Exponents and Radicals Lesson #7:
Rational Exponents - Part Two

Review Complete the following as a review.

Product Law	$x^m x^n =$	x^{m+n}	***Quotient Law***	$x^m \div x^n =$	x^{m-n}
Power of a Power	$(x^m)^n =$	x^{mn}	***Power of a Product***	$(xy)^m =$	$x^m y^m$
Power of a Quotient	$\left(\frac{x}{y}\right)^m =$	$\frac{x^m}{y^m}, y \neq 0$			
Integral Exponent Rule	$x^{-m} =$	$\frac{1}{x^m}$ where $x \neq 0$			
Rational Exponents	$x^{\frac{m}{n}} =$	or $(\quad)^m$			

$\left(\frac{x}{y}\right)^{-m} = \left(\frac{y}{x}\right)^m$

Writing Powers as Radicals

$x^{\frac{m}{n}} = \sqrt[n]{x^m}$ or $\left(\sqrt[n]{x}\right)^m$

Class Ex. #1

Write each power as a radical.

a) $x^{\frac{1}{6}}$ **b)** $-y^{\frac{5}{4}}$ **c)** $(-z)^{\frac{5}{3}}$ **d)** $(-z)^{-\frac{5}{3}}$ **e)** $5t^{\frac{3}{4}}$ **f)** $(5t)^{\frac{3}{4}}$

$\sqrt[6]{x}$ $-\sqrt[4]{y^5}$ $\left(\sqrt[3]{-z}\right)^5$ $\frac{1}{\sqrt[3]{(-z)^5}}$ $5\sqrt[4]{t^3}$ $\left(\sqrt[4]{5t}\right)^3$

$\sqrt[3]{(-z)^5}$

Class Ex. #2

Simplify the following. Write each expression as a power with positive exponents and then as an entire radical.

a) $x^{\frac{3}{2}} \times x$ **b)** $y^{\frac{1}{3}} \div y^{\frac{5}{3}}$ **c)** $(a^{\frac{1}{2}})^{\frac{2}{3}}$ **d)** $\left(\frac{x^2}{y}\right)^{-\frac{1}{2}}$

$x^{\frac{3}{2}+\frac{2}{2}} = x^{\frac{5}{2}}$ $= \sqrt{x^5}$

$y^{\frac{1}{3}-\frac{5}{3}} = y^{-\frac{4}{3}} = \frac{1}{\sqrt[3]{y^4}}$

$= a^{\frac{2}{6}} = a^{\frac{1}{3}} = \sqrt[3]{a}$

$= \left(\frac{y}{x^2}\right)^{\alpha}$

$\sqrt{\frac{y}{x^2}} = \frac{y^{1/2}}{x^{2/2}} = \frac{\sqrt{y}}{x}$

Class Ex. #3

Simplify the following. Write each expression as a power with positive exponents and then as an entire radical.

a) $4x^{\frac{3}{4}} \times 3x^{-\frac{1}{2}}$ **b)** $\frac{5x^{\frac{3}{5}}}{25x^{-\frac{3}{5}}}$ **c)** $(8a^{\frac{1}{2}})^{\frac{4}{3}}$

$= 12x^{3/4 + (-1/2)}$ $-2/4$

$= 12x^{3/4 - 2/4}$

$= 12x^{1/4} = 12\sqrt[4]{x}$

$= \frac{1}{5}x^{\frac{3}{5}-(-\frac{3}{5})}$

$= \frac{1}{5}x^{6/5} = \frac{1}{5}\sqrt[5]{x^6}$

$8^{4/3}a^{4/6} = \left(\sqrt[3]{8}\right)^4 a^{2/3}$

$= 2^4\sqrt[3]{a^2}$

$= 16\sqrt[3]{a^2}$

$= 12x^{1/4} = 12\sqrt[4]{x}$ $= \frac{1}{5}x^{6/5} = \frac{1}{5}\sqrt[5]{x^6}$ $= 16\sqrt[3]{a^2}$

Complete Assignment Questions #1 - #3

Writing Radicals as Powers

We can use the rule $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$ to write radicals as powers.

Radical → exponent form

Class Ex. #4

Write each radical as a power in the form $a^n,\ n \in Q$.

a) $\sqrt[3]{a^5} = a^{5/3}$ **b)** $\sqrt[5]{a^2} = a^{2/5}$ **c)** $\sqrt{a^9} = a^{9/2}$ **d)** $\frac{1}{\sqrt{a^7}} = a^{-7/2}$

Class Ex. #5

Write as a power and evaluate.

a) $\sqrt{\sqrt{1296}} = \left(1296^{1/2}\right)^{1/2} = 1296^{1/4} = 6$

b) $\frac{1}{\sqrt{169}} = 169^{-1/2} = \frac{1}{13} = 13^{-1}$

c) $\sqrt[3]{\sqrt{64}} = \sqrt[3]{8} = 2$ $= \left(64^{1/2}\right)^{1/3} = 64^{1/6} = 2$

Class Ex. #6

Write each expression in the form ax^n, where $a \in I$, and $n \in Q$.

a) $\sqrt[3]{8x^5} = \left(8x^5\right)^{1/3} = 8^{1/3}x^{5/3} = 2x^{5/3}$

b) $\sqrt[5]{32x^3} = \left(32x^3\right)^{1/5} = 32^{1/5}x^{3/5} = 2x^{3/5}$

c) $\sqrt{900x} = (900x)^{1/2} = 900^{1/2}x^{1/2} = 30x^{1/2}$

d) $\left(\sqrt[3]{5}\right)\left(\sqrt[3]{\ }\right)$

d) $\left(\sqrt[3]{x^5}\right)\left(\sqrt[3]{x}\right)$

$= \left(x^{5/3}\right)\left(x^{1/3}\right)$

$= x^{5/3+1/3} = x^{6/3}$

$= x^2$

e) $2\sqrt{x} \times \sqrt[3]{x}$

$= 2x^{1/2} \cdot x^{1/3}$

$= 2x^{3/6} \cdot x^{2/6}$

$= 2x^{3/6+2/6} = \boxed{2x^{5/6}}$

Write an equivalent expression using exponents.

a) $\sqrt{\sqrt{a^3}}$

$= \left(a^{3/2}\right)^{1/2}$

$= a^{3/4}$

b) $\sqrt{\sqrt[3]{64v^6}}$

$= \left(\left(64v^6\right)^{1/3}\right)^{1/2}$

$= \left(64v^6\right)^{1/6}$

$= 64^{1/6} v^{6/6} = \boxed{2v}$

c) $\left(\sqrt[4]{x^5y^3}\right)^{\frac{3}{2}}$

$= \left(\left(x^5y^3\right)^{1/4}\right)^{3/2}$

$= \left(x^5y^3\right)^{3/8}$

$= \boxed{x^{15/8}y^{9/8}}$

Complete Assignment Questions #4 - #15

Assignment

1. Write each power as an entire radical.

a) $a^{\frac{4}{5}}$ **b)** $b^{\frac{3}{2}}$ **c)** $c^{\frac{1}{4}}$ **d)** $x^{-\frac{2}{5}}$ **e)** $y^{-\frac{1}{3}}$

f) $5h^{\frac{2}{3}}$ **g)** $(5h)^{\frac{2}{3}}$ **h)** $-r^{\frac{5}{4}}$ **i)** $(-r)^{\frac{5}{4}}$ **j)** $2x^{-\frac{1}{2}}$

2. Simplify the following. Write each expression as a power with positive exponents and then as an entire radical.

a) $x^{\frac{7}{2}} \times x$ **b)** $y^{\frac{6}{5}} \div y^{\frac{4}{5}}$ **c)** $(a^{\frac{2}{5}})^{\frac{3}{4}}$ **d)** $(e^3f)^{\frac{3}{2}}$

a) $x^{\frac{7}{2}} \times x$ **b)** $y^{\frac{6}{5}} \div y^{\frac{4}{5}}$ **c)** $(a^{\frac{2}{5}})^{\frac{3}{4}}$ **d)** $(e^3 f)^{\frac{3}{2}}$

e) $x^{\frac{1}{2}} \times x^{-1}$ **f)** $y^{\frac{2}{7}} \div y^{\frac{5}{7}}$ **g)** $\left(\frac{x}{y^4}\right)^{\frac{1}{2}}$ **h)** $\left(\frac{x^2}{y}\right)^{-\frac{3}{2}}$

3. Simplify the following. Write each expression as a power with positive exponents and then as an entire radical.

a) $2x^{\frac{3}{8}} \times 5x^{-\frac{1}{8}}$ **b)** $64(a^{\frac{1}{2}})^{\frac{1}{3}}$ **c)** $((64a)^{\frac{1}{3}})^{\frac{1}{2}}$ **d)** $(64a^{\frac{1}{3}})^{\frac{1}{2}}$

e) $\dfrac{y^{\frac{2}{3}} y^{\frac{1}{2}}}{y^{\frac{1}{4}}}$ **f)** $\dfrac{a^3 b^{\frac{1}{2}}}{b^3 (a^{\frac{3}{2}})^2}$ **g)** $\dfrac{10x^{-\frac{3}{5}}}{5x^{\frac{1}{5}}}$ **h)** $\dfrac{(a^4)^{\frac{1}{3}}}{9} \div \dfrac{a}{81^{3/4}}$

4. Write each radical as a power in the form $a^n,\ n \in Q$.

a) $\sqrt[5]{a^3}$ **b)** $\sqrt[5]{a^4}$ **c)** $\sqrt{a^5}$ **d)** $\dfrac{1}{\sqrt[4]{a}}$ **e)** $\dfrac{1}{\sqrt[4]{a^5}}$

5. Write as a power and evaluate.

a) $\sqrt{\sqrt[3]{64}}$ **b)** $\dfrac{1}{\sqrt[4]{625}}$ **c)** $\sqrt{\sqrt{2401}}$

6. Write each expression in the form ax^n, where $a \in I$, and $n \in Q$.

a) $\sqrt[3]{27x^7}$ **b)** $\sqrt[4]{81x^3}$ **c)** $\sqrt[3]{-64x}$

d) $\left(\sqrt[4]{x^3}\right)\left(\sqrt{x}\right)$ **e)** $3\sqrt[3]{x} \times 3\sqrt[3]{x}$ **f)** $\left(\dfrac{25\sqrt[3]{x^5}}{5x^{\frac{1}{3}}}\right)^2$

d) $\left(\sqrt{x}\right)\left(\sqrt{x}\right)$ **e)** $\sqrt[3]{x} \times \sqrt[3]{x}$ **f)** $\left(\frac{}{5x^{\frac{1}{3}}}\right)$

7. Write an equivalent expression using positive exponents.

a) $\sqrt{\sqrt{x^5}}$ **b)** $\sqrt[3]{\sqrt{a^8}}$ **c)** $\sqrt[3]{\sqrt{729y^{12}}}$

d) $\sqrt[3]{\sqrt[4]{x^{\frac{2}{3}}}}$ **e)** $\left(\sqrt[4]{2y-3}\right)^{-3}$ **f)** $\left(\sqrt[4]{x^4y^3}\right)^{\frac{3}{2}}$

g) $-\sqrt[3]{x^2}$ **h)** $\sqrt[3]{(-x)^2}$

Matching Match each item in column 1 with the equivalent item in column 2. Each item in column 2 may be used once, more than once, or not at all.

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Column 1		Column 2	
8.	$\left(\frac{p}{q}\right)^{\frac{4}{3}}$	**A.**	$\sqrt[4]{\frac{q^3}{p^3}}$
9.	$\left(\frac{p}{q}\right)^{\frac{3}{4}}$	**B.**	$\sqrt[4]{\frac{p^3}{q^3}}$
10.	$\left(\frac{q}{p}\right)^{-\frac{4}{3}}$	**C.**	$-\sqrt[4]{\frac{p^3}{q^3}}$
11.	$\left(\frac{p}{q}\right)^{-\frac{3}{4}}$	**D.**	$\sqrt[3]{\frac{p^4}{q^4}}$
12.	$\left(\frac{q}{p}\right)^{\frac{3}{4}}$	**E.**	$\sqrt[3]{\frac{q^4}{p^4}}$
13.	$\left(\frac{p}{q}\right)^{-\frac{4}{3}}$	**F.**	$-\sqrt[3]{\frac{q^4}{p^4}}$

Multiple Choice

14. Which expression is not equivalent to the others?

A. $a^{-\frac{4}{3}}$

B. $\left(\frac{1}{a^4}\right)^3$

C. $\left(\sqrt[3]{a}\right)^{-4}$

D. $\frac{1}{\sqrt[3]{a^4}}$

Numerical Response **15.** The value, to the nearest tenth, of the expression $\left(\sqrt[3]{x^{\frac{4}{5}} - y^{\frac{1}{2}} + \sqrt[3]{z}}\right)^2$ when $x = 32,\ y = 36,$ and $z = 125$ is _____ .

(Record your answer in the numerical response box from left to right)

Answer Key

Unless otherwise indicated in the question, radicals can be given in the form $\sqrt[n]{x^m}$ or $\left(\sqrt[n]{x}\right)^m$ and powers can be given in the form x^{-n} or $\frac{1}{x^{n.}}$. Equivalent versions of some answers are possible.

1. a) $\sqrt[5]{a^4}$ **b)** $\sqrt{b^3}$ **c)** $\sqrt[4]{c}$ **d)** $\frac{1}{\sqrt[5]{x^2}}$ **e)** $\frac{1}{\sqrt[3]{y}}$ **f)** $5\sqrt[3]{h^2}$

g) $\sqrt[3]{(5h)^2}$ **h)** $-\sqrt[4]{r^5}$ **i)** $\sqrt[4]{(-r)^5}$ **j)** $\frac{2}{\sqrt{x}}$

2. a) $x^{\frac{9}{2}} = \sqrt{x^9}$ **b)** $y^{\frac{2}{5}} = \sqrt[5]{y^2}$ **c)** $a^{\frac{3}{10}} = \sqrt[10]{a^3}$ **d)** $e^{\frac{9}{2}}f^{\frac{3}{2}} = \sqrt{e^9f^3}$

e) $\frac{1}{x^{\frac{1}{2}}} = \frac{1}{\sqrt{x}}$ **f)** $\frac{1}{y^{\frac{3}{7}}} = \frac{1}{\sqrt[7]{y^3}}$ **g)** $\frac{x^{\frac{1}{2}}}{y^2} = \frac{\sqrt{x}}{y^2}$ **h)** $\frac{y^{\frac{3}{2}}}{x^3} = \frac{\sqrt{y^3}}{x^3}$

3. a) $10x^{\frac{1}{4}} = 10\sqrt[4]{x}$ **b)** $64a^{\frac{1}{6}} = 64\sqrt[6]{a}$ **c)** $2a^{\frac{1}{6}} = 2\sqrt[6]{a}$ **d)** $8a^{\frac{1}{6}} = 8\sqrt[6]{a}$

e) $y^{\frac{11}{12}} = \sqrt[12]{y^{11}}$ **f)** $\frac{1}{b^{\frac{5}{2}}} = \frac{1}{\sqrt{b^5}}$ **g)** $\frac{2}{x^{\frac{4}{5}}} = \frac{2}{\sqrt[5]{x^4}}$ **h)** $3a^{\frac{1}{3}} = 3\sqrt[3]{a}$

$b^{\frac{3}{2}}$ $\sqrt{b^5}$ $x^{\frac{4}{5}}$ $\sqrt[5]{x^4}$

4. a) $a^{\frac{3}{5}}$ **b)** $a^{\frac{4}{5}}$ **c)** $a^{\frac{5}{2}}$ **d)** $a^{-\frac{1}{4}}$ **e)** $a^{-\frac{5}{4}}$

5. a) $64^{\frac{1}{6}} = 2$ **b)** $625^{-\frac{1}{4}} = \frac{1}{5}$ **c)** $2401^{\frac{1}{4}} = 7$

6. a) $3x^{\frac{7}{3}}$ **b)** $3x^{\frac{3}{4}}$ **c)** $-4x^{\frac{1}{3}}$ **d)** $x^{\frac{5}{4}}$ **e)** $9x^{\frac{2}{3}}$ **f)** $25x^{\frac{8}{3}}$

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7. a) $x^{\frac{5}{4}}$ **b)** $a^{\frac{4}{3}}$ **c)** $3y^2$ **d)** $x^{\frac{1}{18}}$

e) $\dfrac{1}{(2y-3)^{\frac{3}{4}}}$ **f)** $x^{\frac{3}{2}}y^{\frac{9}{8}}$ **g)** $-x^{\frac{2}{3}}$ **h)** $(-x)^{\frac{2}{3}}$

8. D **9.** B **10.** D **11.** A **12.** A **13.** E

14. B **15.**

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