Lesson 3: Permutations with Restrictions

# Permutations and Combinations Lesson #3:
## Permutations with Restrictions

### *Permutations with Restrictions*

In many problems, restrictions are placed on the order in which objects are arranged. In this type of situation, deal with the restrictions first. Many of the problems in this lesson can be solved using the fundamental counting principle.

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*Class Ex. #1*

Answer the following questions using
i) the fundamental counting principle ii) factorials.

In how many ways can all of the letters of the word **ORANGES** be arranged if:

**a)** there are no further restrictions?

i) 7 6 5 4 3 2 1
= 5040

ii) 7! = 5040

**b)** the first letter must be an **N**?

i) N 6 5 4 3 2 1
= 720

ii) 6! = 720

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*Class Ex. #2*

Three girls ($G_1, G_2, G_3$) and four boys ($B_1, B_2, B_3, B_4$) are to be arranged in a row for a photograph.

**a)** In how many ways can this be done if there are no restrictions as to where each person stands?

7! = 5040

**b)** The photographer decides that he could take a better photograph if no two people of the same gender can sit together.

**i)** Show one arrangement of the students in the space below.

$B_1$ $G_1$ $B_2$ $G_2$ $B_3$ $G_3$ $B_4$ 4 3 3 2 2 1 1

**ii)** Determine the total number of arrangements that the photographer could make.

4! for the boys
3! for the girls

4! × 3! for boys and girls
= 144

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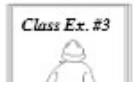
*Class Ex. #3*



Six actors and eight actresses are available for a play with four male roles and three female roles. How many different cast lists are possible?

*Class Ex. #3*

Six actors and eight actresses are available for a play with four male roles and three female roles. How many different cast lists are possible?

$_6P_4 \times {}_8P_3 = 360 \times 336 = 120\,960$

each actor/actress plays a different character so order matters





*Class Ex. #4*

Andrew was asked to determine the number of arrangements of the letters of the word **BRAINS** in which the vowels are together. His reasoning is shown below.

Since the vowels must be kept together, I will treat the "A" and "I" as one element (AI).
This means I now have five elements instead of six to be arranged in order.
The five elements (AI), B, R, N, S, can be arranged in 5! ways.
My answer is 5! = 120.

**a)** Is Andrew correct in saying that "A" and "I" should be combined as one element?

**b)** Andrew's final answer is not correct. Explain what is missing from his reasoning, and determine the correct solution to the problem.

*Class Ex. #5*

Find the number of permutations of the letters in the word **KITCHEN** if

**a)** the letters **K**, **C**, and **N** must be together in the order **K**, **C**, **N**

**b)** the letters **K**, **C**, and **N** must be together but not necessarily in that order

(KCN), I, T, H, E

3!

*Class Ex. #6*

Frank, George, Hanna, Iris, Jacob, and Kim have playoff tickets for a hockey game in adjacent seats A3 to A8.

**a)** In how many different ways could they sit together in seats A3 to A8?

$6! = 720$

**b)** George and Hanna are dating. In how many of the arrangements in a) would they be sitting together?

F, (HG), I, J, K

sitting together?

F, (HG), I, J, K

5!2! = 240

**c)** Just before the game, George and Hanna have a disagreement and decide not to sit together. In how many of the arrangements in a) would they be sitting apart?

720 - 240 = 480 not together

**Complete Assignment Questions #1 - #13** #1-6

# Assignment

**1.** How many arrangements could be made of the word

**a)** **FATHER** if **F** is first?

**b)** **UNCLE** if **C** is first and **L** is last?

**c)** **DAUGHTER** if **UG** is last?

**d)** **MOTHER** if the vowels are first and last?

**2.** How many arrangements of the following words can be made if all the vowels must be kept together?

**a)** **FATHER** **b)** **DAUGHTER** **c)** **EQUATION**

**3.** Determine the number of different arrangements of the six letters in the word **ANSWER**

**a)** without restrictions

**b)** that begin with an **S**

**c)** that begin with a vowel and end with a consonant

**d)** that have the three letters **A**, **N**, and **S** adjacent and in the order **ANS**

**e)** that have the three letters **A**, **N**, and **S** adjacent but not necessarily in that order

**4.** Consider the letters of the word **HEXAGON** .

**a)** In how many ways can the letters of the word **HEXAGON** be arranged using all the letters?

**b)** How many arrangements of the letters of the word **HEXAGON** begin with an 'H'?

**c)** How many arrangements of the letters of the word **HEXAGON** begin with a vowel?

**d)** How many 3-letter "words" can be made from the letters of the word **HEXAGON**?

**e)** How many 3-letter "words" beginning with a vowel can be made from the letters of the word **HEXAGON**?

**f)** How many 3-letter "words" can be made from the letters of the word **HEXAGON** if every "word" must have the pattern consonant-vowel-consonant?

**5.** In how many ways can four adults and five children be arranged in a single line

**a)** without restriction?

**b)** if children and adults are alternated?

**c)** if the adults are all together and the children are all together?

**d)** if the adults are all together?

**6.** Ann, Brian, Colin, Diane, and Eric go to watch a movie and sit in 5 adjacent seats. In how many ways can this be done if

**a)** Brian sits next to Diane? **b)** Ann refuses to sit next to Eric?

*Use the following question to answer the next three questions.*

Absolute Value Vehicles leases new vehicles. The manager of the dealership must design a brochure which illustrates the 11 different models available.
The designer of the brochure decides to use 8 different black models and 3 different silver models all lined up in a row.

Multiple Choice **7.** The number of ways in which the 11 vehicles can be placed in a row is represented by

**A.** $11!$ **B.** $3! \times 8!$

**C.** $3! \times 8! \times 2!$ **D.** ${}_{11}P_3 \times {}_{11}P_8$

**8.** The number of ways in which the 11 vehicles can be placed in a row if all the black vehicles must be together is

**A.** 24

**B.** 48

**C.** 241 920

**D.** 967 680

**9.** The number of ways in which one silver vehicle is at beginning of the row, another silver vehicle is in the middle of the row, and a third silver vehicle is at the end of the row can be represented by

**A.** $3 \times 8$ **B.** $8!$

**C.** ${}_3P_3 \times {}_8P_8$ **D.** ${}_{11}P_3 \times {}_8P_8$

*Use the following information to answer the next three questions.*

The starting line up of a school mixed softball team consists of four boys and five girls. Before the game, the nine students line up in a row for a photograph.

**10.** The number of different arrangements if the girls all stand together is given by

**A.** $4! \times 5!$ **B.** $5! \times 5!$

**C.** $4! \times 5! \times 2!$ **D.** $5! \times 5! \times 2!$

Numerical Response

**11.** The number of different arrangements if the girls and boys alternate in the row is _____ .

(Record your answer in the numerical response box from left to right.) | | | | |

**12.** The number of different arrangements if the girls all stand together in the middle of the row is _____ .

(Record your answer in the numerical response box from left to right.) | | | | |

**13.** The number of different ways that seven basketball players can be seated on a bench so that two specified players are always sitting side by side is _____ .

(Record your answer in the numerical response box from left to right.) | | | | |

***Answer Key***

**1.** a) 120 b) 6 c) 720 d) 48 **2.** a) 240 b) 4320 c) 2880
**3.** a) 720 b) 120 c) 192 d) 24 e) 144
**4.** a) 5040 b) 720 c) 2160 d) 210 e) 90 f) 36
**5.** a) 362 880 b) 2880 c) 5760 d) 17 280 **6.** a) 48 b) 72
**7.** A **8.** D **9.** C **10.** B
**11.** 2 8 8 0 **12.** 2 8 8 0 **13.** 1 4 4 0