### Exponents and Radicals Lesson #3: Entire Radicals and Mixed Radicals - Part One

Recall the following from Lesson #2.

- The product(quotient) of the roots of two numbers is equal to the root of the product (quotient) of the two numbers.
- The sum (difference) of the roots of two numbers is NOT equal to the root of the sum (difference) of the two numbers.

In general  $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$  where  $a, b \ge 0$  and  $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$  where  $a \ge 0, b > 0$ .

In this lesson we use the above rules in reverse:

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$$
 where  $a, b \ge 0$  and  $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$  where  $a \ge 0, b > 0$ .



Write the following as a product or quotient of radicals.

a) 
$$\sqrt{24} = \sqrt{4 \times 1} = 6 \times 4 \times 16$$
 b)  $\sqrt{18} = \sqrt{9 \times 1} = 2 \times 16 \times 12$  =  $3\sqrt{2}$  c)  $\sqrt{\frac{11}{4}} = \frac{\sqrt{1}}{\sqrt{1}} = \frac{\sqrt{1}}{2} = \frac{\sqrt{1}}{2}$ 

#### Entire Radicals and Mixed Radicals

Use a calculator to approximate the value of each radical to 5 decimal places.

Complete the following to explain why the three radicals are equivalent. 
$$\sqrt{80} = \sqrt{4 \times 20} = \sqrt{80} = \sqrt{4 \times 20} = \sqrt{80} = \sqrt{16 \times 5} = \sqrt{1$$

 $\sqrt{80}$  is an example of an **entire radical**; the number is entirely under the root symbol.

 $2\sqrt{20}$  and  $4\sqrt{5}$  are examples of **mixed radicals**.

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Exponents and Radicals Lesson #3: Entire Radicals and Mixed Radicals- Part One

- Entire/Pure Radicals

   Radicals expressed in the form  $\sqrt[n]{b}$ are called entire (or pure) radicals.

#### **Mixed Radicals**

- Radicals expressed in the form  $a\sqrt[n]{b}$ are called mixed radicals.
- For example,  $\sqrt{25}$ ,  $\sqrt{80}$ ,  $\sqrt[3]{17}$ .

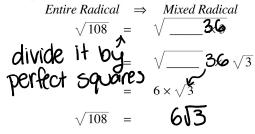
Every mixed radical can be expressed as an entire radical.

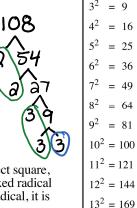
To determine if an entire radical (with an index of 2) can be expressed as a mixed radical, we need to check if the number has a factor which is a perfect square.

#### Converting Entire Radicals (with an index of 2) to Mixed Radicals

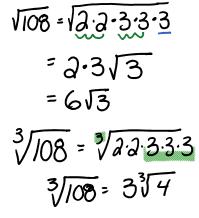
An entire radical of index 2 may be expressed as a mixed radical when the highest perfect square has been factored out of the entire radical.

Complete the following to convert  $\sqrt{108}$  to a mixed radical.





 $14^2 = 196$  $15^2 = 225$ 

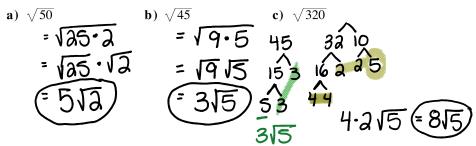




If the perfect square which is factored out is <u>not</u> the highest perfect square, then the process will require more than one step to obtain the mixed radical in simplest form. When converting an entire radical to a mixed radical, it is expected that the answer will be in simplest form.



Convert the following entire radicals to mixed radicals in simplest form.



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Exponents and Radicals Lesson #3: Entire Radicals and Mixed Radicals- Part One 1



Convert the following radicals to mixed radicals in simplest form.

a) 
$$2\sqrt{192}$$
 $\frac{1}{2}$ 
 $\frac$ 

#### Converting Entire Radicals (with an index of 3 or greater) to Mixed Radicals

An entire radical of index 3 may be expressed as a mixed radical when the highest perfect cube has been factored out of the entire radical. An entire radical of index 3 may be expressed as a mixed radical when the highest perfect cube has been factored out of the entire radical.

Complete the following to convert  $\sqrt[3]{54}$  to a mixed radical.

Entire Radical 
$$\Rightarrow$$
 Mixed Radical  $\sqrt[3]{54} = \sqrt[3]{2}$ 

$$= \sqrt[3]{2}$$

$$= \sqrt[3]{2}$$

$$= \sqrt[3]{2}$$

$$= \sqrt[3]{2}$$

$$= \sqrt[3]{2}$$

$$\sqrt[3]{54} = \sqrt[3]{2}$$

 $10^3 = 1000$ 

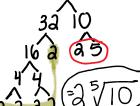


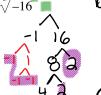
A similar process is involved for indices greater than 3.

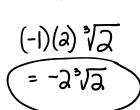


Convert the following radicals to mixed radicals in simplest form.









**Complete Assignment Questions #6 - #9** 

(2,3,4) aceg, (5,6) ac, 7

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Exponents and Radicals Lesson #3: Entire Radicals and Mixed Radicals-Part One

On L1-4

### Extension: Radicals involving Variables (Entire to Mixed)

Since  $x^3 \times x^3 = x^6$ , then  $\sqrt{x^6} =$ \_\_\_\_. Also, since  $x^5 \times x^5 \times x^5 = x^{15}$  then  $\sqrt[3]{x^{15}} =$ \_\_\_\_.

So 
$$\sqrt{x^4} =$$
\_\_\_\_\_.  $\sqrt{y^{10}} =$ \_\_\_\_\_.  $\sqrt{a^8b^6} =$ \_\_\_\_\_.  $\sqrt[3]{x^{24}} =$ \_\_\_\_.  $\sqrt[3]{y^6} =$ \_\_\_\_.

Complete the following to convert  $\sqrt{x^5}$  to a mixed radical.

$$\begin{array}{rcl} \textit{Entire Radical} & \Rightarrow & \textit{Mixed Radical} \\ \sqrt{x^5} & = & \sqrt{x^4 \times x} \\ & = & \sqrt{\times \sqrt{x}} \\ & = & \times \sqrt{x} \\ \sqrt{x^5} & = & \end{array}$$



Convert the following entire radicals to mixed radicals in simplest form.

a) 
$$\sqrt{a^7}$$

**b)** 
$$\sqrt{t^9}$$
 **c)**  $\sqrt[3]{x^5}$ 

**c)** 
$$\sqrt[3]{x^2}$$

**d**) 
$$\sqrt[3]{x}$$





Convert the following entire radicals to mixed radicals in simplest form.

- **a)**  $\sqrt{x^6y^5}$  **b)**  $\sqrt{18x^3}$  **c)**  $\sqrt{32y^7z^8}$  **d)**  $\sqrt[3]{40x^4y^9}$

#### **Complete Assignment Questions #10 - #12**

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# **Assignment**

- 1. Identify whether each radical is written as a mixed radical or an entire radical.
  - a)  $\sqrt{35}$
- **b**)  $2\sqrt{7}$
- **c**)  $\sqrt{81}$
- **d**)  $0.3\sqrt{6}$
- 2. Convert the following radicals to mixed radicals in simplest form.
  - a)  $\sqrt{8}$
- **b**)  $\sqrt{20}$
- **c**)  $\sqrt{75}$
- **d**)  $\sqrt{98}$

- - $3\sqrt{32}$  **f**)  $-5\sqrt{45}$  **g**)  $2\sqrt{54}$  **h**)  $-4\sqrt{48}$

- **3.** Convert the following radicals to mixed radicals in simplest form.
  - **a**)  $\sqrt{96}$
- **b**)  $\sqrt{242}$
- c)  $-\frac{2}{3}\sqrt{180}$  d)  $\frac{1}{8}\sqrt{320}$





- **e)**  $\sqrt{245}$  **f)**  $4\sqrt{338}$
- **g**)  $\sqrt{1250}$  **h**)  $\sqrt{980}$

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- **20** Exponents and Radicals Lesson #3: Entire Radicals and Mixed Radicals- Part One
- **4.** Convert the following radicals to mixed radicals in simplest form. There are two which cannot be converted. Identify them and explain why they cannot be converted to mixed radicals.
  - **a**)  $-\frac{5}{6}\sqrt{304}$  **b**)  $\sqrt{66}$  **c**)  $4\sqrt{272}$  **d**)  $-3\sqrt{288}$

- **e**)  $2\sqrt{369}$  **f**)  $\sqrt{364}$  **g**)  $\frac{2}{5}\sqrt{450}$  **h**)  $\frac{7}{11}\sqrt{341}$

- **5.** Convert the following radicals to mixed radicals where the radicand is a whole number. **a)**  $\sqrt{\frac{2}{9}}$  **b)**  $\sqrt{\frac{5}{4}}$  **c)**  $\sqrt{\frac{18}{25}}$  **d)**  $7\sqrt{\frac{20}{49}}$

- 6. Convert the following radicals to mixed radicals in simplest form. a)  $\sqrt[3]{48}$  b)  $\sqrt[3]{128}$  c)  $\sqrt[3]{2000}$  d)  $5\sqrt[3]{-81}$

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Exponents and Radicals Lesson #3: Entire Radicals and Mixed Radicals- Part One

- 7. Convert the following radicals to mixed radicals in simplest form.
  - **a)**  $\frac{5}{6}\sqrt[3]{108}$
- **b**)  $5\sqrt[4]{162}$

81 wixed radical, and recorded to be simplified.

15次



Multiple Choice 8.  $\sqrt[3]{240}$  is equivalent to A.  $2\sqrt[3]{40}$  B.  $4\sqrt[3]{15}$  C.  $2\sqrt[3]{30}$  D.  $8\sqrt[3]{30}$ 

Numerical Response 9. When  $\sqrt[4]{891}$  is converted to the form  $3\sqrt[4]{x}$ , the value of x is \_\_\_\_\_.

(Record your answer in the numerical response box from left to right)



#### Extension Assignment

- **10.** Express each as a mixed radical in simplest form.

- **a)**  $\sqrt{a^5}$  **b)**  $\sqrt{t^3}$  **c)**  $\sqrt{x^{11}}$  **d)**  $\sqrt[3]{x^4}$  **e)**  $\sqrt[3]{b^8}$  **f)**  $\sqrt[4]{x^6}$

- **a)**  $\sqrt{a^5}$  **b)**  $\sqrt{t^3}$  **c)**  $\sqrt{x^{11}}$  **d)**  $\sqrt[3]{x^4}$  **e)**  $\sqrt[3]{b^8}$  **f)**  $\sqrt[4]{x^6}$

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- Exponents and Radicals Lesson #3: Entire Radicals and Mixed Radicals- Part One
- 11. Express each as a mixed radical in simplest form.
- a)  $\sqrt{8v^2}$
- **b**)  $\sqrt{16p^3}$
- c)  $\sqrt{75y^3z^4}$

- **d)**  $\sqrt[3]{2000x^7}$  **e)**  $5\sqrt{28c^4d^3}$  **f)**  $-6\sqrt{29a^4b^8}$

- Multiple 12.  $\sqrt{3x} \sqrt{2x}$  is equivalent to

  - **A.**  $\sqrt{6x}$  **B.**  $\sqrt{36x^2}$

  - **C.**  $6\sqrt{x}$  **D.**  $x\sqrt{6}$

#### Answer Key

- **1.** a) entire **b)** mixed **c)** entire **d)** mixed **2.** a)  $2\sqrt{2}$  b)  $2\sqrt{5}$  c)  $5\sqrt{3}$  d)  $7\sqrt{2}$  e)  $12\sqrt{2}$  f)  $-15\sqrt{5}$  g)  $6\sqrt{6}$  h)  $-16\sqrt{3}$
- **3.** a)  $4\sqrt{6}$  b)  $11\sqrt{2}$  c)  $-4\sqrt{5}$  d)  $\sqrt{5}$  e)  $7\sqrt{5}$  f)  $52\sqrt{2}$  g)  $25\sqrt{2}$  h)  $14\sqrt{5}$
- **4.** a)  $-\frac{10}{3}\sqrt{19}$  b) cannot be converted because 66 does not have a factor which is a perfect square.
  - **d**)  $-36\sqrt{2}$  **e**)  $6\sqrt{41}$ **f**)  $2\sqrt{91}$
- **c)**  $16\sqrt{17}$  **d)**  $-36\sqrt{2}$  **e)**  $6\sqrt{41}$  **f)**  $2\sqrt{91}$  **g)**  $6\sqrt{2}$  **h)** cannot be converted because 341 does not have a factor which is a perfect square. **5. a)**  $\frac{1}{3}\sqrt{2}$  **b)**  $\frac{1}{2}\sqrt{5}$  **c)**  $\frac{3}{5}\sqrt{2}$  **d)**  $2\sqrt{5}$  **6. a)**  $2\sqrt[3]{6}$  **b)**  $4\sqrt[3]{2}$  **c)**  $10\sqrt[3]{2}$  **d)**  $-15\sqrt[3]{3}$  **7. a)**  $\frac{5}{2}\sqrt[3]{4}$  **b)**  $15\sqrt[4]{2}$  **c)**  $-2\sqrt[5]{6}$  **d)**  $-10\sqrt[3]{5}$
- 8. C 9. 1 1
- **10.a**)  $a^2\sqrt{a}$  **b**)  $t\sqrt{t}$  **c**)  $x^5\sqrt{x}$  **d**)  $x\sqrt[3]{x}$  **e**)  $b^2\sqrt[3]{b^2}$  **f**)  $x\sqrt[4]{x^2}$  **11.a**)  $2y\sqrt{2}$  **b**)  $4p\sqrt{p}$  **c**)  $5yz^2\sqrt{3y}$  **d**)  $10x^2\sqrt[3]{2x}$  **e**)  $10c^2d\sqrt{7d}$  **f**)  $-6a^2b^4\sqrt{29}$

**10.a**) 
$$a^2\sqrt{a}$$
 **b**)  $t\sqrt{t}$  **c**)  $x^3\sqrt{x}$  **d**)  $x\sqrt{x}$  **e**)  $b^2\sqrt{b^2}$  **f**)  $x\sqrt{x^2}$ 
**11.a**)  $2y\sqrt{2}$  **b**)  $4p\sqrt{p}$  **c**)  $5yz^2\sqrt{3y}$  **d**)  $10x^2\sqrt[3]{2x}$ 
**e**)  $10c^2d\sqrt{7d}$  **f**)  $-6a^2b^4\sqrt{29}$ 

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# Exponents and Radicals Lesson #4: Entire Radicals and Mixed Radicals - Part Two

Converting Mixed Radicals (with an index of 2) to Entire Radicals

A mixed radical of index 2 may be expressed as an entire radical. This is done by i) converting the number outside the radical symbol into a radical and then 2) multiplying it by the radicand. The number outside the radical symbol can be converted into a radical by raising it to the power of 2.

Complete the following to convert  $3\sqrt{14}$  to an entire radical.

Mixed Radical 
$$\Rightarrow$$
 Entire Radical  $3\sqrt{14} = \sqrt{\phantom{a}} \times 3\sqrt{34}$ 

$$= \sqrt{\phantom{a}} \times 94$$

$$3\sqrt{14} = \sqrt{\phantom{a}} \times 94$$



Convert the following mixed radicals to entire radicals.

a) 
$$2\sqrt{5}$$
. b)  $4\sqrt{7} = \sqrt{4.4.7}$  c)  $10\sqrt{6}$  =  $\sqrt{3.2.5}$  =  $\sqrt{16}\sqrt{7}$  =  $\sqrt{600}$  =  $\sqrt{4.5}$  =  $\sqrt{16.7}$  =  $\sqrt{600}$  =  $\sqrt{12}$ 



Convert the following mixed radicals to entire radicals.

a) 
$$\frac{3}{2}\sqrt{8}$$
b)  $0.4\sqrt{50}$ 
c)  $-5\sqrt{7}$ 
=  $-1\sqrt{5}.5\sqrt{7}$ 
=  $-1\sqrt{5}.5\sqrt{$ 

**Complete Assignment Questions #1 - #2** 

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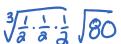
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Converting Mixed Radicals (with an index of 3 or greater) to Entire Radicals

### Converting Mixed Radicals (with an index of 3 or greater) to Entire Radicals

A mixed radical of index 3 may be expressed as an entire radical by 1) converting the number outside the radical symbol into a radical and then 2) multiplying it by the radicand. The number outside the radical symbol can be converted into a radical by raising it to the power of 3.

Complete the following to convert  $\frac{1}{2}\sqrt[3]{80}$  to an entire radical.



Mixed Radical 
$$\Rightarrow$$
 Entire Radical  $\frac{1}{2}\sqrt[3]{80} = \sqrt[3]{\times 80}$   $= \sqrt[3]{\times 80}$   $= \sqrt[3]{80}$ 



Convert the following mixed radicals to entire radicals.

cant go in, even index

a) 
$$2\sqrt[4]{3}$$
 b)  $-4\sqrt[3]{7}$  c)  $2\sqrt[5]{100}$  d)  $-3\sqrt[4]{2}$   $3\sqrt[3]{3}$   $3\sqrt[3$ 

Extension: Radicals involving Variables (Mixed to Entire)

Convert the following mixed radicals to entire radicals.



- **a**)  $2\sqrt{x^3}$
- **b**)  $a^2\sqrt{a}$
- c)  $x^5\sqrt{xy}$
- **d**)  $3xy^3 \sqrt[3]{2z^4}$

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**Complete Assignment Question #9** 

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Exponents and Radicals Lesson #4: Entire Radicals and Mixed Radicals- Part Two

# **Assignment**

1. Convert the following to entire radical form.

## *ASSIGNMENT*

- 1. Convert the following to entire radical form.
  - a)  $2\sqrt{6}$
- **b**)  $3\sqrt{7}$  **c**)  $5\sqrt{15}$  **d**)  $12\sqrt{2}$

- **e**)  $3\sqrt{25}$  **f**)  $-8\sqrt{3}$  **g**)  $9\sqrt{10}$  **h**)  $-4\sqrt{5}$

- 2. Convert the following to entire radical form. a)  $\frac{1}{2}\sqrt{27}$  b) 15 c)  $\frac{3}{2}\sqrt{8}$  d)  $3^2\sqrt{21}$

- **3.** Convert the following mixed radicals to entire radicals.

- **a)**  $2\sqrt[3]{3}$  **b)**  $-4\sqrt[3]{6}$  **c)**  $5\sqrt[4]{2}$  **d)**  $\frac{4}{5}\sqrt[3]{100}$

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- **4.** Convert the following mixed radicals to entire radicals.

- **4.** Convert the following mixed radicals to entire radicals.
- **b**)  $3\sqrt[3]{4}$  **c**)  $-3\sqrt[4]{3}$
- **d**)  $-10\sqrt[3]{5}$

- e)  $2\sqrt[5]{6}$  f)  $\frac{1}{2}\sqrt[3]{16}$  g)  $\frac{3}{10}\sqrt[4]{100\,000}$  h)  $-5\sqrt[3]{9}$

- **5.** Convert the following mixed radicals to entire radicals.
  - a)  $7\sqrt{5}$

- **b**)  $2\sqrt[3]{4}$ 
  - **c**)  $-2\sqrt[4]{3}$

- **d**)  $-10\sqrt[3]{7}$
- **e**)  $8\sqrt{10}$
- **f**)  $\frac{1}{3}\sqrt[3]{9}$

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Multiple 6. Without using a calculator, determine which of the following radicals is not equal to the

- A.  $12\sqrt{2}$
- $\sqrt{288}$
- $6\sqrt{8}$
- $4\sqrt{72}$
- 7. Consider the following two statements:

**Statement 1:**  $-3\sqrt[4]{8} = 3\sqrt[4]{-8}$ .

**Statement 2:**  $-2\sqrt[3]{7} = 2\sqrt[3]{-7}$ .

Which of the following is correct?

- Α. Both statements are true.
- В. Both statements are false.
- C. Statement 1 is true, and statement 2 is false.
- D. Statement 1 is false, and statement 2 is true.

Numerical	١,
Response	•

**8.** The mixed radical  $\frac{1}{12}\sqrt[3]{128}$  can be converted to a mixed radical in simplest form  $a\sqrt[3]{b}$ .

The value of a + b, to the nearest tenth, is \_

(Record your answer in the numerical response box from left to right)



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## Extension Assignment

- **9.** Express as an entire radical.

  - **a**)  $6\sqrt{y}$  **b**)  $8\sqrt{c^2}$
- c)  $10\sqrt{2yz^3}$  d)  $-3\sqrt[3]{x^2}$

- **e)**  $c\sqrt{c}$  **f)**  $x\sqrt{3y^3}$  **g)**  $11c^2\sqrt{c^2d}$  **h)**  $5a^3b\sqrt{3a^2b}$
- i)  $4\sqrt{3} \ a^2b$  j)  $2p^2q \sqrt[3]{5pq^2}$  k)  $7p^8q^9\sqrt{p^2r}$  l)  $2xy^3 \sqrt[4]{9x}$

#### Answer Key

- **1.** a)  $\sqrt{24}$  b)  $\sqrt{63}$  c)  $\sqrt{375}$  d)  $\sqrt{288}$  e)  $\sqrt{225}$  f)  $-\sqrt{192}$  g)  $\sqrt{810}$  h)  $-\sqrt{80}$

- **2.** a)  $\sqrt{3}$  b)  $\sqrt{225}$  c)  $\sqrt{18}$  d)  $\sqrt{1701}$

- **3. a)**  $\sqrt[3]{24}$  **b)**  $\sqrt[3]{-384}$  or  $-\sqrt[3]{384}$  **c)**  $\sqrt[4]{1250}$  **d)**  $\sqrt[3]{\frac{256}{5}}$
- **4.** a)  $\sqrt[4]{32}$  b)  $\sqrt[3]{108}$  c)  $-\sqrt[4]{243}$  d)  $-\sqrt[3]{5000}$  or  $\sqrt[3]{-5000}$  e)  $\sqrt[5]{192}$  f)  $\sqrt[3]{2}$  g)  $\sqrt[4]{810}$ **h**)  $-\sqrt[3]{1125}$  or  $\sqrt[3]{-1125}$

- **5.** a)  $\sqrt{245}$  b)  $\sqrt[3]{32}$  c)  $-\sqrt[4]{48}$  d)  $-\sqrt[3]{7000}$  or  $\sqrt[3]{-7000}$  e)  $\sqrt{640}$  f)  $\sqrt[3]{\frac{1}{3}}$

- 7. D 8. 2 .

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