Financial Mathematics Lesson #2:
Investments Using Compound Interest

Compound Interest Formula Where Interest is Not Compounded Annually

The compound interest formula in the previous lesson can be adapted for cases where the compounding does not occur annually, e.g. semi-annually (twice a year), quarterly (four times a year), monthly (twelve times a year), etc.

The formula is the same, but i and n are defined in terms of the compounding period.

$$A = P(1+i)^n$$

where
- A represents the final amount
- P represents the initial principal
- i represents the interest rate per compounding period
- n represents the number of compounding periods

- Note that i does **NOT** always represent the annual interest rate.
  i is calculated as $\rightarrow$ $\dfrac{\text{annual interest rate}}{\text{number of compounding periods per year}}$

- Note that n does **NOT** always represent the number of years.
  n is calculated as $\rightarrow$ (number of years) × (number of compounding periods per year)

If the annual rate of interest is 6% per annum, state the interest rate per compounding period and the total number of compounding periods in each case.

a) compounded semi-annually for 5 years

interest rate $\frac{6\%}{2} = 3\%$

# of periods $5 \times 2 = 10$

b) compounded monthly for 6 years ← 12

interest rate $\frac{6\%}{12} = 0.5\%$

# of periods $6 \times 12 = 72$

$3 000 is invested for 5 years at an annual interest rate of 7%. Complete the table to calculate the final value of the investment if interest is compounded according to the period of time given in the table.

Compounding Period	*Number of Compounding Periods Per Year*	*Total Number of Compounding Periods*	*Interest Rate per Compounding Period*	*Formula $A = P(1+i)^n$*	*Amount*
Annually	1	$5 \times 1 = 5$	$\frac{7\%}{1} = 7\%$	$3000(1.07)^5$	$4207.66
Semi-Annually	2	$5 \times 2 = 10$	$\frac{7\%}{2} = 3.5\%$	$3000(1.035)^{10}$	$4231.80
Quarterly	4	$5 \times 4 = 20$	$\frac{7\%}{4} = 1.75\%$	$3000(1.0175)^{20}$	$4244.33
Monthly	12	$5 \times 12 = 60$	$\frac{7\%}{12} = 0.58\overline{3}\%$	$3000(1.00583\overline{3})^{60}$	$4252.88

Complete Assignment Questions #1 - #2

Using TVM to Solve Non-Annual Compound Interest Problems

REMEMBER:

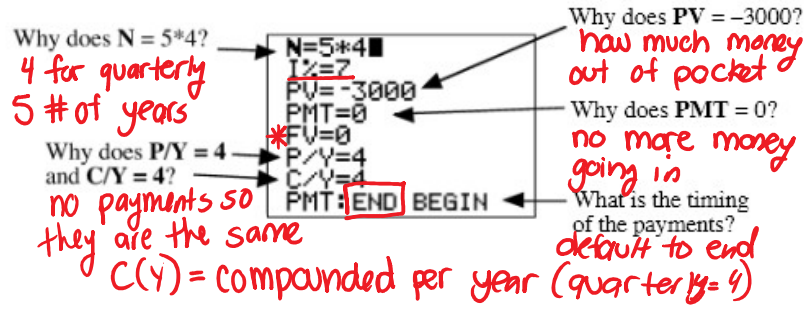
P/Y represents the number of payments (PMT) made to the investment or loan.

- For questions with no payments, make P/Y the same as the C/Y value.
- For questions **with payments**, the P/Y value is equal to the number of payments made in one year.

Consider the compounded quarterly part of the table from Class Ex. #2.

"$3000 is invested for 5 years at an annual interest rate of 7% compounded quarterly. Determine the value of the investment at the end of the term."

a) Complete by answering the questions.



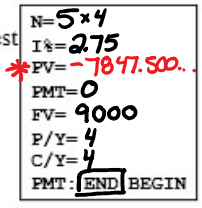
b) Calculate the FV value. Compare this solution with the compounded quarterly part of Class Ex. #2.

*FV = 4244.334597

future value is $4244.33

Minuete would like to invest money so that she will have $9 000 in 5 years. The bond she would like to invest in predicts an annual interest rate of 2.75% compounded quarterly. How much should her initial investment be?

she needs to invest $7847.50 to have $9000 in 5 years.



Complete Assignment Questions #3 - #7

fncalculator.com →

- end
- *PV
- PMT 0
- FV 9000
- % 2.75
- periods 20
- comp Quarterly

#1-7

Annuity

An **annuity** is a series of **equal payments** made at **regular time intervals**.
There are different forms of annuities. For example;

- A **regular savings** plan can be considered as an investment annuity.
- Paying off a **loan** (next lesson) or a **mortgage** (in Lesson 4) with **regular payments** is a form of an annuity.
- **Retirement income** is often received in the form of an annuity.

Using TVM for Basic Annuity Investment Applications

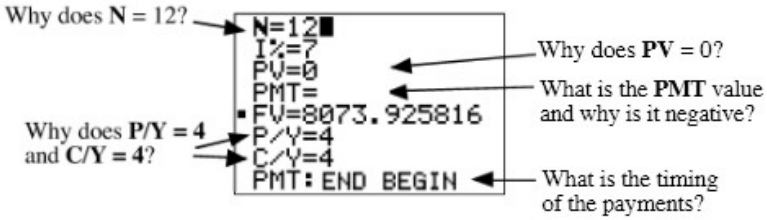
Because an annuity involves making periodic payments, keep the following in mind:

- The P/Y value is equal to the number of payments made in one year (unlike questions with no periodic payments).
- The PV (present value, or initial amount) may be zero, or not. If PV is zero, it indicates there is no initial amount, but there are regular periodic payments. If PV is a number other than zero, then an initial sum of money was invested (or borrowed) together with regular periodic payments.

Consider the following investment problem:
" Byron invests $600 in a savings account quarterly. If the account pays interest at 7% per year compounded quarterly, what is the value of the investment at the end of three years?"

a) Why would this type of investment be considered an annuity?

b) Complete the following to answer the question in the scenario using TVM Solver. The diagram below shows the screen display after the FV value has been determined.



c) What is the value of the annuity investment at the end of three years?

Banker Bill wanted to set up a fund for his granddaughters Becky and Bonnie. The girls had to choose between the following two options:

Option A: Two thousand dollars will be invested on each birthday between age 27 and age 64 inclusive. The money will stay in the account until age 65.

Option B: Two thousand dollars will be invested on each birthday between age 19 and age 26 inclusive. The money will stay in the account until age 65.

Both options will receive interest at the rate of 10% per year compounded annually until age 65.

Becky recognized that more money will be invested under Option A and chose this option.

Bonnie chose Option B to be different from Becky.

a) How many payments were made on Becky's behalf? How much money was invested for her?

b) How many payments were made on Bonnie's behalf? How much money was invested for her?

c) At age 65, what was:

i) Becky's investment worth?

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

ii) Bonnie's investment worth?

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

d) What conclusion can you draw from this class example?

Complete Assignment Questions #8 - #12

Assignment

1. If the annual rate of interest is 9% per annum, state the interest rate per compounding period and the total number of compounding periods in each case:

a) compounded semi-annually for 4 years **b)** compounded quarterly for 3 years

c) compounded monthly for $4\frac{1}{2}$ years **d)** compounded annually for 6 years

2. $4000 is invested for 4 years at an annual interest rate of 7.2%.

Complete the table to calculate the final value of the investment if interest is compounded for the time periods given in the table.

Compounding Period	*Number of Compounding Periods Per Year*	*Total Number of Compounding Periods*	*Interest Rate per Compounding Period*	*Formula A =*	*Amount*
Annually					
Semi-Annually					
Quarterly					
Monthly					

3. Determine the final amount of each investment, to the nearest cent, given the initial investment, the annual rate of interest, the compounding period, and the length of the investment.

a) An investment of $2 000 at 7% per annum compounded annually for 6 years.

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

b) An investment of $7 800 at 5.7% per annum compounded monthly for five years.

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

4. Given the following annual interest rates, compounding periods, and number of years for the investment, how much money, to the nearest dollar, would you need to invest today for the investment to accumulate to at least $10 000?

a) 6.1% per annum, compounded quarterly, for three years.

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

b) 5.2% per annum, compounded semi-annually, for four years.

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

5. Determine the annual rate of interest, to the nearest tenth of a percent, given the initial investment, the final amount, the compounding period, and the length of the investment.

a) An investment of $7 500 compounded semi-annually for five years to produce a final amount of $12 500.

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

b) An investment of $2 800 compounded monthly for three years to produce a final amount of $3 750.

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

6. Determine the annual rate of interest to the nearest tenth of a percent, given the initial investment, the interest paid, the compounding period, and the length of the investment.

a) An investment of $6 100 compounded quarterly for three years to produce $995 interest.

```
N= 3x4
I%=
PV= -6100
PMT=0
FV= 7095
P/Y= 4
C/Y= 4
PMT: END BEGIN
```

FV = 6100 + 995 = 7095

b) An investment of $180 000 compounded semi-annually for five years to produce $65 000 interest.

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

#1-7

7. Determine the length of the investment, given the initial investment, the final amount, the annual interest rate, and the compounding period.

a) An investment of $9 000 compounded quarterly at 7% p.a. to produce a final amount of $12 300. (*Answer to the nearest quarter of a year.*)

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

b) An investment of $550 compounded daily at 7.3% per year to produce a final amount of $675. (*Answer to the nearest day.*)

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

8. Assunta invests \$300 in a savings account at the beginning of each month. If the account pays interest at 7.2% per year compounded monthly, what is the value of the investment at the end of five years?

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

9. Tony deposited \$275 into his savings account at the beginning of each month for a period of two years. If the account pays interest at 5.2% per year compounded monthly, how much interest has he earned at the end of two years?

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

10. On January 1, Nicole had \$9 500 in an RRSP account. She decided to make monthly payments of \$100 into that account starting on that date. If the rate of interest is 6.8% per annum compounded monthly, determine

a) the value of the RRSP after three years of payments
b) the interest earned

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

11. Helen wants to invest some money so that her grandson Tim will have at least \$25, 000 for college education in eighteen years' time. The bank offers an annual rate of 5.75% compounded monthly.

How much, to the nearest dollar, should her initial investment be?

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

12. Gabriel invested \$7 500 in a GIC for a three year term at 7.15% compounded semi-annually. At the end of the term, he transferred the money into a savings account paying 4.95% per annum compounded monthly. He also made monthly payments of \$250 each month into the savings account, starting on the day he opened it. What was the value of his investment five years after the savings account was opened?

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

Answer Key

1. **a)** 4.5%, 8 **b)** 2.25%, 12 **c)** 0.75%, 54 **d)** 9%, 6

2. See the table below.

Compounding Period	*Number of Compounding Periods Per Year*	*Total Number of Compounding Periods*	*Interest Rate per Compounding Period*	*Formula*	*Amount*
Annually	1	4	7.2%	$A = 4000(1.072)^4$	$5282.50
Semi-Annually	2	8	3.6%	$A = 4000(1.036)^8$	$5308.09
Quarterly	4	16	1.8%	$A = 4000(1.018)^{16}$	$5321.38
Monthly	12	48	0.6%	$A = 4000(1.006)^{48}$	$5330.44

3. **a)** $3001.46 **b)** $10 365.15

4. **a)** $8340 **b)** $8144

5. **a)** 10.5% **b)** 9.8% **6.** **a)** 5.1% **b)** 6.3% **7.** **a)** 4.5 years **b)** 1024 days

8. $21 718.96 **9.** $369.67 **10.a)** $15 646.72 **b)** $2546.72 **11.** $8903.00

12. $28 903.69