4}{6-4}$; $= \dfrac{10-4\sqrt{6}}{2} = 5-2\sqrt{6}$

c) $\dfrac{1+\sqrt{x}}{(1+\sqrt{x})}$; $= \dfrac{1+\sqrt{x}}{1-x}$

Class Ex. #2

Express $\dfrac{\sqrt{8} - \sqrt{3}}{4\sqrt{3} - \sqrt{2}}$ with a whole number in the denominator.

conjugate: $4\sqrt{3}+\sqrt{2}$

$\dfrac{\sqrt{8}-\sqrt{3}}{4\sqrt{3}-\sqrt{2}} \cdot \dfrac{(4\sqrt{3}+\sqrt{2})}{(4\sqrt{3}+\sqrt{2})}$

$\dfrac{4\sqrt{24}+\sqrt{16}-4\sqrt{9}-\sqrt{6}}{16(3)-2} = \dfrac{8\sqrt{6}+4-4(3)-\sqrt{6}}{46}$

$4\sqrt{24} = 4(2)\sqrt{6} = 8\sqrt{6}$

$= \dfrac{7\sqrt{6}-8}{46}$

(1-5)ac, 6

Class Ex. #3

The area of a trapezoid is given by the formula $A = \frac{1}{2}h(a + b)$ where a and b are the lengths of the parallel sides and h is the shortest distance between the sides. If the area of a trapezoid is 20 cm^2 and the parallel sides are of lengths $\sqrt{6}$ cm and $\sqrt{5}$ cm, determine the exact value of the distance between the parallel sides. Answer with a rational denominator.

Complete Assignment Questions #1 - #12

Assignment

1. Simplify by rationalizing the denominator.

a) $\dfrac{4}{\sqrt{5} - 1}$ b) $\dfrac{1}{\sqrt{6} + 2}$ c) $\dfrac{3}{3 - \sqrt{3}}$

d) $\dfrac{\sqrt{7}}{\sqrt{7} - 2}$ e) $\dfrac{3}{\sqrt{2} - \sqrt{3}}$ f) $\dfrac{\sqrt{2}}{\sqrt{6} + \sqrt{2}}$

2. Simplify by rationalizing the denominator.

a) $\dfrac{2\sqrt{3}}{3\sqrt{2} + \sqrt{3}}$ b) $\dfrac{3\sqrt{11}}{3\sqrt{11} + 10}$

c) $\dfrac{\sqrt{2}}{\sqrt{12} - \sqrt{8}}$ d) $\dfrac{\sqrt{7}}{4 - \sqrt{14}}$

3. Simplify, leaving an integer in the denominator.

a) $\dfrac{\sqrt{3} - 1}{\sqrt{3} + 1}$ b) $\dfrac{\sqrt{5} - 2}{\sqrt{5} - 1}$

c) $\dfrac{\sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}}$ d) $\dfrac{5 - \sqrt{10}}{3 + \sqrt{10}}$

4. Simplify, leaving a whole number in the denominator.

a) $\dfrac{\sqrt{11} + 5\sqrt{2}}{\sqrt{11} - 2\sqrt{2}}$ b) $\dfrac{2\sqrt{6} - \sqrt{3}}{3\sqrt{3} + \sqrt{6}}$

c) $\dfrac{\sqrt{30} + 3\sqrt{3}}{\sqrt{30} - 3\sqrt{3}}$ d) $\dfrac{3\sqrt{5} - 2\sqrt{3}}{3\sqrt{5} + 2\sqrt{3}}$

c) $\dfrac{\sqrt{30}+3\sqrt{3}}{\sqrt{30}-3\sqrt{3}}$

d) $\dfrac{3\sqrt{5}-2\sqrt{3}}{3\sqrt{5}+2\sqrt{3}}$

5. Simplify by rationalizing the denominator.

a) $\dfrac{3}{2\sqrt{x}+3}$ **b)** $\dfrac{x+\sqrt{10}}{x-\sqrt{10}}$ **c)** $\dfrac{\sqrt{k}+\sqrt{2}}{\sqrt{k}-\sqrt{2}}$

6. The area of a rectangle is 5 m^2 and the length is $3+\sqrt{3}$ m. Calculate the width of the rectangle, expressing the answer

i) as an exact value with a whole number in the denominator
ii) as a decimal to the nearest hundredth

7. A triangle has area $\left(2\sqrt{15}-3\sqrt{6}\right)$ square units and base $(\sqrt{15}+\sqrt{6})$ units. Determine the exact value of the height of the triangle, giving the answer with a rational denominator.

Multiple Choice

8. The fraction $\dfrac{2}{\sqrt{5}-\sqrt{3}}$ expressed with a rational denominator is

A. $\dfrac{\sqrt{5}+\sqrt{3}}{4}$ **B.** $\dfrac{\sqrt{5}+\sqrt{3}}{8}$
C. $\sqrt{5}+\sqrt{3}$ **D.** $\dfrac{2\sqrt{5}+\sqrt{3}}{2}$

9. When $\dfrac{1}{2(2+\sqrt{3})}$ is expressed with a rational denominator, the result is

A. $\dfrac{2-\sqrt{3}}{2}$ **B.** $\dfrac{2-\sqrt{3}}{-1}$
C. $\dfrac{2-\sqrt{3}}{14}$ **D.** $\dfrac{2-\sqrt{3}}{-10}$

10. $\dfrac{3\sqrt{5}+\sqrt{3}}{2\sqrt{5}+\sqrt{3}}$, expressed with a rational denominator in simplest form, is

A. $\dfrac{33+5\sqrt{15}}{23}$
B. $\dfrac{33+5\sqrt{15}}{17}$
C. $\dfrac{27-\sqrt{15}}{23}$
D. $\dfrac{27-\sqrt{15}}{17}$

11. $\dfrac{p}{q-\sqrt{r}}$, expressed with a rational denominator, may be written as

A. $\dfrac{p}{q^2-r}$
B. $\dfrac{p\left(q+\sqrt{r}\right)}{q^2-r^2}$
C. $\dfrac{p\left(q+\sqrt{r}\right)}{q^2-r}$
D. $\dfrac{p\left(q-\sqrt{r}\right)}{q^2+r}$

Numerical Response

12. When the denominator is rationalized, $\dfrac{\sqrt{10}-\sqrt{2}}{\sqrt{10}+\sqrt{2}}$ can be expressed in the form $a-b\sqrt{5}$, where $a, b \in Q$. The value of $a+b$, to the nearest tenth, is _____.

(Record your answer in the numerical response box from left to right.)

Answer Key

1. **a)** $\sqrt{5}+1$ **b)** $\frac{\sqrt{6}-2}{2}$ **c)** $\frac{3+\sqrt{3}}{2}$ **d)** $\frac{7+2\sqrt{7}}{3}$ **e)** $-3\sqrt{2}-3\sqrt{3}$ **f)** $\frac{\sqrt{3}-1}{2}$

2. **a)** $\frac{2\sqrt{6}-2}{5}$ **b)** $30\sqrt{11}-99$ **c)** $\frac{\sqrt{6}+2}{2}$ **d)** $\frac{4\sqrt{7}+7\sqrt{2}}{2}$

3. **a)** $2-\sqrt{3}$ **b)** $\frac{3-\sqrt{5}}{4}$ **c)** $2+\sqrt{3}$ **d)** $8\sqrt{10}-25$

4. **a)** $\frac{31+7\sqrt{22}}{3}$ **b)** $\sqrt{2}-1$ **c)** $19+6\sqrt{10}$ **d)** $\frac{19-4\sqrt{15}}{11}$

5. **a)** $\frac{6\sqrt{x}-9}{4x-9}$ **b)** $\frac{x^2+2\sqrt{10}\,x+10}{x^2-10}$ **c)** $\frac{k+2\sqrt{2k}+2}{k-2}$

6. **i)** $\frac{15-5\sqrt{3}}{6}$ m. **ii)** 1.06 m. **7.** $\frac{32-10\sqrt{10}}{3}$ units.

8. C **9.** A **10.** D **11.** C **12.**

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