Operations on Radicals Lesson #1:
Adding and Subtracting Radicals

Investigation 1

In the last unit we verified that addition of radicals cannot be done by adding the radicands.

In order to develop a rule for adding and subtracting radicals, complete the work below.

a) Use a calculator to investigate which of the following radical statements are true. Circle the statements which are true and place a single line through the expressions which are false.

i) $\sqrt{2} + 5\sqrt{2} = 6\sqrt{2}$ — *true*

iv) $7\sqrt{5} + 7\sqrt[3]{5} = 14\sqrt[5]{5}$ — *false*

ii) $4\sqrt[3]{5} - 7\sqrt[3]{5} = -3\sqrt[3]{5}$ — *true*

v) $\sqrt[3]{3} \pm \sqrt[3]{2} = \sqrt[3]{5}$ — *false*

iii) $5\sqrt{8} - 2\sqrt{8} + 7\sqrt{8} = 10\sqrt{8}$ — *true*

b) Use the results in a) to suggest a rule for adding and subtracting radicals.

Radicals can be added or subtracted if they have the same radicand and the same index

c) Simplify the following. Express the answer as a mixed radical.

i) $8\sqrt{7} - 3\sqrt{7} + 15\sqrt{7}$

$= 20\sqrt{7}$

ii) $18\sqrt[5]{10} + 12\sqrt[5]{10} - 7\sqrt[5]{10}$

$= 23\sqrt[5]{10}$

iii) $5\sqrt{x} - 4\sqrt{x}$

$= \sqrt{x}$

Investigation 2

$8x - 3x + 15x = 20x$

$8x^2 + 3x - 2x^2 = 6x^2 + 3x$

a) Use a calculator to verify that the following statements are true.

i) $\sqrt{2} + \sqrt{8} = 3\sqrt{2}$ — *True*

ii) $5\sqrt{12} + 6\sqrt{48} = 34\sqrt{3}$ — *True*

b) Does this appear to contradict the rule you wrote in Investigation #1 b)? *yes*

c) Complete the following by writing each radical in simplest mixed form to show that the rule can be modified.

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i) $\sqrt{2} + \sqrt{8}$ — *not simpliest form*

$= \sqrt{2} + \sqrt{4 \cdot 2}$

$= \sqrt{2} + 2\sqrt{2}$

$= 3\sqrt{2}$

ii) $5\sqrt{12} + 6\sqrt{48}$

$= 5\sqrt{4 \cdot 3} + 6\sqrt{16 \cdot 3}$

$= 5(2)\sqrt{3} + 6(4)\sqrt{3}$

$= 10\sqrt{3} + 24\sqrt{3}$

$\boxed{= 34\sqrt{3}}$

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Adding and Subtracting Radicals

In order to add and subtract radicals, they must be able to be expressed as **like radicals**, i.e. radicals with the SAME radicand **and** the SAME index.

Class Ex. #1 Write each expression in terms of a single radical.

a) $\sqrt{80} - \sqrt{20}$

$\sqrt{16 \cdot 5}$

$= 4\sqrt{5} - 2\sqrt{5} = 2\sqrt{5}$

b) $\sqrt[3]{80} + \sqrt[3]{270}$

$2\sqrt[3]{10} + 3\sqrt[3]{10}$

$= 5\sqrt[3]{10}$

c) $7\sqrt{27} - 3\sqrt{75} + 2\sqrt{147}$

$= 7(3)\sqrt{3} - 3(5)\sqrt{3} + 2(7)\sqrt{3}$

$= 21\sqrt{3} - 15\sqrt{3} + 14\sqrt{3}$

$= 20\sqrt{3}$

Class Ex. #2 Simplify by combining like radicals.

a) $-5\sqrt{108} + \dfrac{3}{4}\sqrt{8} - \dfrac{5}{4}\sqrt{48} + \dfrac{1}{2}\sqrt{50}$

$= -5(6)\sqrt{3} + \dfrac{3}{4}(2)\sqrt{2} - \dfrac{5}{4}(4)\sqrt{3} + \dfrac{1}{2}(5)\sqrt{2}$

$= -30\sqrt{3} + \dfrac{36}{24}\sqrt{2} - 5\sqrt{3} + \dfrac{5}{2}\sqrt{2}$

$= -35\sqrt{3} + \dfrac{8}{2}\sqrt{2} = \boxed{-35\sqrt{3} + 4\sqrt{2}}$

b) $\dfrac{\sqrt[3]{64}}{8} + 2\sqrt[3]{375} - \dfrac{2\sqrt[3]{54}}{2} - \dfrac{5\sqrt[3]{24}}{2}$

b) $\dfrac{\sqrt[3]{64}}{8} + 2\sqrt[3]{375} - \dfrac{2\sqrt[3]{54}}{3} - \dfrac{5\sqrt[3]{24}}{2}$

$= \frac{1}{8}\sqrt[3]{64} + 2\sqrt[3]{375} - \frac{2}{3}\sqrt[3]{54} - \frac{5}{2}\sqrt[3]{24}$

$= \frac{1}{8}(4) + 2(5)\sqrt[3]{3} - \frac{2}{3}(3)\sqrt[3]{2} - \frac{5}{2}(2)\sqrt[3]{3}$

$= \frac{4}{8} + 10\sqrt[3]{3} - 2\sqrt[3]{2} - 5\sqrt[3]{3}$
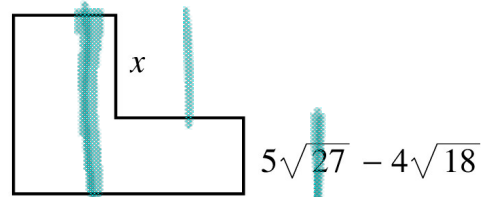
$= \frac{1}{2} + 5\sqrt[3]{3} - 2\sqrt[3]{2}$

Complete Assignment Questions #1 - #5

Class Ex. #3

Find the length of x

a) as an exact value
b) as a decimal to the nearest tenth

$8\sqrt{2} + 2\sqrt{12}$

$5\sqrt{27} - 4\sqrt{18}$

x

$x = 8\sqrt{2} + 2\sqrt{12} - \left(5\sqrt{27} - 4\sqrt{18}\right)$

$= 8\sqrt{2} + 2(2)\sqrt{3} - \left(5(3)\sqrt{3} - 4(3)\sqrt{2}\right)$

$= 8\sqrt{2} + 4\sqrt{3} - 15\sqrt{3} + 12\sqrt{2} = \boxed{20\sqrt{2} - 11\sqrt{3}}$

a) $x = 20\sqrt{2} - 11\sqrt{3}$

b) 9.2

Complete Assignment Questions #6 - #13

Assignment

1. Simplify.

a) $5\sqrt{7} - 2\sqrt{7}$ **b)** $9\sqrt[3]{13} + 2\sqrt[3]{13}$ **c)** $4\sqrt{11} - 9\sqrt{11} + \sqrt{11}$

d) $4\sqrt{5} - 2\sqrt{2} + 8\sqrt{2}$ **e)** $13\sqrt[4]{a} + 7\sqrt[4]{a}$ **f)** $-3\sqrt{2} + 6\sqrt{3} - 9\sqrt{3} + 4\sqrt{2}$

d) $4\sqrt{5} - 2\sqrt{2} + 8\sqrt{2}$ **e)** $13\sqrt[4]{a} + 7\sqrt[4]{a}$ **f)** $-3\sqrt{2} + 6\sqrt{3} - 9\sqrt{3} + 4\sqrt{2}$

2. Write each expression in terms of a single radical.

  a) $\sqrt{125} - \sqrt{5}$ **b)** $\sqrt{27} + \sqrt{12}$ **c)** $\sqrt{24} - \sqrt{54} + 2\sqrt{6}$

  d) $\sqrt{150} + \sqrt{216}$ **e)** $\sqrt[3]{16} + \sqrt[3]{128}$ **f)** $-3\sqrt{175} + 8\sqrt{28} - \sqrt{63}$

  g) $\sqrt[4]{16} + \sqrt[4]{162}$ **h)** $2\sqrt{700} - 6\sqrt{63}$ **i)** $-7\sqrt[3]{54} - 2\sqrt[3]{250}$

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3. Simplify by combining like radicals.
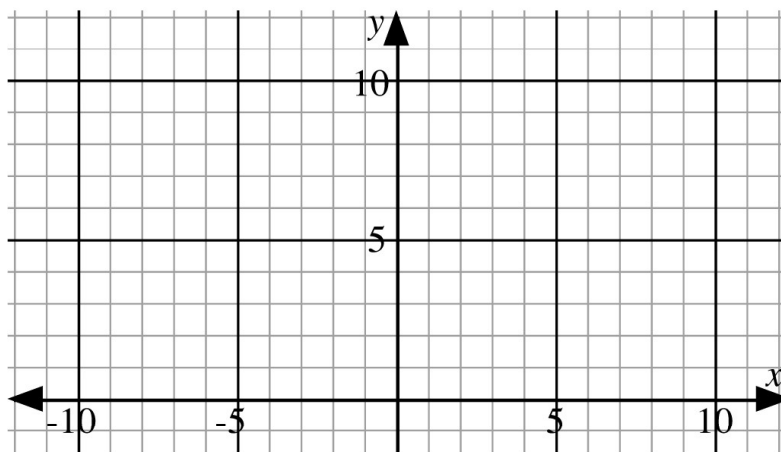
  a) $\sqrt{20} + \sqrt{72} - \sqrt{45}$ **b)** $\sqrt{27} + \sqrt{12} - \sqrt{32} - \sqrt{8}$

  c) $\sqrt{98} - \sqrt{20} + \sqrt{18}$ **d)** $2\sqrt{252} - \sqrt{726} - 5\sqrt{63}$

e) $2\sqrt[3]{108} + \sqrt[3]{32} + 3\sqrt[3]{256}$

f) $12\sqrt{150} - 5\sqrt{54} + 3\sqrt{24}$

4. Plot the points $A(-6, 0)$, $B(10, 0)$, and $C(2, 4)$. and determine, in simplest radical form, the perimeter of $\triangle ABC$.
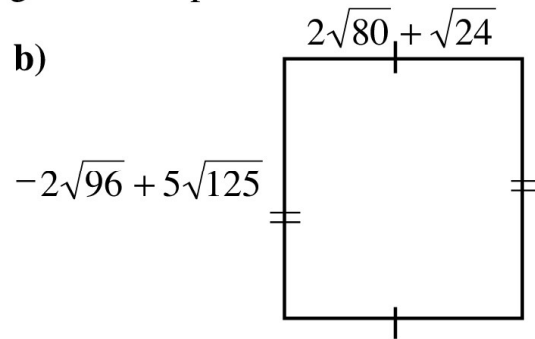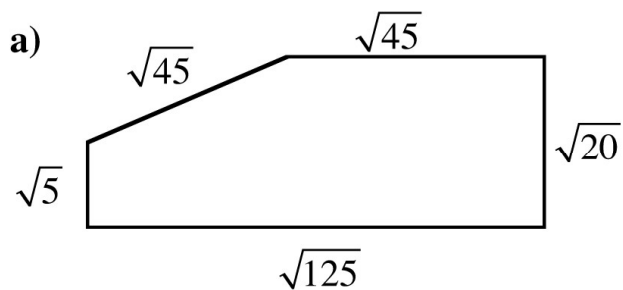
5. Write in simplest radical form.

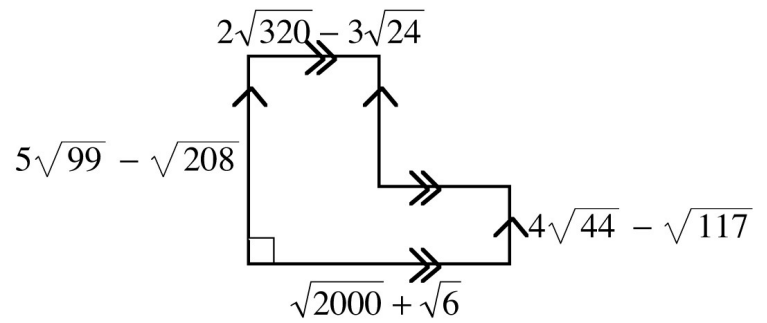**a)** $\dfrac{1}{3}\sqrt{63} + \dfrac{2}{5}\sqrt{700} - \dfrac{2}{3}\sqrt{112} + \dfrac{3}{2}\sqrt{28}$

b) $\dfrac{7\sqrt[3]{1024}}{2} + \dfrac{5\sqrt[3]{2000}}{12} - 3\sqrt[3]{686} + \dfrac{1}{8}\sqrt[3]{128}$

6. Determine the perimeter of the following figures in simplest radical form.

a)

$\sqrt{45}$ $\sqrt{45}$ $\sqrt{20}$ $\sqrt{5}$ $\sqrt{125}$

b)

$2\sqrt{80} + \sqrt{24}$ $-2\sqrt{96} + 5\sqrt{125}$

7. Determine a radical expression for the length of each of the missing sides.

$2\sqrt{320} - 3\sqrt{24}$

$5\sqrt{99} - \sqrt{208}$

$4\sqrt{44} - \sqrt{117}$

$\sqrt{2000} + \sqrt{6}$

8. Determine the next two terms of the following sequences.

a) $4 + 2\sqrt{2}$, $6 + 3\sqrt{2}$, $8 + 4\sqrt{2}$, ...

b) $6 + 2\sqrt{3}$, $3 + \sqrt{3}$, $0, \ldots$.

9. $\sqrt{75} + \sqrt{3}$ equals

    A. $6\sqrt{3}$
    B. $26\sqrt{3}$
    C. $\sqrt{78}$
    D. $3\sqrt{5} + \sqrt{3}$

10. Given that $x - 2\sqrt{5} = \sqrt{45}$, then $\sqrt{5} + x$ is equal to

    A. $2\sqrt{5}$
    B. $3\sqrt{5}$
    C. $4\sqrt{5}$
    D. $6\sqrt{5}$

11. In simplest radical form the perimeter of $\Delta\,PQR$ is

    A. $\sqrt{252}$
    B. $6\sqrt{7}$
    C. $10\sqrt{3} + 4\sqrt{6}$
    D. $52\sqrt{3} + 16\sqrt{6}$
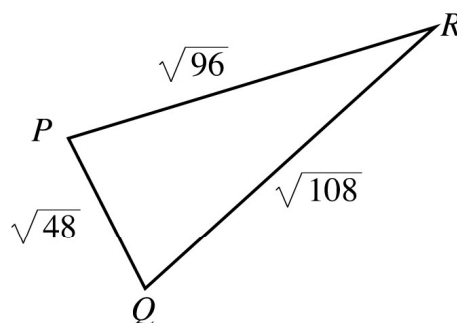
12. When simplified, the expression $\sqrt{52} + \sqrt{208} - \sqrt{13} + \sqrt{169}$ can be written in

12. When simplified, the expression $\sqrt{52} + \sqrt{208} - \sqrt{13} + \sqrt{169}$ can be written in the form $p\sqrt{13} + q$. The value of pq is _____.

(Record your answer in the numerical response box from left to right.)