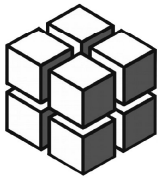
Prime Factorization and Exponents Lesson #3:
Powers with Whole Number Exponents

In this lesson we review numbers written as powers, and the exponent laws applied to powers with numerical bases and whole number exponents.
We extend the work to consider bases which are variable.

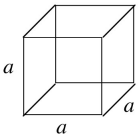
Exponents

In mathematics, exponents are used as a short way to write repeated multiplication.



The number of small cubes in the diagram can be calculated by the **repeated multiplication** $2 \times 2 \times 2$.
This can be written in **exponential form** as 2^3.

Exponents can also be used with variables.

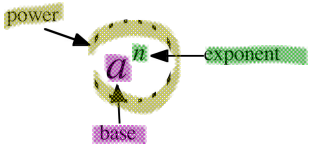


The volume of the cube to the left can be determined by repeated multiplication $a \times a \times a$ or in exponential form a^3.

Powers

A **power** is a number written in exponential form.

It consists of a **base** and an **exponent**.



State the base and the exponent in each of the following powers.

a) 4^5 base: 4 exponent: 5 **b)** $(-3)^6$ b = -3 e = 6 **c)** x^y b = x e = y

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A number that multiplies a variable is called a **coefficient**.
In the expression $7p^3$ the coefficient is 7. base: p exponent: 3

State the coefficient in each of the of the following.

a) $8x^2$ 8 **b)** $-3z^9$ -3 **c)** $\frac{a^8}{7}$ $\frac{1}{7}$

Note that written as a repeated multiplication $7p^3 = 7 \times p \times p \times p$,
whereas $(7p)^3 = 7p \times 7p \times 7p = 7 \times p \times 7 \times p \times 7 \times p = 7 \times 7 \times 7 \times p \times p \times p = 343p^3$.

Write each of the following as a repeated multiplication.

a) $3a^4b$ 3aaaab

c) $3(ab)^4$ 3ababababab

b) $3ab^4$ 3abbbb

d) $(3ab)^4$ 3ab 3ab 3ab 3ab

Evaluating Powers

$10^3 = 10 \times 10 \times 10 =$ 1000

$3^4 = 3 \times 3 \times 3 \times 3 =$ 81

$(-6)^2 = (-6) \times (-6) =$ 36

$-6^2 = -(6 \times 6) =$ -36

The Zero Exponent

Complete the patterns below by adding one more row.

$10^4 = 10000$	$3^4 = 81$
$10^3 = 1000$	$3^3 = 27$
$10^2 = 100$	$3^2 = 9$
$10^1 = 10$	$3^1 = 3$
$10^0 =$ 1	$3^0 =$ 1

The results above are examples of a general rule when a base is raised to the exponent zero.
Complete: $a^0 =$ __1__ .

Evaluate the following. **a)** 6^0 1 **b)** $(-9)^0$ 1 **c)** -9^0 -1 **d)** $2(6^2)^0$ $= 2(36)^0$ $= 2(1)$ $= 2$

Complete Assignment Questions #1 - #7

The Exponent Laws

The exponent laws with whole number exponents and numerical bases were covered in previous math courses.

The chart below extends the exponent laws to bases which are variables.

Complete the table as a review of the exponent laws.

Numerical Bases	Variable Bases	Exponent Laws
$8^3 \times 8^2 = (8 \cdot 8 \cdot 8)(8 \cdot 8)$ $= 8^5$ or 8^{3+2}	$a^3 \times a^2 = (a \cdot a \cdot a)(a \cdot a)$ $= a^5$ or a^{3+2}	**Product Law** $(a^m)(a^n) = a^{m+n}$
$8^3 \div 8^2 = \frac{8 \cdot 8 \cdot 8}{8 \cdot 8}$ $= 8^1$ or 8^{3-2}	$a^3 \div a^2 = \frac{a \cdot a \cdot a}{a \cdot a}$ $= a^1$ or a^{3-2}	**Quotient Law** $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$ $(a \neq 0)$
$(8 \cdot 7)^3 = (8 \cdot 7)(8 \cdot 7)(8 \cdot 7)$ $= (8 \cdot 8 \cdot 8)(7 \cdot 7 \cdot 7)$ $= 8^3 \cdot 7^3$	$(a \cdot b)^3 = (a \cdot b)(a \cdot b)(a \cdot b)$ $= (a \cdot a \cdot a)(b \cdot b \cdot b)$ $= a^3 b^3$	**Power of a Product Law** $(ab)^m = a^m b^m$
$\left(\frac{8}{7}\right)^3 = \left(\frac{8}{7}\right)\left(\frac{8}{7}\right)\left(\frac{8}{7}\right)$ $= \frac{8^3}{7^3}$	$\left(\frac{a}{b}\right)^3 = \left(\frac{a}{b}\right)\left(\frac{a}{b}\right)\left(\frac{a}{b}\right)$ $= \frac{a^3}{b^3}$	**Power of a Quotient Law** $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ $(b \neq 0)$
$(8^3)^2 = (8^3)(8^3)$ $= (8 \cdot 8 \cdot 8)(8 \cdot 8 \cdot 8)$ $= 8^6$ or $8^{3 \times 2}$	$(a^3)^2 = (a^3)(a^3)$ $= (a \cdot a \cdot a)(a \cdot a \cdot a)$ $= a^6$ or $a^{3 \times 3}$	**Power of a Power Law** $(a^m)^n = a^{mn}$

Use the exponent laws to simplify and then evaluate.

a) $3^4 \cdot 3^2$ $3^{4+2} = 3^6 = 729$

b) $\frac{(-2)^5}{(-2)^3}$ $(-2)^{5-3} = (-2)^2 = 4$

c) $(5^2)^3$ $5^{2 \cdot 3} = 5^6 = 15625$

calculator: 3^6 or 3x^y 6

Use the exponent laws to simplify.

a) $(a)^4(a)^3$ $a^{4+3} = a^{12}$

b) $x^6 x$ $x^{6+1} = x^7$

c) $b^7 \times b^0 \times b^4$ $b^{7+0+4} = b^{11}$

d) $\frac{x^8}{x^4}$ $x^{8-4} = x^4$

e) $\frac{y^{10}}{y^2}$ $y^{10-2} = y^8$

f) $(a^4)^3$ a^{12}

g) $(y^5)^5$ $= y^{5 \cdot 5} = y^{25}$

h) $\left(\frac{x}{y}\right)^9$ $= \frac{x^9}{y^9}$

i) $\left(\frac{c}{4}\right)^2$ $= \frac{c^2}{4^2} = \frac{c^2}{16}$

j) $(st)^6$ $= s^6 t^6$

k) $(2a)^5$ $= 2^5 a^5$ $= 32a^5$

l) $(-3pq)^4$ $(-3)^4 p^4 q^4$ 3 · -3 (+ ✓ − ✓ +) $+3^4 p^4 q^4 = 81 p^4 q^4$

Complete Assignment Questions #8 - #22

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Assignment

Complete Assignment Questions #8 - #22

Assignment

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$+3^4p^4q^4 = 81p^4q^4$

Assignment 1de, 2be, 3bdfhj, 4bcf, 5, 6, 7be, 15-18, 21

1. State the base and the exponent in each of the following powers.

a) 8^3 **b)** k^{15} **c)** 2^x **d)** $(-x)^4$ **e)** $\left(\frac{3}{4}\right)^6$

2. State the coefficient in each of the following.

a) $5x^7$ **b)** $-6z^2$ **c)** a^3 **d)** $\frac{y^3}{4}$ **e)** $\frac{5y^9}{8}$

3. Write each of the following as a repeated multiplication.

a) c^4 **b)** $5x^3$ **c)** $(ab)^2$ **d)** $(-5)^3$ **e)** s^2t

f) $2\left(\frac{5}{4}\right)^3$ **g)** $(4a)^3$ **h)** $3cd^2$ **i)** $3(cd)^2$ **j)** $(3cd)^2$

4. Evaluate.

a) 3^8 **b)** -5^2 **c)** $(-5)^2$

d) $(-5)^3$ **e)** -5^3 **f)** $\left(\frac{3}{5}\right)^3$

5. Evaluate without using a calculator.

a) -10^2 **b)** $(-10)^2$ **c)** -10^3 **d)** $(-10)^3$

6. Explain why -8^0 and $(-8)^0$ have different values.

7. Evaluate without using a calculator.

a) 32^0 **b)** -1^0 **c)** $\left(-\frac{1}{2}\right)^0$ **d)** $\frac{1}{2}(4)^0$ **e)** $\frac{1}{2}(4^2)^0$

8. Write in a simpler form and evaluate.

a) $9^3 \cdot 9^6$ **b)** $(7^2)^3$ **c)** $\frac{8^{15}}{8^{13}}$ **d)** $\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)^3$

e) $\frac{1.5^7}{1.5^5}$ **f)** $(-3^3)^2$ **g)** $(-5)^6 \times (-5)^2$ **h)** $4^3 \cdot 4^4 \cdot 4^2$

9. Explain using factors why $(x^2)(x^3) \neq (x^2)^3$.

10. Use the Product Law to simplify.

a) $a^4 \times a^2$ **b)** $m^6 \cdot m^3$ **c)** $(s^5)(s^5)$ **d)** $x^6 x^5$ **e)** $y^{10} \times y^2$

11. Use the Quotient Law to simplify.

a) $\frac{t^8}{t^2}$ **b)** $x^6 \div x^4$ **c)** $\frac{p^{10}}{p^9}$ **d)** $d^{18} \div d^9$ **e)** $p^8 \div p$

12. Use the Power of a Product Law to simplify.

a) $(xy)^5$ **b)** $(mn)^4$ **c)** $(3x)^3$ **d)** $(10z)^3$ **e)** $\left(\frac{1}{2}c\right)^2$

f) $(2b)^4$ **g)** $(-x)^3$ **h)** $(-3y)^4$ **i)** $(-4pq)^2$ **j)** $(-4pq)^3$

13. Use the Power of a Quotient Law to simplify.

a) $\left(\frac{x}{y}\right)^2$ **b)** $\left(\frac{a}{b}\right)^6$ **c)** $\left(\frac{5}{c}\right)^4$ **d)** $\left(\frac{b}{5}\right)^3$ **e)** $\left(\frac{z}{y}\right)^{10}$

14. Use the Power of a Power law to simplify.

a) $(p^2)^2$ **b)** $(h^4)^5$ **c)** $(b^4)^3$ **d)** $(s^9)^{10}$ **e)** $(z^7)^3$

15. State the value of x in each of the following.

a) $(a^3)(a^x) = a^9$ **b)** $b^x \cdot b^4 = b^8$ **c)** $c^x \div c^4 = c^{12}$

15. State the value of x in each of the following.

a) $(a^3)(a^x) = a^9$ **b)** $b^x \cdot b^4 = b^8$ **c)** $c^x \div c^4 = c^{12}$

d) $\dfrac{d^{10}}{d^x} = d^2$ **e)** $(e^x)^5 = e^{15}$ **f)** $(f^7)^x = f^7$

16. Use the exponent laws to simplify.

a) $\dfrac{x^{12}}{x^3}$ **b)** $(xy)^7$ **c)** $(t^3)^3$ **d)** t^3t^3

e) $y^4 \times y^8$ **f)** $\left(\dfrac{a}{b}\right)^{11}$ **g)** $\left(\dfrac{d}{2}\right)^3$ **h)** $(2st)^6$

17. Simplify.

a) $g^{12}g^3$ **b)** $\dfrac{a^7}{a^5}$ **c)** $(3bc)^4$ **d)** $\left(\dfrac{5}{y}\right)^2$

e) $(-a)^4$ **f)** $\left(-\dfrac{1}{3}pq\right)^4$ **g)** $(a^3)(a^4)(a^5)$ **h)** $\dfrac{x^6}{x^0}$

18. Simplify.

a) $\frac{y^7}{y^7}$ **b)** $(-ab)^5$ **c)** $(m^6)(m^6)$ **d)** $(r^0)^3$

e) $c^3c^4c^5c^6$ **f)** $(-ab)^6$ **g)** $\frac{1}{a^3a^5}$ **h)** $2(xy)^3$

19. After marking an exponents quiz a teacher recorded the most common errors made by students. In each case, identify the error made by the students and, where possible, provide the correct simplification.

a) $2^3 \times 2^4 = 2^{12}$ **b)** $(4^3)^2 = 4^9$

c) $3^4 \times 3^5 = 9^9$ **d)** $3^2 \times 2^3 = 6^5$

e) $(-5a^2b)^3 = -5a^6b^3$ **f)** $\left(\frac{1}{2}pq\right)\left(\frac{1}{2}pq\right) = p^2q^2$

Matching **20.** Match each item in column 1 on the left with the equivalent item in column 2 on the right. Each item in column 2 may be used once, more than once, or not at all.

Column 1		Column 2	
i)	$(-a^2)^3$	**A.**	a^4
ii)	$(-a^3)^2$	**B.**	a^5
iii)	$a^3 \times a^2$	**C.**	a^6
iv)	$a^8 \div a^2$	**D.**	a^{24}
v)	$a^{30} \div a^6$	**E.**	$-a^5$
vi)	$-(a^2)^3$	**F.**	$-a^6$
vii)	$-(a^3)^2$		

Use the following information to answer the next question.

$(2^3)^p = 2^{12}$	$\frac{4^{10}}{4^q} = 4^2$
$2^r \cdot 2^r = 2^{16}$	$(3^s)^2 = 1$

Numerical Response **21.** Write the value of p in the first box.
Write the value of q in the second box.
Write the value of r in the third box.
Write the value of s in the fourth box.
(Record your answer in the numerical response box from left to right)

22. If $(a^n)\,(a^n)\,(a^n) = a^{27}$, where n is a whole number, then the value of n is _____ .

(Record your answer in the numerical response box from left to right)

Answer Key

1. **a)** base 8, exponent 3 **b)** base k, exponent 15 **c)** base 2, exponent x
d) base $-x$, exponent 4 **e)** base $\frac{3}{4}$, exponent 6

2. **a)** 5 **b)** -6 **c)** 1 **d)** $\frac{1}{4}$ **e)** $\frac{5}{8}$

3. **a)** $c \times c \times c \times c$ **b)** $5 \times x \times x \times x$ **c)** $a \times a \times b \times b$ **d)** $(-5) \times (-5) \times (-5)$
e) $s \times s \times t$ **f)** $2 \times \frac{5}{4} \times \frac{5}{4} \times \frac{5}{4}$ **g)** $4 \times 4 \times 4 \times a \times a \times a$
h) $3 \times c \times d \times d$ **i)** $3 \times c \times c \times d \times d$ **j)** $3 \times 3 \times c \times c \times d \times d$

4. **a)** 6561 **b)** -25 **c)** 25 **d)** -125 **e)** -125 **f)** $\frac{27}{125}$

5. **a)** -100 **b)** 100 **c)** -1000 **d)** -1000

6. $-8^0 = -1$ since the exponent applies only to the base 8.
$(-8)^0 = 1$ since the exponent applies to the base -8.

7. **a)** 1 **b)** -1 **c)** 1 **d)** $\frac{1}{2}$ **e)** $\frac{1}{2}$

8. **a)** $9^9 = 387\,420\,489$ **b)** $7^6 = 117\,649$ **c)** $8^2 = 64$ **d)** $\left(\frac{2}{3}\right)^4 = \frac{16}{81}$
e) $1.5^2 = 2.25$ **f)** $3^6 = 729$ **g)** $(-5)^8 = 390\,625$ **h)** $4^9 = 262\,144$

9. $(x^2)(x^3) = (x \times x)(x \times x \times x) = (x \times x \times x \times x \times x) = x^5$
$(x^2)^3 = x^2 \times x^2 \times x^2 = x \times x \times x \times x \times x \times x = x^6$
Therefore $(x^2)(x^3) \neq (x^2)^3$.

10. **a)** a^6 **b)** m^9 **c)** s^{10} **d)** x^{11} **e)** y^{12}

11. **a)** t^6 **b)** x^2 **c)** p **d)** d^9 **e)** p^7

12. **a)** x^5y^5 **b)** m^4n^4 **c)** $27x^3$ **d)** $1000z^3$ **e)** $\frac{1}{4}c^2$
f) $16b^4$ **g)** $-x^3$ **h)** $81y^4$ **i)** $16p^2q^2$ **j)** $-64p^3q^3$

13. **a)** $\frac{x^2}{y^2}$ **b)** $\frac{a^6}{b^6}$ **c)** $\frac{625}{c^4}$ **d)** $\frac{b^3}{125}$ **e)** $\frac{z^{10}}{y^{10}}$

14. **a)** p^4 **b)** h^{20} **c)** b^{12} **d)** s^{90} **e)** z^{21}

15. **a)** 6 **b)** 4 **c)** 16 **d)** 8 **e)** 3 **f)** 1

16. **a)** x^9 **b)** x^7y^7 **c)** t^9 **d)** t^6 **e)** y^{12} **f)** $\frac{a^{11}}{b^{11}}$ **g)** $\frac{d^3}{8}$ **h)** $64s^6t^6$

17. **a)** g^{15} **b)** a^2 **c)** $81b^4c^4$ **d)** $\frac{25}{y^2}$ **e)** a^4 **f)** $\frac{1}{81}p^4q^4$ **g)** a^{12} **h)** x^6

18. **a)** $y^0 = 1$ **b)** $-a^5b^5$ **c)** m^{12} **d)** $r^0 = 1$ **e)** c^{18} **f)** a^6b^6 **g)** $\frac{1}{a^8}$ **h)** $2x^3y^3$

19. **a)** The student multiplied 3 and 4, instead of adding 3 and 4. The correct simplification is $2^{3+4} = 2^7$.
b) The student squared 3 instead of multiplying 3 and 2. The correct simplification r is $4^{3\times 2} = 4^6$.
c) The student multiplied the bases together. The correct simplification is 3^9.
d) The student multiplied the bases together. The exponent laws are only valid when the bases are the same. No simplification.
e) The student did not cube the 5. The correct simplification is $-125a^6b^3$.
f) The student added $\frac{1}{2}$ and $\frac{1}{2}$ instead of multiplying $\frac{1}{2}$ and $\frac{1}{2}$. The correct simplification is $\frac{1}{4}p^2q^2$.

20. **i)** F **ii)** C **iii)** B **iv)** C **v)** D **vi)** F **vii)** F

21.

4	8	8	0

22.

9			