# Lesson 1: Simple Interest and Compound Interest

# Financial Mathematics Lesson #1:
## Simple Interest and Compound Interest


**This unit is required for students and teachers following the WNCP curriculum for Foundations of Mathematics Grade 12. It is not required for students and teachers following the Alberta version of the curriculum, but could be included for completeness or enrichment.**

### *Investing Money*

If you deposit money in a financial institution, such as a bank, you are in effect lending money to the bank. In exchange, the bank pays you interest. There are two types of interest: **simple interest** and **compound interest**.

### *Simple Interest*

Simple interest is usually applicable to short term investments of one year or less or to long term investments where the annual interest is paid to the investor and not reinvested.

**a)** If you invest \$500, earning interest at a rate of 6% per year, how much interest would you earn in

$6 \div 100 = 0.06$

**i)** one year? $500(0.06)(1) = \$30$ **ii)** half a year? $500(0.06)\left(\frac{1}{2}\right) = \$15$ **iii)** one month? $500(0.06)\left(\frac{1}{12}\right) = \$2.50$

**b)** If $r$ is the annual interest rate (expressed as a decimal) and \$$P$ is the initial investment, calculate how much interest would be earned in

**i)** one year **ii)** half a year **iii)** $t$ years

**The formula to calculate simple interest is** $I = Prt$

where

$I$ represents the amount of interest
$P$ represents the principal (the initial investment)
$r$ represents the annual rate of interest - expressed as a decimal
$t$ represents the time in years for which the money is invested

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Millie invests in \$2 350 at 7% per year for six months. Calculate, after six months,

**a)** the simple interest on Millie's investment $2350(0.07)\left(\frac{6}{12}\right) = \$82.25$

**b)** the value of Millie's investment $2350 + 82.25 = \$2432.25$

### *Compound Interest*

In simple interest, the principal at the beginning of the second year is the same as the principal at the beginning of the first year.

In compound interest, the interest earned during the first year is added to the original principal to form a new principal.

To understand the comparison between simple interest and compound interest, do the investigation on the next three pages.

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## *Exploring Simple Interest and Compound Interest*

A bank offers two types of savings bond:

- "Regular Savings Bond", which pays simple interest at 9% p.a. (per year, or per annum).
- "Compound Savings Bond", which pays interest at 9% p.a., compounded annually.

Consider an initial investment of \$15 000 in each bond.

***Simple Interest***

**a)** The simple interest each year

is 9% of \$15 000 = 1350 .

0.09 x 15000

**b)** Complete the table, which gives the value of the bond at the end of each of the first 8 years.

| End of Year | Amount (\$) |
|---|---|
| 1 | 16 350 |
| 2 | 17 700 |
| 3 | 19 050 |
| 4 | 20 400 |
| 5 | 21 750 |
| 6 | 23100 |
| 7 | 24 450 |
| 8 | 25 800 |

+1350 + + + +

***Compound Interest***

**c)** Complete the following to determine the compound interest and the value of the bond.

End of Year 1: Value of Bond = Principal + Interest

$= 15\,000 + 15\,000(0.09)$

$= 15\,000(1 + 0.09)$ factor out 15 000

$= 15\,000(1.09)$

End of Year 2: Value of Bond = Principal + Interest

$= 15\,000(1.09) + 15\,000(1.09)(0.09)$

$= 15\,000(1.09)(1 + 0.09)$ factor out $15\,000(1.09)$

$= 15\,000(1.09)(1.09)$

$= 15\,000(1.09)^2$

End of Year 3: Value of Bond = Principal + Interest

$= 15\,000(1.09)^2 + 15\,000(1.09)^2(0.09)$

$= 15\,000(1.09)^2(1 + 0.09)$ factor out $15\,000(1.09)^2$

=

$= 15\,000(1.09)^3$

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**d)** The value for the bond at the end of each year is 1.09 times the value at the end of the previous year. Complete the table below.

| End of Year | Value of Bond | Amount ($) |
|---|---|---|
| 1 | 15 000(1.09) | 16 350.00 |
| 2 | $15\,000(1.09)^2$ | 17 821.50 |
| 3 | $15\,000(1.09)^3$ | 19425.44 |
| 4 | $15000(1.09)^4$ | 21 173 |
| 5 | $15000(1.09)^5$ | 23079.36 |
| 6 | $15000(1.09)^6$ | 25 156.50 |
| 7 | $15000(1.09)^7$ | 27 420.59 |
| 8 | $15000(1.09)^8$ | 29 888.44 |
| $n$ | $15000(1.09)^n$ | |

15000 x 1.09 $y^x$

The value at the end of $n$ years is $15\,000(1.09)^n$.

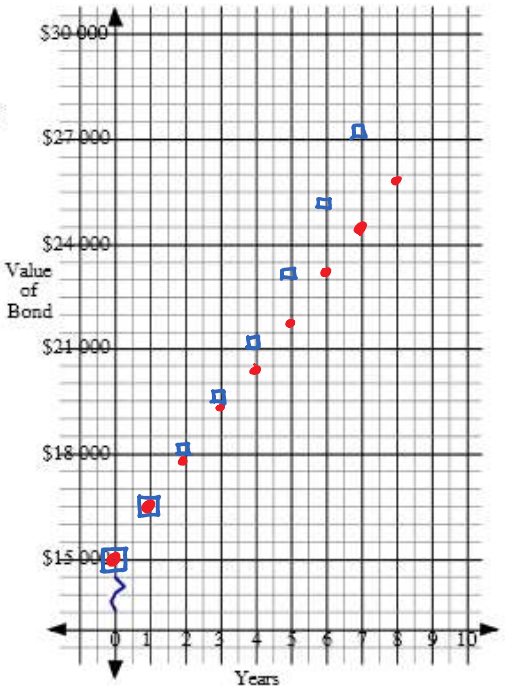
**e)** Plot the data from the simple interest and compound interest tables. Do not join the points.

**f)** The graph shows that the simple interest bond is growing in a **linear** pattern and the compound interest bond is growing more quickly, or **exponentially**.

The final amount, $A$, can be written as an exponential function of the number of years, $n$, in the form $A = ab^n$.

- State the values of $a$ and $b$ and write the function in this form.

- How do the values of $a$ and $b$ relate to the scenario in this question?

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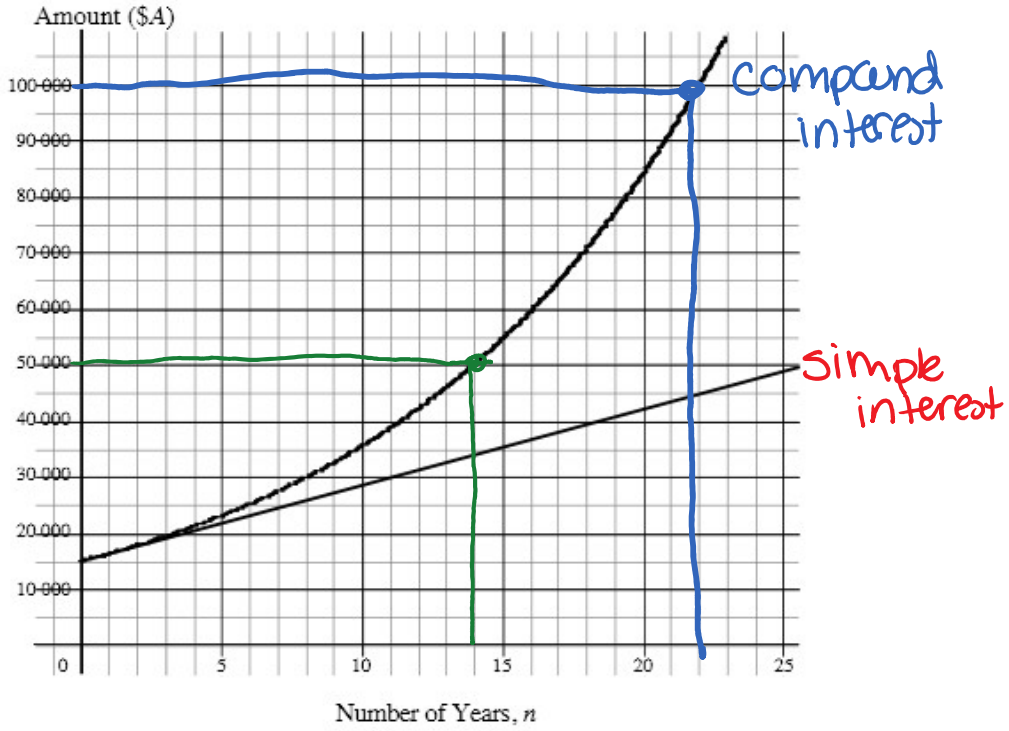
**g)** The difference between exponential growth and linear growth is more apparent if we extend the graph in f) to more than 20 years.




Use the compound interest graph to estimate the number of years it would take for the initial investment of $15 000 to increase in value to **i)** $50 000 **ii)** $100 000

i) 14 years ii) 22 years

**h)** Consider an investment of $30 000 compounded annually at 5% p.a.
Write an exponential function which will represent the final amount $A after $n$ years.

**i)** Consider an investment of $750 compounded annually at 3.5% p.a.
Write an exponential function which will represent the final amount $A after $n$ years.

**j)** In general, if $A$ = the final amount (or the value of the bond), $P$ = the principal (the initial investment), $i$ = the annual interest rate, and $n$ = the number of years, we have the formula:

$A =$

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## *Compound Interest Formula Where Interest is Compounded Annually*

In the previous exploration, interest was compounded on an annual basis. In practice, compounding can take place over any period of time, e.g. semi-annually, monthly, daily, continuously, etc.

In this lesson, we will focus on interest that is compounded annually.

The formula that can be used to calculate compound interest where the interest is compounded on an annual basis is an exponential function of the form $y = ab^x$.

$A = P(1 + i)^n$ where

- $A$ represents the final amount
- $P$ represents the principal (or initial investment)
- $i$ represents the annual interest rate
- $n$ represents the number of years

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$4 000 is invested in a 3-year GIC, compounded yearly at a rate of 3.5% per annum.

**a)** Identify values for $P$, $i$, and $n$.

3.5 ÷ 100 = 0.035

$P = 4000$ $i = 0.035$ $n = 3$

**b)** Determine the value of the investment at the end of the term.

$A = P(1+i)^n$

$A = 4000(1+0.035)^3$ $4000(1.035)^3 = 4434.87$

---


Reina has invested $8 000 in a 5-year bond, compounded annually at a rate of 3.2% per annum. Determine the interest earned at the end of the 5-year period.

$P = 8000$ $i = 0.032$ $n = 5$

$A = 8000(1.032)^5$

$= 9364.58$

$-8000$

$1364.58 interest earned

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Amber Lynn invested $8 000 in an investment for 5 years compounded annually at a rate of 7.25% per year.

**a)** Determine the value of her investment at the end of the term.

**b)** Calculate the profit made on her investment.

**c)** Suppose that Amber Lynn has not decided how long she will keep the investment. Let $x$ represent the number of years that her money is invested.
Write an equation of the form $y = ab^x$ to represent her balance after $x$ years.

**d)** Graph the equation in part c) with the window $x$: [0, 10, 1] $y$:[8000, 18000, 1000].

**i)** What do the numbers on the $x$-axis represent?

**ii)** What do the numbers on the $y$-axis represent?

**e)** Amber Lynn wants to know which of the following options will increase her investment more:

- increasing her initial investment by $500, or,
- finding an investment with an annual interest rate of 8%.

For each of these two scenarios, write an equation to represent her balance after $x$ years.

Initial investment of $8500 Interest rate of 8%

**f)** Graph the two equations on the same axes as your graph from part d).

**g)** Describe the differences in the graphs.

**h)** If Amber Lynn wants to cash out her investment after 5 years, which is the better option?

**i)** If Amber Lynn wants to cash out her investment after 10 years, which is the better option?

**Complete Assignment Questions #1 - #7**

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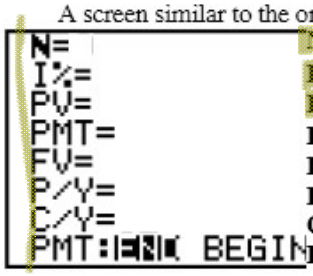
## *Using TVM to Solve Annual Compound Interest Problems*

Consider Class Ex. #4: " Amber Lynn invested $8 000 in an investment for 5 years compounded annually at a rate of 7.25% per year. Determine the value of her investment at the end of the term."

Using TVM Solver

Determine the value of the investment at the end of the term using TVM Solver.

**1.** Access the TVM Solver by pressing [APPS] [1] [1] [ENTER].
A screen similar to the one shown will appear.




**N** represents the total number of payment periods.
**I%** represents the annual interest rate.
**PV** represents the present value, or inital value (***see note below***).
**PMT** represents the payment amount (***see note below***).
**FV** represents the future value (***see step 3 below***).
**P/Y** represents the number of payments per year (***see note below***).
**C/Y** represents the number of compounding periods per year.
**PMT:** END BEGIN represents the timing of the payments.

**2.** Enter the given values using the [ENTER] or arrow key after each entry.

```
N=5          ( This value changes based on the compounding period.)
I%=7.25
PV=-8000     ( See the note below as to why this is a negative value.)
PMT=0        ( Since no additional payments are made after initial investment.)
FV=0         ( See step 3 below.)
P/Y=1        ( See note below.)
C/Y=1
PMT:END BEGIN( Assume the default, END, unless otherwise stated.)
```


For the TI-83/84 series Financial functions, take note of the following points:

- cash received is a positive value (money entering your "pocket")
- cash paid is a negative value (money leaving your "pocket")
- P/Y represents the number of payments (PMT) made to the investment or loan. If **no** additional payments are made to the investment or loan, then enter this value so that it is equal to the C/Y value. The TI calculator will not accept 0 for P/Y (which it should for this type of question, because no additional payments were made into the investment). **For questions with no payments, make P/Y the same as the C/Y value.**

The amount $8000 is entered as a negative value because it is the initial principal that has been paid from you to the financial institution.

**3.** The final value of the bond is represented by **FV**. Although **FV** is not known at this time, the calculator will not proceed unless a value is entered - we have used **0**. To find the final value of the bond, place the cursor over the value of **FV** and then press [ALPHA] [ENTER]. An indicator square on the left of **FV** indicates that the future value has been determined.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## *Rule of 72*

**The Rule of 72** determines the approximate time taken for money to double in value in an investment. This rule states that for money compounded annually, the annual interest rate multiplied by the number of years is approximately 72. It may be expressed in one of the following ways:

Annual Interest Rate × # of years = 72 OR # of years = $\frac{72}{\text{Annual Interest Rate}}$

Investigate the rule by comparing the results of the Rule of 72 to the results with TVM Solver.

**"Rule of 72"**

| | Interest Rate (Annual) | Time (Years) |
|---|---|---|
| a) | 9% $\frac{72}{9}$ | 8 |
| b) | 6% | 12 |
| c) | 4% $\frac{72}{4}$ | 18 |

**TVM Solver**

| | Interest Rate (Annual) | Time (Years) |
|---|---|---|
| a) | 9% | 8.04 |
| b) | 5.946% | (12) |
| c) | 4% | 17.67 |

a) years = $\frac{72}{9}$ = 8

b) interest = $\frac{72}{12}$ = 6%

Use PV = -1000 and FV = 2000
↳ $1000 into investment and $2000 (double the money) out
- Solve for N in a) and c) and solve I% in b)


- Use the "Rule of 72" to estimate the number of years it would take for an investment of $1000 to grow to $8000 if the annual interest rate was 7.2%.

1000 → 2000 → 4000 → 8000 doubles 3 times

$\frac{72}{7.2}$ = 10 x 3 = 30 years

- Check the accuracy of the answer using TVM Solver.

PV -1000
FV 8000
% 7.2
periods 29.91 solved

estimate is very close

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

**Complete Assignment Questions #8 - #9**

#1-5, 7-9

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# Assignment

**1.** Calculate the simple interest in each case:

**a)** \$785 is invested at 8% per annum for one year.

**b)** \$840 is invested at 6% per year for six months.

**2.** \$7 800 is invested in a 5 year GIC compounded yearly at a rate of 5% per annum. Determine the value of the investment at the end of the term.

**3.** Eight years ago, Julian invested \$25 000 in technology stocks. The return on his investment was equivalent to a rate of 7.75% per annum, compounded yearly. Determine the amount of interest he made on the investment.

**4.** Dorean has invested \$15 750 in a 7-year investment compounded yearly at a rate of 3.75% per annum. Determine the value of the investment at the end of the term.

**5.** Three years ago, Candice invested \$10 000 in high-risk stocks. The return on her investment was equivalent to a rate of 17% per annum, compounded annually. Determine the amount of interest she made on the investment.

**6.** Andre invested \$7 000 in an investment paying 4.5% per annum compounded over eight years. Would the interest earned on the investment after four years be one-half of the interest earned on the investment in eight years?

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**7.** A bond has been purchased for $1 500 and invested in an account bearing an annual interest rate of 6.25%.

**a)** If the bond is a **Regular Interest Bond** then

- complete the table below
- write an equation that models this scenario
- sketch a graph without numerical values that represents this investment.

| **Year** | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| **Value** | $1 500.00 | $1 593.75 | $1 687.50 | | | |

**b)** If the bond is a **Compound Interest Bond** then

- complete the table below
- write an equation that models this scenario
- sketch a graph without numerical values that represents this investment.

| **Year** | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| **Value** | $1 500.00 | $1 593.75 | $1 693.36 | | | |

**8.** Use the TVM Solver to verify the answers to questions #2 - #5 of this assignment. Place your values in the charts below.

#2

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

#3

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

#4

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

#5

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN
```

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**9.** Complete the following charts by using the "Rule of 72" and then the TVM Solver feature of a graphing calculator. Use \$5 000 as the initial investment. Round off to the nearest hundredth where necessary.

**"Rule of 72"**

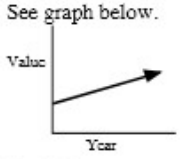
| | Interest Rate (Annual) | Time (Years) |
|---|---|---|
| **a)** | 10% | |
| **b)** | | 14 |
| **c)** | 7% | |

**TVM Solver**

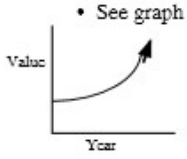
| | Interest Rate (Annual) | Time (Years) |
|---|---|---|
| **a)** | 10% | |
| **b)** | | 14 |
| **c)** | 7% | |

## *Answer Key*

**1.** **a)** \$62.80 **b)** \$25.20 **2.** \$9955.00 **3.** \$20 423.25
**4.** \$20 379.68 **5.** \$6016.13
**6.** No, because the interest is compounded, which means it follows an exponential pattern. If the investment paid regular interest (or simple interest), the interest earned would follow a linear pattern and then the statement would be true.
**7.** **a)**
- year 3, \$1781.25
  year 4, \$1875.00
  year 5, \$1968.75
- $y = 93.75x + 1500$
- See graph below.




**b)**
- year 3, \$1799.19
  year 4, \$1911.64
  year 5, \$2031.12
- $y = 1500(1.0625)^x$
- See graph below.




**9.** See tables below.

**"Rule of 72"**

| | Interest Rate (Annual) | Time (Years) |
|---|---|---|
| **a)** | 10% | 7.2 |
| **b)** | 5.14% | 14 |
| **c)** | 7% | 10.29 |

**TVM Solver**

| | Interest Rate (Annual) | Time (Years) |
|---|---|---|
| **a)** | 10% | 7.27 |
| **b)** | 5.08% | 14 |
| **c)** | 7% | 10.24 |

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