## Exponents and Radicals Lesson #7: Rational Exponents - Part Two

Review

Complete the following as a review.

Product Law

$$x^m x^n =$$

Quotient Law  $x^m \div x^n =$ 

$$(x^m)^n =$$

Power of a Power  $(x^m)^n = \chi^m$  Power of a Product  $(xy)^m = \chi^m \chi^m$ 

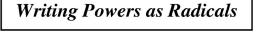
$$\left(\frac{x}{y}\right)^m = \int$$

Power of a Quotient  $\left(\frac{x}{y}\right)^m = \int \begin{array}{c} \chi^m \\ \chi^m \\ \chi^m \\ \chi^m \\ \end{array}$ Integral Exponent Rule  $x^{-m} = \begin{array}{c} \chi^m \\ \chi^m \\ \chi^m \\ \end{array}$ or  $()^m$ 

$$x^{\frac{m}{n}} =$$

$$\sqrt{\chi_{\rm w}}$$







Write each power as a radical.

**a**) 
$$x^{\frac{1}{6}}$$

**b**) 
$$-v^{\frac{5}{4}}$$

b) 
$$-y^{\frac{5}{4}}$$
 c)  $(-z)^{\frac{5}{3}}$  d)  $(-z)^{-\frac{5}{3}}$  e)  $5t^{\frac{3}{4}}$  f)  $(5t)^{\frac{3}{4}}$   $(5t)^{\frac{3}{4}}$   $(5t)^{\frac{3}{4}}$ 

$$\mathbf{f)} \quad (5t)$$

$$\left(\sqrt[3]{-Z}\right)^{2}$$



Simplify the following. Write each expression as a power with positive exponents and then as an entire radical.

**b**) 
$$y^{\frac{1}{3}} \div y^{\frac{5}{3}}$$

**c**) 
$$(a^{\frac{1}{2}})^{\frac{2}{3}}$$

$$\left(\frac{x^2}{y}\right)$$

$$\int_{a}^{b} \sqrt{x^{5}}$$

$$\sqrt{\frac{y}{X^2}}$$
 =

a) 
$$x^{\frac{3}{2}} \times x$$
  $= \frac{3}{2}$  b)  $y^{\frac{1}{3}} \div y^{\frac{5}{3}}$  c)  $(a^{\frac{1}{2}})^{\frac{2}{3}}$   $= Q^{\frac{3}{4}} = Q^{\frac{1}{3}}$   $= Q^{\frac{3}{4}} = Q^{\frac{1}{3}}$   $= Q^{\frac{1}{2}} = Q^{\frac{1}{3}}$   $= Q^{\frac{1}{3}} = Q^{\frac{1}{3}} = Q^{\frac{1}{3}}$   $= Q^{\frac{1}{3}} = Q^{\frac{1}{3}} = Q^{\frac{1}{3}} = Q^{\frac{1}{3}}$   $= Q^{\frac{1}{3}} = Q$ 

Class Ex. #3

Simplify the following. Write each expression as a power with positive exponents and then as an entire radical.

as an entire radical.  
a) 
$$4x^{\frac{3}{4}} \times 3x^{-\frac{1}{2}}$$

$$= | 2 | x^{\frac{3}{4}} - \frac{2}{4} |$$

$$= | 2 | x^{\frac{1}{4}} = | 2 | 4 | 4 |$$

b) 
$$\frac{5x^{\frac{5}{5}}}{25x^{-\frac{3}{5}}}$$
  
=  $\frac{1}{5}\chi^{\frac{3}{5}-(-\frac{3}{5})}$   
=  $\frac{1}{5}\chi^{\frac{6}{5}} = \frac{1}{5}\sqrt[5]{\chi^{6}}$ 

$$= \frac{1}{5} x^{\frac{1}{3}} = 13 \sqrt[4]{x}$$

$$\int = |6|^3 \sqrt{a^2}$$

#### **Complete Assignment Questions #1 - #3**

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### Writing Radicals as Powers

We can use the rule  $a^{\frac{m}{n}} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$  to write radicals as powers.



# Rodical -> exponent form

Write each radical as a power in the form  $a^n$ ,  $n \in Q$ .





 $= \int_{3}^{2}$   $= \int_{3}^{2}$ 

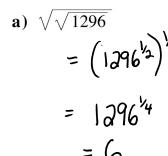
c) 
$$\sqrt{a^9}$$







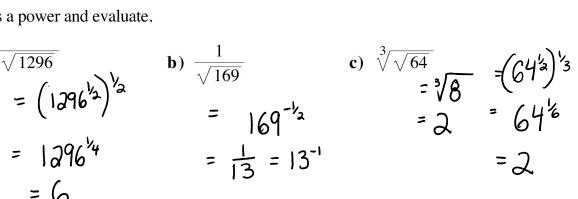
Write as a power and evaluate.



b) 
$$\frac{1}{\sqrt{169}}$$

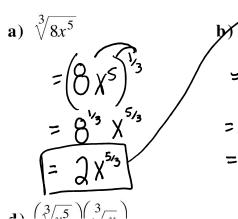
$$= |69^{-1/2}]$$

$$= \frac{1}{13} = 13^{-1}$$





Write each expression in the form  $ax^n$ , where  $a \in I$ , and  $n \in Q$ .



c) 
$$\sqrt{900x}$$
  
=  $(900x)^{1/2}$   
=  $900^{1/2}x^{1/2}$   
=  $30x^{1/2}$ 

$$\frac{1}{d} \left( \sqrt[3]{x^5} \right) \left( \sqrt[3]{x} \right)$$

$$= \left( \sqrt[5]{3} \right) \left( \sqrt[1]{3} \right)$$

$$= \left( \sqrt[5]{3} \right) \left( \sqrt[1]{3} \right)$$

$$= \sqrt[5]{3} + \sqrt[1]{3}$$

$$= \sqrt[5]{3} + \sqrt[3]{3}$$

e) 
$$2\sqrt{x} \times \sqrt[3]{x}$$

$$= 2\sqrt{x^3/6} \cdot \chi^{1/3}$$

$$= 2\sqrt{x^3/6} \cdot \chi^{3/6}$$

$$= 2\sqrt{x^3/6} \cdot \chi^{3/6}$$

$$= 2\sqrt{x^3/6} \cdot \chi^{3/6}$$

$$= 2\sqrt{x^3/6} \cdot \chi^{3/6}$$

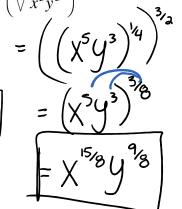
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Write an equivalent expression using exponents.

- b)  $\sqrt[3]{64v^6}$   $= ((64v^6)^{v_3})^{v_3}$   $= (64v^6)^{v_6}$   $= (44v^6)^{v_6}$   $= (44v^6)^{v_6}$



**Complete Assignment Questions #4 - #15** 

# Assignment

- 1. Write each power as an entire radical.

- **a)**  $a^{\frac{4}{5}}$  **b)**  $b^{\frac{3}{2}}$  **c)**  $c^{\frac{1}{4}}$  **d)**  $x^{-\frac{2}{5}}$  **e)**  $y^{-\frac{1}{3}}$

- **f**)  $5h^{\frac{2}{3}}$  **g**)  $(5h)^{\frac{2}{3}}$  **h**)  $-r^{\frac{5}{4}}$  **i**)  $(-r)^{\frac{5}{4}}$  **j**)  $2x^{-\frac{1}{2}}$
- 2. Simplify the following. Write each expression as a power with positive exponents and then as an entire radical.

  - **a)**  $x^{\frac{7}{2}} \times x$  **b)**  $y^{\frac{6}{5}} \div y^{\frac{4}{5}}$  **c)**  $(a^{\frac{2}{5}})^{\frac{3}{4}}$  **d)**  $(e^3 f)^{\frac{3}{2}}$

$$\mathbf{a)} \ \ x^{\frac{7}{2}} \times x$$

**a)** 
$$x^{\frac{7}{2}} \times x$$
 **b)**  $y^{\frac{6}{5}} \div y^{\frac{4}{5}}$  **c)**  $(a^{\frac{2}{5}})^{\frac{3}{4}}$  **d)**  $(e^3 f)^{\frac{3}{2}}$ 

**c**) 
$$(a^{\frac{2}{5}})^{\frac{3}{2}}$$

**d**) 
$$(e^3f)^{\frac{3}{2}}$$

**e**) 
$$x^{\frac{1}{2}} \times x^{-1}$$

**f**) 
$$y^{\frac{2}{7}} \div y^{\frac{5}{7}}$$

$$\mathbf{g}) \left(\frac{x}{y^4}\right)^{\frac{1}{2}}$$

**e)** 
$$x^{\frac{1}{2}} \times x^{-1}$$
 **f)**  $y^{\frac{2}{7}} \div y^{\frac{5}{7}}$  **g)**  $\left(\frac{x}{y^4}\right)^{\frac{1}{2}}$  **h)**  $\left(\frac{x^2}{y}\right)^{-\frac{3}{2}}$ 

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- 3. Simplify the following. Write each expression as a power with positive exponents and then as an entire radical.

**a)** 
$$2x^{\frac{3}{8}} \times 5x^{-\frac{1}{8}}$$
 **b)**  $64(a^{\frac{1}{2}})^{\frac{1}{3}}$  **c)**  $((64a)^{\frac{1}{3}})^{\frac{1}{2}}$  **d)**  $(64a^{\frac{1}{3}})^{\frac{1}{2}}$ 

**b**) 
$$64(a^{\frac{1}{2}})^{\frac{1}{3}}$$

**c**) 
$$((64a)^{\frac{1}{3}})^{\frac{1}{2}}$$

**d**) 
$$(64a^{\frac{1}{3}})^{\frac{1}{2}}$$

**e**) 
$$\frac{y^{\frac{2}{3}}y^{\frac{1}{2}}}{v^{\frac{1}{4}}}$$

$$\mathbf{f)} \ \frac{a^3b^{\frac{1}{2}}}{b^3(a^{\frac{3}{2}})^2}$$

$$\mathbf{g}) \ \frac{10x^{-\frac{3}{5}}}{5x^{\frac{1}{5}}}$$

**e**) 
$$\frac{y^{\frac{2}{3}}y^{\frac{1}{2}}}{y^{\frac{1}{4}}}$$
 **f**)  $\frac{a^3b^{\frac{1}{2}}}{b^3(a^{\frac{3}{2}})^2}$  **g**)  $\frac{10x^{-\frac{3}{5}}}{5x^{\frac{1}{5}}}$  **h**)  $\frac{(a^4)^{\frac{1}{3}}}{9} \div \frac{a}{81^{\frac{3}{4}}}$ 

- **4.** Write each radical as a power in the form  $a^n$ ,  $n \in Q$ .

- **a)**  $\sqrt[5]{a^3}$  **b)**  $\sqrt[5]{a^4}$  **c)**  $\sqrt{a^5}$  **d)**  $\frac{1}{\sqrt[4]{a}}$  **e)**  $\frac{1}{\sqrt[4]{a^5}}$

- **5.** Write as a power and evaluate.
  - **a**)  $\sqrt[3]{64}$

**b**)  $\frac{1}{\sqrt[4]{625}}$ 

**c**)  $\sqrt{\sqrt{2401}}$ 

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- **6.** Write each expression in the form  $ax^n$ , where  $a \in I$ , and  $n \in Q$ .
  - **a)**  $\sqrt[3]{27x^7}$

- **b**)  $\sqrt[4]{81x^3}$
- **c**)  $\sqrt[3]{-64x}$

$$\mathbf{d}) \left( \sqrt[4]{x^3} \right) \left( \sqrt{x} \right)$$

$$e) \quad 3\sqrt[3]{x} \times 3\sqrt[3]{x}$$

$$\mathbf{f)} \ \left( \frac{25\sqrt[3]{x^5}}{5x^{\frac{1}{3}}} \right)^2$$

$$\mathbf{u} \in (\forall x ) (\forall x)$$

e) 
$$3 \vee x \times 3 \vee x$$

$$\begin{array}{c|c} & & \\ \hline & 5x^{\frac{1}{3}} \end{array}$$

7. Write an equivalent expression using positive exponents.

a) 
$$\sqrt{\sqrt{x^5}}$$

**b**) 
$$\sqrt[3]{\sqrt{a^8}}$$

**c)** 
$$\sqrt[3]{\sqrt{729y^{12}}}$$

**d**) 
$$\sqrt[3]{\sqrt[4]{x^{\frac{2}{3}}}}$$

**e**) 
$$\left(\sqrt[4]{2y-3}\right)^{-3}$$

$$\mathbf{f)} \quad \left(\sqrt[4]{x^4y^3}\right)^{\frac{3}{2}}$$

**g**) 
$$-\sqrt[3]{x^2}$$

**h**) 
$$\sqrt[3]{(-x)^2}$$

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Matching Match each item in column 1 with the equivalent item in column 2. Each item in column 2 may be used once, more than once, or not at all.

Match each item in column 1 with the equivalent item in column 2. Each item in column 2 may be used once, more than once, or not at all.

Column 1

8. 
$$\left(\frac{p}{q}\right)^{\frac{4}{3}}$$

9. 
$$\left(\frac{p}{q}\right)^{\frac{3}{4}}$$

$$10. \quad \left(\frac{q}{p}\right)^{-\frac{4}{3}}$$

$$11. \quad \left(\frac{p}{q}\right)^{-\frac{3}{4}}$$

12. 
$$\left(\frac{q}{p}\right)^{\frac{3}{4}}$$

13. 
$$\left(\frac{p}{q}\right)^{-\frac{4}{3}}$$

Column 2

$$\mathbf{A.} \quad \sqrt[4]{\frac{q^3}{p^3}}$$

$$\mathbf{B.} \quad \sqrt[4]{\frac{p^3}{q^3}}$$

$$\mathbf{C.} \quad -\sqrt[4]{\frac{p^3}{q^3}}$$

$$\mathbf{D.} \quad \sqrt[3]{\frac{p^4}{q^4}}$$

$$\mathbf{E.} \quad \sqrt[3]{\frac{q^4}{p^4}}$$

$$\mathbf{F.} \quad -\sqrt[3]{\frac{q^4}{p^4}}$$

Multiple 14. Choice

**14.** Which expression is not equivalent to the others?

**A.** 
$$a^{-\frac{4}{3}}$$

**B.** 
$$\left(\frac{1}{a^4}\right)^3$$

C. 
$$\left(\sqrt[3]{a}\right)^{-4}$$

**D.** 
$$\frac{1}{\sqrt[3]{a^4}}$$



The value, to the nearest tenth, of the expression  $\left(\sqrt[3]{x^{\frac{4}{5}} - y^{\frac{1}{2}} + \sqrt[3]{z}}\right)^2$ when x = 32, y = 36, and z = 125 is \_\_\_\_\_.

(Record your answer in the numerical response box from left to right)



#### Answer Key

Unless otherwise indicated in the question, radicals can be given in the form  $\sqrt[n]{x^m}$  or  $\left(\sqrt[n]{x}\right)^m$  and powers can be given in the form  $x^{-n}$  or  $\frac{1}{x^n}$ . Equivalent versions of some answers are possible.

1. a) 
$$\sqrt[5]{a^4}$$

**b**) 
$$\sqrt{b^3}$$

c) 
$$\sqrt[4]{c}$$

**1.** a) 
$$\sqrt[5]{a^4}$$
 b)  $\sqrt{b^3}$  c)  $\sqrt[4]{c}$  d)  $\frac{1}{\sqrt[5]{x^2}}$  e)  $\frac{1}{\sqrt[3]{y}}$  f)  $5\sqrt[3]{h^2}$ 

e) 
$$\frac{1}{\sqrt[3]{v}}$$

**f**) 
$$5\sqrt[3]{h^2}$$

**g**) 
$$\sqrt[3]{(5h)^2}$$

**h**) 
$$-\sqrt[4]{r^5}$$

**g**) 
$$\sqrt[3]{(5h)^2}$$
 **h**)  $-\sqrt[4]{r^5}$  **i**)  $\sqrt[4]{(-r)^5}$  **j**)  $\frac{2}{\sqrt{r}}$ 

$$\mathbf{j})\frac{2}{\sqrt{x}}$$

**b**) 
$$y^{\frac{2}{5}} =$$

**c**) 
$$a^{\frac{3}{10}} = \sqrt[10]{a^3}$$

**d**) 
$$e^{\frac{9}{2}f^{\frac{3}{2}}} = \sqrt{e^9f^3}$$

$$\mathbf{e)} \quad \frac{1}{x^{\frac{1}{2}}} = \frac{1}{\sqrt{x}}$$

$$\mathbf{f)} \quad \frac{1}{\sqrt[3]{7}} = \frac{1}{\sqrt[7]{y^3}}$$

**2.** a) 
$$x^{\frac{9}{2}} = \sqrt{x^9}$$
 b)  $y^{\frac{2}{5}} = \sqrt[5]{y^2}$  c)  $a^{\frac{3}{10}} = \sqrt[10]{a^3}$  d)  $e^{\frac{9}{2}f^{\frac{3}{2}}} = \sqrt{e^9f^3}$  e)  $\frac{1}{x^{\frac{1}{2}}} = \frac{1}{\sqrt{x}}$  f)  $\frac{1}{y^{\frac{3}{7}}} = \frac{1}{\sqrt[7]{y^3}}$  g)  $\frac{x^{\frac{1}{2}}}{y^2} = \frac{\sqrt{x}}{y^2}$  h)  $\frac{y^{\frac{3}{2}}}{x^3} = \frac{\sqrt{y^3}}{x^3}$ 

**h**) 
$$\frac{y^{\frac{3}{2}}}{x^3} = \frac{\sqrt{y^3}}{x^3}$$

**3.** a) 
$$10x^{\frac{1}{4}} = 10\sqrt[4]{x}$$
 b)  $64a^{\frac{1}{6}} = 64\sqrt[6]{a}$  c)  $2a^{\frac{1}{6}} = 2\sqrt[6]{a}$  d)  $8a^{\frac{1}{6}} = 8\sqrt[6]{a}$ 

**b**) 
$$64a^{\frac{1}{6}} = 64\sqrt[6]{a}$$

**c**) 
$$2a^{\frac{1}{6}} = 2\sqrt[6]{a}$$

**d**) 
$$8a^{\frac{1}{6}} = 8\sqrt[6]{a}$$

**e**) 
$$y^{\frac{11}{12}} = \sqrt[12]{y^{11}}$$

$$\mathbf{f}) \quad \frac{1}{b^{\frac{5}{2}}} = \frac{1}{\sqrt{b^5}}$$

**e**) 
$$y^{\frac{11}{12}} = \sqrt[3]{y^{\frac{11}{11}}}$$
 **f**)  $\frac{1}{b^{\frac{5}{2}}} = \frac{1}{\sqrt{b^5}}$  **g**)  $\frac{2}{x^{\frac{4}{5}}} = \frac{2}{\sqrt[5]{x^4}}$  **h**)  $3a^{\frac{1}{3}} = 3\sqrt[3]{a}$ 

**h**) 
$$3a^{\frac{1}{3}} = 3\sqrt[3]{a}$$

$$b^{\frac{3}{2}}$$
  $\sqrt{b^5}$   $x^4$ 

**4.** a)  $a^{\frac{3}{5}}$  b)  $a^{\frac{4}{5}}$  c)  $a^{\frac{5}{2}}$  d)  $a^{-\frac{1}{4}}$  e)  $a^{-\frac{5}{4}}$ 

**5.** a)  $64^{\frac{1}{6}} = 2$  b)  $625^{-\frac{1}{4}} = \frac{1}{5}$  c)  $2401^{\frac{1}{4}} = 7$ 

**6. a)**  $3x^{\frac{7}{3}}$  **b)**  $3x^{\frac{3}{4}}$  **c)**  $-4x^{\frac{1}{3}}$  **d)**  $x^{\frac{5}{4}}$  **e)**  $9x^{\frac{2}{3}}$  **f)**  $25x^{\frac{8}{3}}$ 

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**a)** x **b)**  $a^{\frac{1}{3}}$  **c)**  $3y^{\frac{2}{3}}$  **d)**  $x^{\frac{1}{18}}$  **e)**  $\frac{1}{(2y-3)^{\frac{3}{4}}}$  **f)**  $x^{\frac{3}{2}}y^{\frac{9}{8}}$  **g)**  $-x^{\frac{2}{3}}$  **h)**  $(-x)^{\frac{2}{3}}$ 

8. D

9. B

10.D

11.A

12.A

13.E

14.B

**15.** 6

