

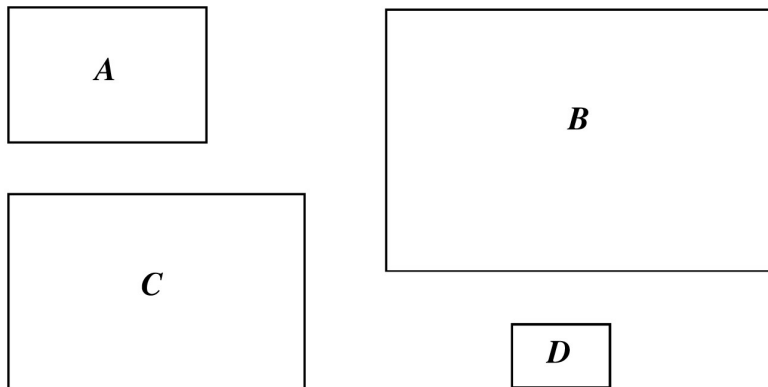
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## Measurement Lesson #4: Linear Scale Factors and Perimeter

In earlier math courses we studied the concept of **similar** objects - objects which have the same shape but not the same size. One way of describing the enlargement or reduction of an object proportionally is by **scale factors**.

### Linear Scale Factor Investigation

Rectangles *B*, *C*, and *D* are all similar to rectangle *A*.



1. Measure in cm, and then complete the following.

- First answer as a reduced fraction.
- Second, express the reduced fraction to one decimal place.

a) i)  $\frac{\text{Length of } \square B}{\text{Length of } \square A} = \frac{6}{3} = \frac{2}{1} = 2$     ii)  $\frac{\text{Width of } \square B}{\text{Width of } \square A} = \frac{4}{2} = \frac{2}{1} = 2$

b) i)  $\frac{\text{Length of } \square C}{\text{Length of } \square A} = \frac{4.5}{3} = \frac{3}{2} = 1.5$     ii)  $\frac{\text{Width of } \square C}{\text{Width of } \square A} = \frac{3}{2} = 1.5$

c) i)  $\frac{\text{Length of } \square D}{\text{Length of } \square A} = \frac{1.5}{3} = \frac{1}{2} = 0.5$     ii)  $\frac{\text{Width of } \square D}{\text{Width of } \square A} = \frac{1}{2} = 0.5$

2. Use your results from #1 to complete the following.

- a) i) The length of *B* is 2 times the length of *A*.  
 ii) The width of *B* is 2 times the width of *A*.
- b) i) The length of *C* is 1.5 times the length of *A*.  
 ii) The width of *C* is 1.5 times the width of *A*.
- c) i) The length of *D* is 0.5 times the length of *A*.  
 ii) The width of *D* is 0.5 times the width of *A*.

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**Linear Scale Factor**

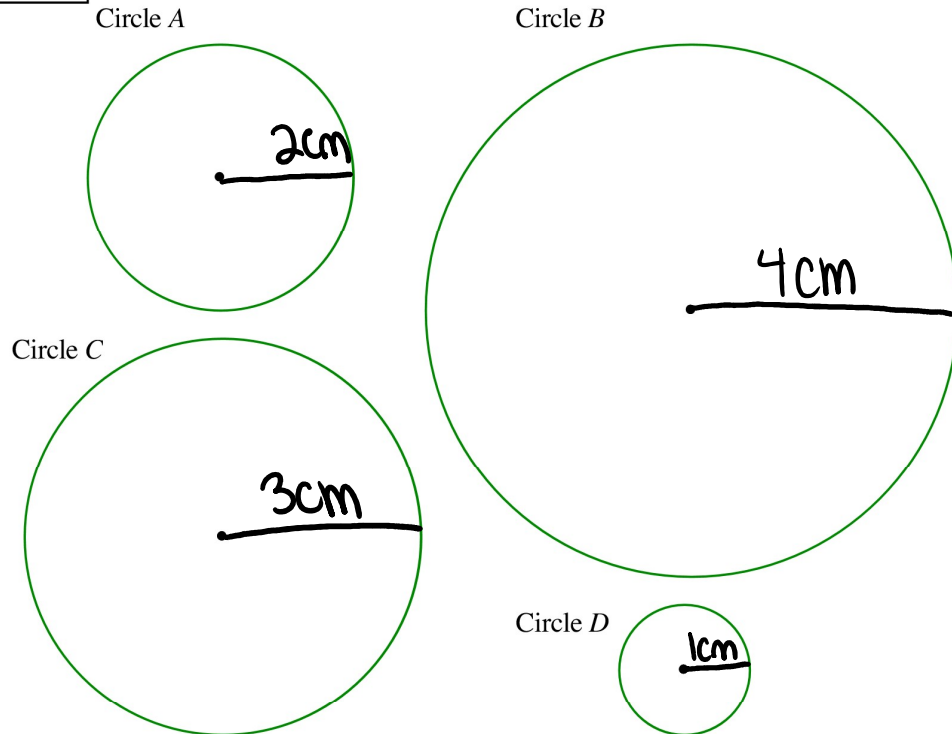
A linear scale factor describes the enlargement or reduction of length. It is described as a ratio in the form  $a : b$  or as a rational number  $\frac{a}{b}$ , or as a percent. For example

- The **linear scale factor** of □B from □A is 2 : 1 or 2 or 200%
  - The **linear scale factor** of □C from □A is 3 : 2 or 1.5 or 150%
  - The **linear scale factor** of □D from □A is 1 : 2 or 0.5 or 50%
- } enlarge  
} reduction

A scale factor **greater than 1** describes an **enlargement**.  
 A scale factor **between 0 and 1** describes a **reduction**.

**Investigating Relationship Between Linear Scale Factor and Perimeter Scale Factor**

**Part 1**



- a) Measure, to the nearest cm, the radius of each circle and write the measure on the diagram.
- b) Circles B, C, and D are enlargements or reductions of Circle A. Determine the linear scale factor in each case.
- The **linear scale factor** of circle B from circle A is 2 (4:2)
  - The **linear scale factor** of circle C from circle A is 1.5 (3:2)
  - The **linear scale factor** of circle D from circle A is 0.5 (1:2)

- c) The perimeter of a circle is called the circumference, which can be calculated using the formula  $C = 2\pi r$ . Calculate the circumference of each circle as an exact value (i.e. as a multiple of  $\pi$ ). The first one has been completed.

Circle A: Circumference =  $2\pi(2) = 4\pi$       Circle B: Circumference =  $2\pi(4) = 8\pi$   
 Circle C: Circumference =  $2\pi(3) = 6\pi$       Circle D: Circumference =  $2\pi(1) = 2\pi$

- d) Determine the perimeter scale factors by completing the following.

i) Perimeter scale factor of B from A =  $\frac{\text{Circumference of B}}{\text{Circumference of A}} = \frac{8\pi}{4\pi} = \frac{2}{1} = 2$

ii) Perimeter scale factor of C from A =  $\frac{\text{Circumference of C}}{\text{Circumference of A}} = \frac{6\pi}{4\pi} = \frac{3}{2} = 1.5$

iii) Perimeter scale factor of D from A =  $\frac{\text{Circumference of D}}{\text{Circumference of A}} = \frac{2\pi}{4\pi} = \frac{1}{2} = 0.5$

- e) What do you notice about the linear scale factors in part b) and the perimeter scale factors in part d)?

**Part 2**

- a) Complete the following to investigate the relationship between linear scale factor and perimeter scale factor. The first one has been done.

	Original Dimensions of Rectangle (cm)	Original Perimeter (cm)	Linear Scale Factor Applied to Rectangle	New Dimensions of Rectangle (cm)	New Perimeter (cm)	Perimeter Scale Factor $\frac{\text{New Perimeter}}{\text{Original Perimeter}}$
P	3 x 5	16 cm	2 : 1 or 2	6 x 10	32 cm	$\frac{\text{New Perimeter}}{\text{Original Perimeter}} = \frac{32}{16} \Rightarrow \frac{2}{1} \Rightarrow 2$
Q	2 x 6	16 cm	3 : 1 or 3	6 x 18	48 cm	$\frac{48}{16} = \frac{3}{1} = 3$
R	9 x 6	30 cm	1 : 3 or $\frac{1}{3}$	3 x 2	10 cm	$\frac{10}{30} = \frac{1}{3}$
S	3 x 12	30 cm	2 : 3 or $\frac{2}{3}$	2 x 8	20 cm	$\frac{20}{30} = \frac{2}{3}$

- b) Compare the linear scale factors to the perimeter scale factors on the chart in a). Use these results to make the following statement true:

Perimeter scale factor = linear scale factor

- c) Complete the table using the linear scale factors in a).

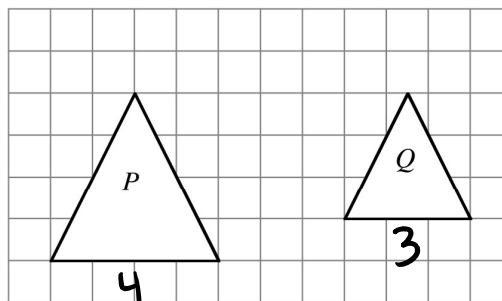
<input type="checkbox"/>	Linear Scale Factor	Enlargement or Reduction
P	2	e
Q	3	e
R	$\frac{1}{3}$	r
S	$\frac{2}{3}$	(



Class Ex. #1

Determine the scale factor that will transform diagram P to diagram Q. Give your answer as

- a) a ratio  $3:4$   
 b) a rational number  $\frac{3}{4}$  or  $0.75$   
 c) a percent  $75\%$



Class Ex. #2

Sara increased the length and width of a rectangular 8" x 10" photograph by a factor of 5:2.

- a) Is this an enlargement or reduction?

enlargement

- b) Calculate the new dimensions of the picture.

$$\text{length} = 8 \times \frac{5}{2} = 20''$$

$$\text{width} = 10 \times \frac{5}{2} = 25''$$

The new picture is 20" x 25"

- c) Show that the ratio  $\frac{\text{new perimeter}}{\text{original perimeter}} = \frac{5}{2}$

$$\text{new perimeter} = 2(25) + 2(20) = 90''$$

$$\text{original perimeter} = 2(10) + 2(8) = 36''$$

$$\text{ratio} = \frac{90}{36} = \frac{5}{2}$$



Complete Assignment Questions #1 - #10

#3, 4, 5, 6, 8, 9

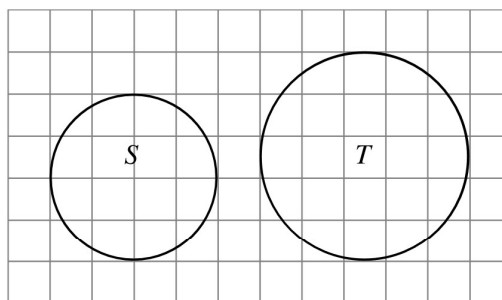
## Assignment

1. State whether the following scale factors represent enlargements or reductions.

- a) scale factor =  $\frac{2}{3}$       b) scale factor = 1.2      c) scale factor = 80%

2. Determine the scale factor that will transform circle *S* to circle *T*. Give your answer as

- a) a ratio  
b) a rational number  
c) a percent

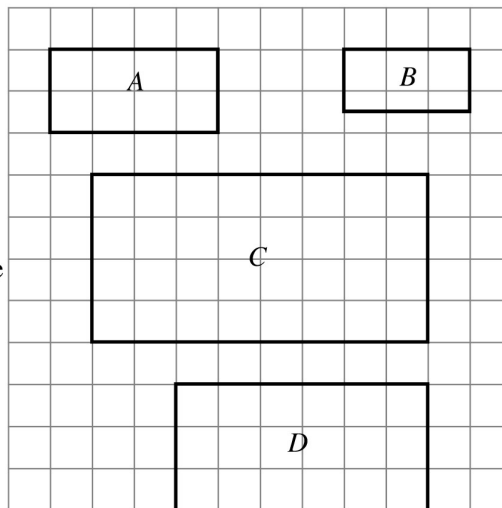


3. a) Determine, as a ratio, the scale factor that was used to transform

- i) *A* to *B*      ii) *B* to *A*

b) Determine, as a rational number, the scale factor that was used to transform

- i) *A* to *C*      ii) *D* to *A*



c) Determine, as a percent, the scale factor that was used to transform

- i) *C* to *A*      ii) *D* to *B*      iii) *B* to *D*      iv) *C* to *B*

4. Complete the table.

	Original Dimensions of Figure (cm)	New Dimensions of Figure (cm)	Linear Scale Factor	Enlargement or Reduction
a)	2 x 5	6 x 15		
b)	3 x 7	1.5 x 3.5		
c)	12 x 8	18 x 12		
d)	9 x 6	6 x 4		
e)	8 x 10	14 x 17.5		
f)	15 x 11	11.25 x 8.25		

5. Diagram 2 is a reduction of Diagram 1.

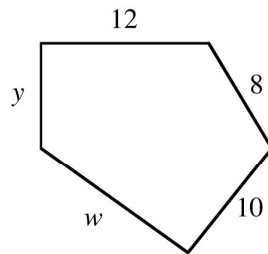


Diagram 1

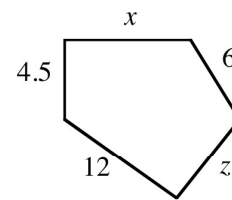


Diagram 2

- State the scale factor of the reduction.
- Use proportional reasoning to determine the values of  $w$ ,  $x$ ,  $y$ , and  $z$ .

6. A triangular object has a perimeter 87 m. What is the new perimeter if a linear scale factor of 4:1 is applied to it?
  
7. A corn field is rectangular in shape and has a perimeter of 8 km. Based on last year's demand, the farmer decides to increase his corn production by dedicating more land to growing corn. He increases the dimensions of his original corn field by a linear scale factor of 2.5. Determine the perimeter of the new cornfield.
  
8. The circumference of a circle is reduced from 35 cm to 14 cm. What is the linear scale factor of the reduction?
  
  
9. Emillie has a rectangular-shaped political cartoon drawing measuring 16 cm x 24 cm.
  - a) She needs to enlarge the dimensions by 25% for inclusion in her Social Studies presentation.
    - i) Determine the linear scale factor of the enlargement in simplest ratio form.
  
    - ii) Calculate the perimeter of the enlargement.
  
  - b) Emillie's Social Studies teacher also asked for a written report after the presentation. Emillie decides to reduce the original political drawing by 75%.
    - i) What is the scale factor of the reduction?
  
    - ii) Determine the perimeter of the reduced drawing.



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Use the following information to answer the next question.

A circle has a diameter of 20 cm. It is transformed into a smaller circle whose diameter is 12 cm smaller. Students were asked to determine the scale factor of the reduction. Their answers are given below:

**Lien:** 0.4      **Kendra:** 60%      **Helen:**  $\frac{3}{5}$       **Bonnie:** 40%

**Multiple Choice**

10. The correct answer was given by
- A. Kendra and Helen only
  - B. Lien and Kendra only
  - C. Lien and Bonnie only
  - D. some other combination of the students

**Answer Key**

1. a) reduction    b) enlargement    c) reduction      2. a) 5:4    b) 1.25    c) 125%
3. a) i) 3:4    ii) 4:3    b) i) 2    ii)  $\frac{2}{3}$     c) i) 50%    ii) 50%    iii) 200%    iv) 37.5%
4. See chart below

	Original Dimensions of Figure (cm)	New Dimensions of Figure (cm)	Linear Scale Factor	Enlargement or Reduction
a)	2 x 5	6 x 15	3:1 or 3	Enlargement
b)	3 x 7	1.5 x 3.5	1:2 or 0.5	Reduction
c)	12 x 8	18 x 12	3:2 or 1.5	Enlargement
d)	9 x 6	6 x 4	2:3 or $\frac{2}{3}$	Reduction
e)	8 x 10	14 x 17.5	7:4 or 1.75	Enlargement
f)	15 x 11	11.25 x 8.25	3:4 or 0.75	Reduction

5. a)  $\frac{3}{4}$     b)  $w = 16, x = 9, y = 6, z = 7.5$     6. 348 m    7. 20 km    8. 2:5 or 0.4
9. a) i) 5:4    ii) 100 cm    b) i) 1:4    ii) 20 cm    10. C

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