Operations on Radicals Lesson #1: Adding and Subtracting Radicals

Investigation 1

In the last unit we verified that addition of radicals cannot be done by adding the radicands.

In order to develop a rule for adding and subtracting radicals, complete the work below.

 a) Use a calculator to investigate which of the following radical statements are true. Circle the statements which are true and place a single line through the expressions which are false.

i)
$$\sqrt{2} + 5\sqrt{2} = 6\sqrt{2}$$
 for v iv) $7\sqrt{5} + 7\sqrt[3]{5} = 14\sqrt[5]{5}$ for v
ii) $4\sqrt[3]{5} = -3\sqrt[3]{5}$ for v iv) $\sqrt[3]{3} \pm \sqrt[3]{2} = \sqrt[3]{5}$
iii) $5\sqrt{8} - 2\sqrt[8]{8} \pm 7\sqrt{8} = 10\sqrt{8}$ for v fo

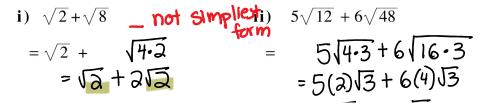
- b) Use the results in a) to suggest a rule for adding and subtracting radicals. Radicals can be added or subtracted if they have the same radicand and the same index
- c) Simplify the following. Express the answer as a mixed radical.
 - i) $8\sqrt{7} 3\sqrt{7} + 15\sqrt{7}$ ii) $18\sqrt[5]{10} + 12\sqrt[5]{10} 7\sqrt[5]{10}$ iii) $5\sqrt{x} 4\sqrt{x}$ = $20\sqrt{7}$ = $23\sqrt[5]{10}$ = $\sqrt{\chi}$

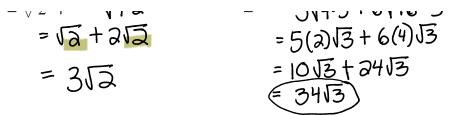
Investigation 2

8x - 3x + 15x = a0x $8x^{a} + 3x - 2x^{2} = 6x^{2} + 3x$

yes

- a) Use a calculator to verify that the following statements are true.
 - i) $\sqrt{2} + \sqrt{8} = 3\sqrt{2}$ True $5\sqrt{12} + 6\sqrt{48} = 34\sqrt{3}$ The
- **b**) Does this appear to contradict the rule you wrote in Investigation #1 b)?
- c) Complete the following by writing each radical in simplest mixed form to show that the rule can be modified.

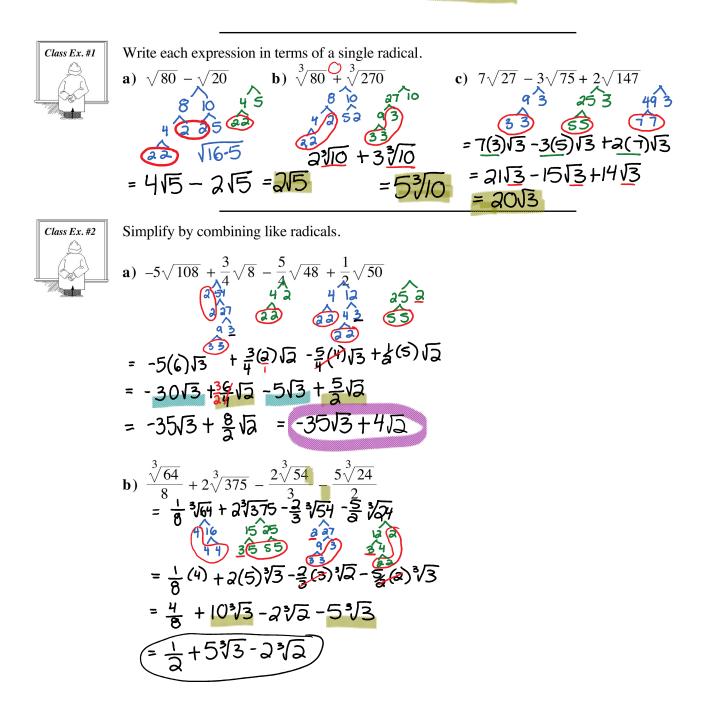




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Adding and Subtracting Radicals

In order to add and subtract radicals, they must be able to be expressed as **like radicals**, i.e. radicals with the SAME <u>radicand</u> and the SAME <u>index</u>.



$$=\frac{1}{2}+5\sqrt[3]{3}-2\sqrt[3]{2}$$

Complete Assignment Questions #1 - #5

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59 Operations on Radicals Lesson #1: Adding and Subtracting Radicals Class Ex. #3 Find the length of *x* х a) as an exact value $8\sqrt{2} + 2\sqrt{12}$ **b**) as a decimal to the <u>nearest</u> tenth $5\sqrt{27} - 4\sqrt{18}$ X= 812+2112- (5127-4118) $X = 8\sqrt{a} + 2\sqrt{1a} - 5\sqrt{a7} + 4\sqrt{18}$ $X = 8\sqrt{a} + 2\sqrt{13} - 5\sqrt{37} + 4\sqrt{18}$ $X = 8\sqrt{a} + 2\sqrt{37} - 5\sqrt{37} + 4\sqrt{37}$ x= 812+413-1513+1212 b) 9.2317... = 9.2 a) X= 2012-1113 60,7 **Complete Assignment Questions #6 - #13** Assignment 1. Simplify. **a)** $5\sqrt{7} - 2\sqrt{7}$ **b)** $9\sqrt[3]{13} + 2\sqrt[3]{13}$ **c)** $4\sqrt{11} - 9\sqrt{11} + \sqrt{11}$ **d**) $4\sqrt{5} - 2\sqrt{2} + 8\sqrt{2}$ **e**) $13\sqrt[4]{a} + 7\sqrt[4]{a}$ **f**) $-3\sqrt{2} + 6\sqrt{3} - 9\sqrt{3} + 4\sqrt{2}$ 2. Write each expression in terms of a single radical. c) $\sqrt{24} - \sqrt{54} + 2\sqrt{6}$ **a)** $\sqrt{125} - \sqrt{5}$ **b)** $\sqrt{27} + \sqrt{12}$

d)
$$\sqrt{150} + \sqrt{216}$$
 e) $\sqrt[3]{16} + \sqrt[3]{128}$ **f**) $-3\sqrt{175} + 8\sqrt{28} - \sqrt{63}$

g) $\sqrt[4]{16} \pm \sqrt[4]{162}$ **b**) $2\sqrt{700} = 6\sqrt{63}$ **i**) $-7\sqrt[3]{54} = 2\sqrt[3]{250}$

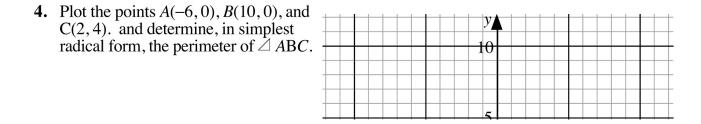
g)
$$\sqrt[4]{16} + \sqrt[4]{162}$$
 h) $2\sqrt{700} - 6\sqrt{63}$ **i**) $-7\sqrt[3]{54} - 2\sqrt[3]{250}$

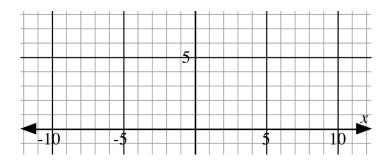
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3. Simplify by combining like radicals. **a)** $\sqrt{20} + \sqrt{72} - \sqrt{45}$ **b)** $\sqrt{27} + \sqrt{12} - \sqrt{32} - \sqrt{8}$

c)
$$\sqrt{98} - \sqrt{20} + \sqrt{18}$$
 d) $2\sqrt{252} - \sqrt{726} - 5\sqrt{63}$

e)
$$2\sqrt[3]{108} + \sqrt[3]{32} + 3\sqrt[3]{256}$$
 f) $12\sqrt{150} - 5\sqrt{54} + 3\sqrt{24}$





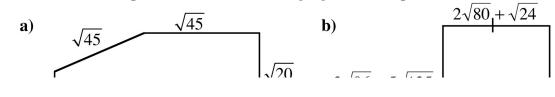
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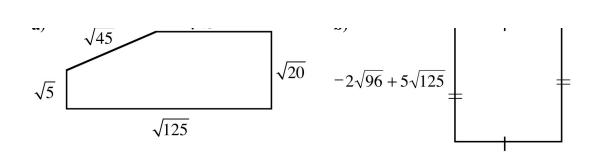
- 5. Write in simplest radical form. **a**) $\frac{1}{3}\sqrt{63} + \frac{2}{5}\sqrt{700} \frac{2}{3}\sqrt{112} + \frac{3}{2}\sqrt{28}$

b)
$$\frac{7\sqrt[3]{1024}}{2} + \frac{5\sqrt[3]{2000}}{12} - 3\sqrt[3]{686} + \frac{1}{8}\sqrt[3]{128}$$

6. Determine the perimeter of the following figures in simplest radical form.

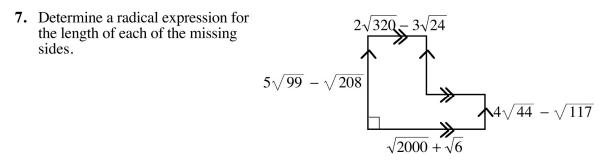


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- 8. Determine the next two terms of the following sequences.
 - **a**) $4 + 2\sqrt{2}$, $6 + 3\sqrt{2}$, $8 + 4\sqrt{2}$, **b**) $6 + 2\sqrt{3}$, $3 + \sqrt{3}$, $0, \ldots$

Multiple 9. $\sqrt{75} + \sqrt{3}$ equals Choice A. $6\sqrt{3}$

- **B.** $26\sqrt{3}$
- **C.** $\sqrt{78}$ **D.** $3\sqrt{5} + \sqrt{3}$

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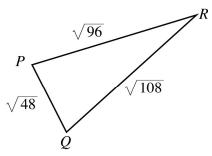
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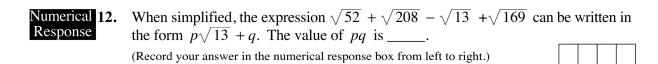
10. Given that $x - 2\sqrt{5} = \sqrt{45}$, then $\sqrt{5} + x$ is equal to

- A. $2\sqrt{5}$
- **B.** $3\sqrt{5}$ C. $4\sqrt{5}$
- **D.** $6\sqrt{5}$

11. In simplest radical form the perimeter of $\triangle PQR$ is

- **A.** $\sqrt{252}$
- **B.** $6\sqrt{7}$
- **C.** $10\sqrt{3} + 4\sqrt{6}$
- **D.** $52\sqrt{3} + 16\sqrt{6}$

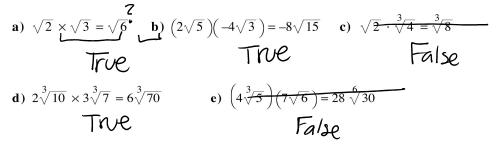




Operations on Radicals Lesson #2: Multiplying Radicals

Investigation Investigating Multiplication Properties of Radicals

Use a calculator to determine whether the following statements are true or false.



Based on the results from a) - e), write a rule which describes the process of multiplying radicals.

-Same index - multiply radicand by radicand - multiply coefficient by coefficient

Multiplying Radicals

To multiply radicals, the index must be the same in each radical.

- Multiply numerical coefficients by numerical coefficients.
- Multiply radicand by radicand.

Multiply and simplify where possible.

• Simplify into mixed radical form if possible.



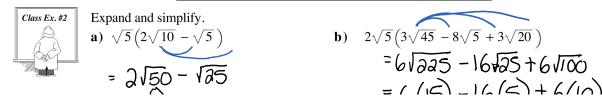
It is usually easier to convert each radical to its simplest mixed form before multiplying.

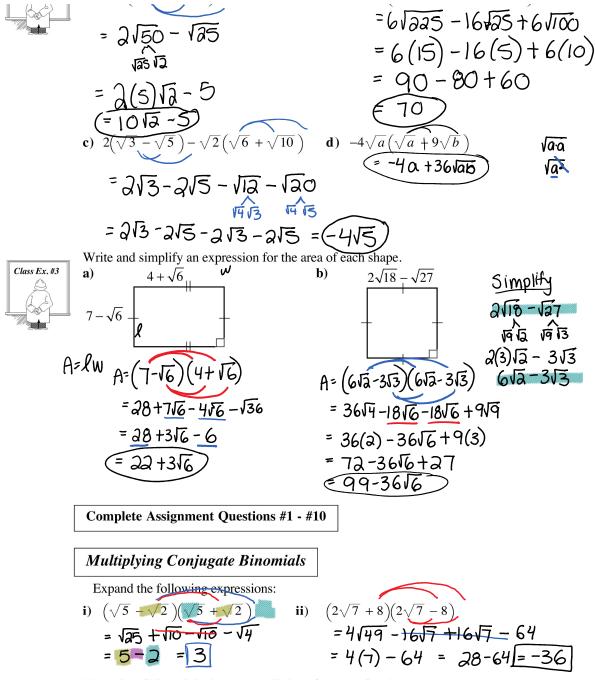


a)
$$\sqrt{8} \cdot \sqrt{8}$$
 b) $(4\sqrt{5})(3\sqrt{6})$ c) $(4\sqrt{x})(3\sqrt{y})$ d) $-2\sqrt{8} \times 5\sqrt{12}$
 $=\sqrt{64}$ $=12\sqrt{30}$ $=\sqrt{4\sqrt{x}}(3\sqrt{y})$ $=\sqrt{10}\sqrt{96}$ $\sqrt{14}\sqrt{3}$
 $=\sqrt{64}$ $=\sqrt{30}\sqrt{30}$ $=\sqrt{10}\sqrt{4}\sqrt{3}$ $\sqrt{10}\sqrt{3}$
 $=\sqrt{64}$ $=\sqrt{10}\sqrt{6}$ $\sqrt{10}\sqrt{3}$
 $=\sqrt{64}$ $\sqrt{10}\sqrt{3}$ $=\sqrt{10}\sqrt{6}$ $-\sqrt{10}\sqrt{3}$ $\sqrt{10}\sqrt{3}$
 $=\sqrt{10}\sqrt{6}$ $-\sqrt{10}\sqrt{6}$ $=\sqrt{10}\sqrt{6}$

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66 Operations on Radicals Lesson #2: *Multiplying Radicals*





The pairs of binomials above are called **conjugates** of each other. What do you notice about the product of two conjugate binomials?

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- Conjugate binomials are pairs of binomials in the form $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} c\sqrt{d}$
- The product of conjugate binomials is always a rational number of the form $a^2b c^2d$.



Write the conjugate of each, then multiply each pair.

a) $4\sqrt{6} + 3 = 0$ Conjugate: $4\sqrt{6} = 3$ $\sqrt{11} + \sqrt{2}$ conjugate: $-3\sqrt{11} + \sqrt{2}$ conjugate: -3

Assignment

- 1. Multiply and simplify where possible. Do not use a calculator.
 - a) $(\sqrt{7})(\sqrt{3})$ b) $4\sqrt{3} \times 2\sqrt{5}$ c) $-3\sqrt{5} \times 2\sqrt{2}$ d) $6\sqrt{p} \times 8\sqrt{q}$
 - e) $(\sqrt{15})(\sqrt{3})$ f) $10\sqrt{5} \times 9\sqrt{5}$ g) $3\sqrt{6} \cdot 5\sqrt{10}$ h) $\sqrt{a} \times 10\sqrt{a}$

i)
$$7\sqrt{54} \cdot 2\sqrt{6}$$
 j) $(\sqrt{32})(\sqrt{6})$ k) $\sqrt{15} \times 3\sqrt{27}$

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- 2. In each case, write each radical as the product of two mixed radicals in two different ways.
 a) 15√18
 b) 35√6
- **3.** Express in simplest form. Do not use a calculator. **a**) $(\sqrt{3})^2$ **b**) $(4\sqrt{2})^2$ **c**) $(-3\sqrt{5})^2$ **d**) $-(\sqrt{12})^2$ **e**) $(\sqrt{5})^3$
- 4. Express in simplest form.

a) $\sqrt{5} \times 2\sqrt{3} \times 3\sqrt{2}$ b) $2\sqrt{6} \times 2\sqrt{3} \times 3\sqrt{2}$ c) $(-2\sqrt{6})(2\sqrt{3})(-3\sqrt{5})$

4. Express in simplest form.

a)
$$\sqrt{5} \times 2\sqrt{3} \times 3\sqrt{2}$$
 b) $2\sqrt{6} \times 2\sqrt{3} \times 3\sqrt{2}$ c) $(-2\sqrt{6})(2\sqrt{3})(-3\sqrt{5}) =$
 $= 12\sqrt{90} = 12(3)\sqrt{10}$
 $\sqrt{90} = 36\sqrt{10}$
 $\sqrt{90} = 36\sqrt{10$

- **5**. Consider the product $6\sqrt{5} \times 3\sqrt{8}$.
 - **a**) Use a **two decimal place approximation** for each radical to determine a two decimal place approximation for the product.
 - **b**) Determine the **exact value** of the product as a mixed radical in simplest form.
 - c) Determine a two decimal place approximation to the answer in b).
 - d) Which of the two decimal place approximations is more accurate? Explain.

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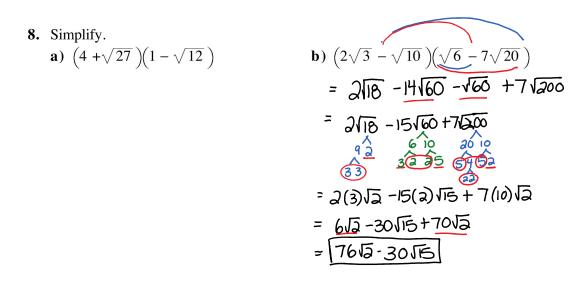
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6. Expand and simplify where possible.
a)
$$\sqrt{6} \left(2\sqrt{6} - \sqrt{5} \right)$$
 b) $\sqrt{2} \left(1 - \sqrt{2} \right)$ c) $2\sqrt{3} \left(2\sqrt{7} - 4\sqrt{5} \right)$

7. Expand and simplify.

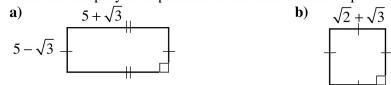
a)
$$\sqrt{3}(2\sqrt{6} - \sqrt{12})$$
 b) $\sqrt{8}(\sqrt{6} - \sqrt{2})$ **c)** $\sqrt{y}(\sqrt{x} - 9\sqrt{y})$

d)
$$2\sqrt{11} \left(3\sqrt{2} - \sqrt{50} + 3\sqrt{32} \right)$$
 e) $\sqrt{5} \left(3\sqrt{5} - \sqrt{75} + 3\sqrt{3} \right)$



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9. Write and simplify an expression for the area of each shape.



c) rectangle $2\sqrt{10}$ by $\left(\sqrt{6} + 4\sqrt{5}\right)$

d) square with sides
$$3\sqrt{208} - 8$$

 $A = lw$
 $(3\sqrt{208} - 8)(3\sqrt{208} - 8)$
 $(12\sqrt{13} - 8)(2\sqrt{13} - 8)$
 $= 144(13) - 96\sqrt{13} - 96\sqrt{13} + 64$
 $= 1872 - 192\sqrt{13} + 64$
 $= 1936 - 192\sqrt{13}$

10. Expand and simplify.

a) $(5\sqrt{3}-2)^2$ **b**) $(4\sqrt{6}-\sqrt{2})^2$

c)
$$2(\sqrt{15} - 3\sqrt{5})^2$$
 d) $(7\sqrt{x} - 2\sqrt{y})^2$

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- **11.** Expand and simplify. **a)** $(\sqrt{5}+1)(\sqrt{5}-1)$ **b**) $(\sqrt{8}+\sqrt{7})(\sqrt{8}-\sqrt{7})$ **c)** $(2\sqrt{6}-\sqrt{2})(2\sqrt{6}+\sqrt{2})$
- **12.** Write the conjugate of each. **a)** $\sqrt{2} - \sqrt{5}$ **b)** $4 + \sqrt{7}$ **c)** $-3\sqrt{8} - 15$
- 13. Write the conjugate of each, then multiply each pair. a) $\sqrt{3} - 1$ b) $2 + \sqrt{5}$ c) $2\sqrt{6} - \sqrt{3}$

13. Write the conjugate of each, then multiply each pair.

a)
$$\sqrt{3} - 1$$
 b) $2 + \sqrt{5}$ **c**) $2\sqrt{6} - \sqrt{3}$

d)
$$2\sqrt{8} + \sqrt{27}$$
 e) $\sqrt{32} - \sqrt{3}$ **f**) $-3\sqrt{40} + 2\sqrt{10}$

Multiple
Choice 14. For all values of a and b,
$$(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})$$
 is equal to
A. $\sqrt{(a-b)(a+b)}$
B. $a-b$
C. $a+b$
D. $a^2 - b^2$

15. $(\sqrt{2})^5$ is equal to **A.** $\sqrt{10}$ **B.** $5\sqrt{2}$ **C.** $4\sqrt{2}$ **D.** 32

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16. The expression $\sqrt{5}(\sqrt{10} + 12\sqrt{5}) - \sqrt{7}(\sqrt{7} - 2\sqrt{14})$ can be simplified to the form $a + b\sqrt{c}$, where *a*, *b* and *c* are integers. The value of a + b + c is _____. (Record your answer in the numerical response box from left to right.)

17. If $p \oplus q$ means "(p-q) multiplied by q" then the value of $\sqrt{6} \oplus \sqrt{3}$ can be simplified to the form $a + b\sqrt{c}$, where a, b and c are integers.

17. If $p \oplus q$ means "(p-q) multiplied by q" then the value of $\sqrt{6} \oplus \sqrt{3}$ can be simplified to the form $a + b\sqrt{c}$, where a, b and c are integers. The value of c is _____.

(Record your answer in the numerical response box from left to right.)

Answer Key

1. a) $\sqrt{21}$ **b**) $8\sqrt{15}$ **c**) $-6\sqrt{10}$ **d**) $48\sqrt{pq}$ **e**) $3\sqrt{5}$ **f**) 450 **g**) $30\sqrt{15}$ **h**) 10*a* i) 252 j) $8\sqrt{3}$ **k**) $27\sqrt{5}$ 2. Answers may vary. a) $(3\sqrt{3})(5\sqrt{6})$ or $(5\sqrt{3})(3\sqrt{6})$ **b**) $(5\sqrt{2})(7\sqrt{3})$ or $(7\sqrt{2})(5\sqrt{3})$ **3.** a) 3 **b**) 32 **c**) 45 **d**) −12 e) $5\sqrt{5}$ f) $48\sqrt[3]{2}$ **c**) $36\sqrt{10}$ **d**) $6\sqrt{2}$ **e**) 8 **4.** a) $6\sqrt{30}$ **b**) 72 **b**) $36\sqrt{10}$ **5.** a) 113.94 **c**) 113.84 **d**) c) because rounding is not done until the last step. **6.** a) $12 - \sqrt{30}$ b) $\sqrt{2} - 2$ c) $4\sqrt{21} - 8\sqrt{15}$ **b**) $4\sqrt{3} - 4$ e) $15 - 2\sqrt{15}$ **7.** a) $6\sqrt{2} - 6$ c) $\sqrt{xy} - 9y$ **d**) $20\sqrt{22}$ **8.** a) $-14 - 5\sqrt{3}$ **b**) $76\sqrt{2} - 30\sqrt{15}$ c) $4\sqrt{15} + 40\sqrt{2}$ **b**) $5 + 2\sqrt{6}$ **d**) 1936 – 192 $\sqrt{13}$ **9.** a) 22 **10.a**) 79 - 20 $\sqrt{3}$ **b**) $98 - 16\sqrt{3}$ **c**) $120 - 60\sqrt{3}$ **d**) $49x - 28\sqrt{xy} + 4y$ **b**) 1 **11.a**) 4 **c**) 22 **12.a)** $\sqrt{2} + \sqrt{5}$ **b**) $4 - \sqrt{7}$ c) $-3\sqrt{8} + 15$ **13.a**) $\sqrt{3} + 1, 2$ **b**) $2 - \sqrt{5}$, -1 c) $2\sqrt{6} + \sqrt{3}$, 21 **d**) $2\sqrt{8} - \sqrt{27}$, 5 e) $\sqrt{32} + \sqrt{3}$, 29 f) $-3\sqrt{40} - 2\sqrt{10}$, 320 14.B 15. C 16. 7 4 17. 2

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