Lesson 4: Mortgages

Financial Mathematics Lesson #4:
Mortgages

Mortgages

A **mortgage** is a special type of loan that is used to purchase property. The property itself is used as **collateral** - which the lending institution uses to recover its loan in the event that the borrower does not pay back the loan. The person borrowing the money is called the **mortgagor** and the institution lending the money is called the **mortgagee**.

Amortization

To **amortize** a mortgage is to repay a mortgage in equal periodic payments over a given period of time.

The **amortization period** is the time it would take to completely pay off the mortgage if the interest and periodic payments remained constant. It is common to have amortization periods of 15, 20, or 25 years.

The **term** of the mortgage is the length of time the mortgagee agrees to make the mortgage payments to the bank. In the past, the term of the mortgage and the amortization period were the same thing.

Today, with fluctuating interest rates and the desire of lending institutions to increase profits, the **term** of a mortgage has taken a different meaning. An amortization period is still given as the period of time it would take to pay off the loan, but the term of the loan can be a much shorter period of time. **At the end of the term, a new mortgage agreement is negotiated with the lending institution and a new amortization period may result, with a different interest rate**.

- By Canadian law, mortgage interest must be calculated at most semi-annually. Therefore, for mortgages, C/Y will always be 2, and most of the time P/Y is 12, which represents monthly payments.
- Payments are made at the end of the month for mortgages and loans.

Complete the chart.

Scenario	Amortization Period	Term
a) A three year mortgage at 5.25% amortized over 25 years.	25	3
b) A mortgage negotiated for five years at 4.20%. If payments are continued at this rate, the loan will be paid off in 15 years.	15	5

Justin and Jennifer bought a house in Ridge Acres for $300,000. They made a 20% down payment and negotiated a five year mortgage at 3.82% per annum compounded semi-annually. The mortgage is amortized over 25 years.

a) Determine:

- the amortization period
  25 year
- the term of the mortgage
  5 years ← mortgage amount
- the amount of the down payment
  20% of 300 000
  (0.2)(300000) = $60,000
- the amount to be financed
  300,000 − 60,000 = 240,000

b) Complete the following using the TVM Solver to determine what the monthly mortgage payment will be.

What is the **PMT** value and why is it negative?

PMT = −1239.14
neg because out-of-pocket payment

```
N=300
I%=3.82
PV=240000
PMT=-1239.14
FV=0
P/Y=12
C/Y=2
PMT:END BEGIN
```

Explain why the entry for N is 25*12 = 300.
25 years → 12 payments per year

What is the **PV** value and why is it positive?
it is in-pocket, bank is giving that to you

c) What would their house really cost if the term was continuously renegotiated to be the same throughout the amortization period?

300 (1239.14)
= 371,742.00 mortgage payments
+ down payment 60000 = $431,742

d) After the five year term, the mortgage is up for renewal. What is the amount on the mortgage left to be renewed?

```
N= 5 x12
I%= 3.82
PV= 240000
PMT=-1239.14
FV= -208269.38
P/Y= 12
C/Y= 2
PMT: END BEGIN
```

$208 269.38

e) Describe the changes to the TVM solver if Justin and Jennifer choose to pay their mortgage weekly. Calculate their weekly payments under this scenario.

change N to 52 x 25
P/Y = 52
F(V) = 0

PMT = −286.65
pay $286.65 weekly

f) Most banks offer an accelerated weekly payment plan. The weekly payment is calculated by dividing the monthly payment by 4, then the resulting amount is paid weekly.

- Explain why the accelerated weekly payment plan will pay the mortgage off faster.

- What would be the weekly payment using the accelerated weekly payment plan?

- Assuming that the interest rate remains the same throughout the amortization period, determine how long it will take to pay off the mortgage using the accelerated weekly payment plan.

Class Ex. #3

James and Courtney purchased a house for $280,000. They made a 30% down payment and negotiated a three year mortgage at 5.25% amortized over 20 years.

a) Calculate their monthly payments.

N=	20 x 12
I%=	5.25
PV=	196000
*PMT=	-1314.553...
FV=	0
P/Y=	12
C/Y=	2
PMT:	END BEGIN

$1314.55

280000(0.3) = 84000 dwnpy
280000 - 84000 = 196000

b) After the three year term was completed, they re-negotiated a new term for seven years at 3.17% amortized over 15 years. Determine their new monthly payments.

N=	3 x 12
I%=	5.25
PV=	196000
PMT=	-1314.55
*FV=	-177877.88
P/Y=	12
C/Y=	2
PMT:	END BEGIN

↑ first 3 years

N=	15 x 12
I%=	3.17
PV=	177877.88
*PMT=	-1241.20
FV=	0
P/Y=	12
C/Y=	2
PMT:	END BEGIN

$1241.20

Complete Assignment Questions #1 - #10

Assignment

#(1-3)b, 4, 7, 8, 10
Quiz Thursday
L1-4

1. Calculate the monthly mortgage payment in each of the following.

a) $185 000 at 8.25% per annum for three years amortized over 15 years.

N=	
I%=	
PV=	
PMT=	
FV=	
P/Y=	
C/Y=	
PMT:	END BEGIN

b) $315 549 at 3.35% per year for five years amortized over 25 years.

N=	
I%=	
PV=	
PMT=	
FV=	
P/Y=	
C/Y=	
PMT:	END BEGIN

c) $215 725 at 5.14% per annum for four years amortized over 20 years.

N=	
I%=	
PV=	
PMT=	
FV=	
P/Y=	
C/Y=	
PMT:	END BEGIN

2. Calculate the remaining amount left on the loan after the term has been completed.

a) \$205 000 at 7.15% per annum for three years amortized over 20 years.

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

b) \$159 549 at 3.35% per year for five years amortized over 15 years.

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

c) \$50 800 at 6.14% per annum for two years amortized over 20 years.

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

3. Calculate the interest rate, to the nearest 0.01%, on the term of the mortgage.

a) \$110 000 mortgage with a monthly payment of \$820. Amortization is 25 years.

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

b) \$215 000 mortgage with a monthly payment of \$1 795. Amortization is 20 years.

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

c) \$575 000 mortgage with a monthly payment of \$4 995.52. Amortization is 15 years.

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

4. A land development company purchased a rental property for \$279 000. They made a 10% down payment and negotiated a five year mortgage at 6.95% amortized over 25 years.

a) Determine:

- the amortization period
- the term of the mortgage
- the amount of the down payment
- the amount to be financed

b) Determine the monthly payment.

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

c) What would the rental property really cost if the term was continuously renegotiated to be the same throughout the amortization period?

d) After the five year term, the mortgage is up for renewal. What is the amount of the mortgage left to be renewed?

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

5. Audrey bought a condominium for $132 500. She made a 15% down payment and negotiated a three year mortgage at 7.75% per annum compounded semi-annually. The mortgage is amortized over 25 years. Determine:

a) the term of the mortgage

b) the amortization period

c) the amount to be financed

d) the monthly mortgage payment

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

e) the total amount paid for the condominium if the interest rate remained at 7.75% throughout the amortization period

6. Vancouver Credit Union offers a $115 000 mortgage amortized over 20 years at 8% per year compounded semi-annually.

a) Determine the monthly payments.

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

b) Calculate the total amount to be paid.

c) Calculate the amount of interest to be paid.

d) What would happen to the monthly payments and the total to be paid if the mortgage was amortized over 25 years? (No calculations are required.)

7. Kim is shopping for a $105 000 mortgage. Two banks gave her the following options:

- Grand Bank: a two year term at 7.72% amortized over 10 years
- Bank for Less: a five year term at 7.35% amortized over 15 years

a) Which option has the lower monthly payment?

Grand Bank

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

Bank For Less

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

b) Assuming the interest rate remains the same over the amortization period of each loan, determine the total amount of interest paid for each option.

c) Even though the monthly payments are higher, why might Kim choose Grand Bank?

d) Even though the total amount of interest paid is higher, why might Kim choose Bank for Less?

8. Tera and Bill purchased an acreage with a large house close to the city limits of a major city. The acreage and home were sold to them for $752 000. From the sale of their old house plus savings they had accumulated, they were able to make an 86% down payment. Tera and Bill negotiated a seven year mortgage at 4.10% amortized over 20 years for the remaining amount.

a) State: **i)** the amortization period **ii)** the term of the mortgage

b) Determine the monthly payment.

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

c) What would their house really cost if the interest rate remained at 4.1% throughout the amortization period?

d) The seven year term of the mortgage is up for renewal. What is the amount on the mortgage left to be renewed?

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

9. In 2011, Phil and Stephanie moved into a new home and assumed a $137 000 mortgage amortized to the year 2026 at 5.89% per annum compounded semi-annually for a three year term. At the end of the three year term, the mortgage rate had increased by 1.5%. They renegotiated their mortgage for a further three year term at this new rate. Calculate the monthly payment for their renegotiated mortgage if the mortgage was still to be paid in full by the year 2026.

N= I%= PV= PMT= FV= P/Y= C/Y= PMT: END BEGIN	N= I%= PV= PMT= FV= P/Y= C/Y= PMT: END BEGIN	N= I%= PV= PMT= FV= P/Y= C/Y= PMT: END BEGIN

10. Andrea would like to take out a mortgage for $125 000 amortized over 25 years at 6.5% per annum. The bank has presented her with two options.

The first option is to make 12 monthly payments per year for 25 years.
The second option is to make 52 weekly payments per year for 25 years.

How much will Andrea save over the full amortization by using the weekly payments instead of the monthly payments?

N= I%= PV= PMT= FV= P/Y= C/Y= PMT: END BEGIN	N= I%= PV= PMT= FV= P/Y= C/Y= PMT: END BEGIN

Answer Key

1. **a)** $1779.90 **b)** $1550.56 **c)** $1433.94
2. **a)** $189 362.67 **b)** $114 917.10 **c)** $48 022.98
3. **a)** 7.72% **b)** 8.11% **c)** 6.55%
4. **a)** amortization is 25 years, term is 5 years, down payment is $27 900, financed amount is $251 100
b) $1751.00 **c)** $553 200 **d)** $228 468.35
5. **a)** 3 years **b)** 25 years **c)** $112 625 **d)** $841.67 **e)** $272 376
6. **a)** $952.61 **b)** $228 626.40 **c)** $113 626.40
d) monthly payments decrease, overall cost of house increases
7. **a)** Bank for Less **b)** Grand Bank → $45 213.60, Bank for Less → $67 423.80
c) Because the overall interest paid out will be $22 210.20 less.
d) Because she may only be able to afford the lower monthly payment.
8. **a)i)** 20 year amortization **ii)** 7 year term **b)** $641.61 **c)** $800 706.40 **d)** $77 645.89
9. $1233.52 **10.** $518.00