

Lesson 3: Venn Diagrams - Part One

Logical Reasoning and Set Theory Lesson #3: Venn Diagrams - Part One

In the last lesson, we used the connecting words “and”, “or”, and “not” in set theory. In this lesson we will continue to use these connecting words when we introduce **Venn Diagrams** as a visual aid in illustrating the relationships between sets.

Review

The following is taken from Class Ex. #2 of the last lesson.

Consider the following two sets, subsets of the universal set of natural numbers less than 20.

$$A = \{\text{natural numbers less than 20 that are divisible by 3}\}$$

$$B = \{\text{natural numbers less than 20 that are divisible by 5}\}$$

Using the appropriate notation, list the set of whole numbers less than 20 that are

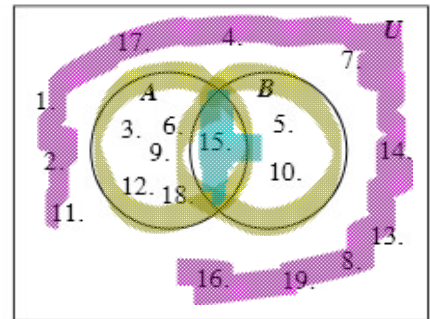
a) divisible by 3 b) divisible by 5 c) divisible by 3 and 5 d) divisible by 3 or 5

$$A = \{3, 6, 9, 12, 15, 18\} \quad B = \{5, 10, 15\} \quad A \cap B = \{15\} \quad A \cup B = \{3, 5, 6, 9, 10, 12, 15, 18\}$$

Venn Diagrams

The relationships between set A , set B , and the universal set U can be illustrated visually in a **Venn Diagram**.

The elements of each set are marked with a dot to distinguish them from the number of elements in each set (see Class Ex. #4).



Notice the following points:

- The number divisible by 3 and 5 is located in the intersection (overlap) of the two circles.
- The numbers divisible by 3 or 5 are located within both circles.
- The numbers in the universal set that are not in set A or set B are located within the rectangle but outside the two circles.

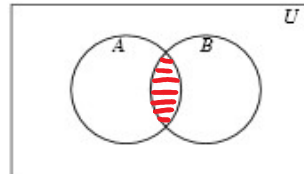
Class Ex. #1



In each of the following Venn diagrams, shade the region representing the given set.

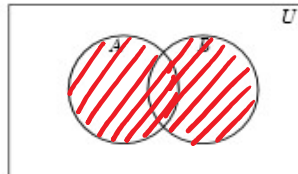
a) The **intersection** of the two sets, i.e. the set A and B .

$$A \cap B$$



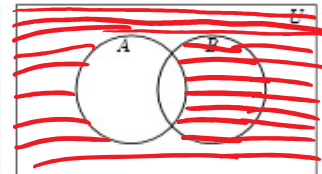
b) The **union** of the two sets, i.e. the set A or B .

$$A \cup B$$



c) The **complement** of the set A , i.e. the set not A .

$$A'$$



Copyright © by Absolute Value Publications. This book is **NOT** covered by the Cancopy agreement.

20 Logical Reasoning and Set Theory Lesson #3: Venn Diagrams - Part One



Class Ex. #2

Consider the following two sets from the review on the previous page.

$A = \{\text{natural numbers less than 20 that are divisible by 3}\}$

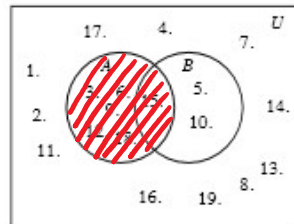
$B = \{\text{natural numbers less than 20 that are divisible by 5}\}$

In each of the following Venn diagrams

- lightly shade the region representing the given set
- describe each set in terms of A and/or B using connecting words where appropriate
- determine the number of elements in the set.

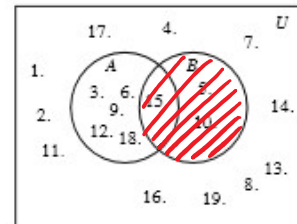
Some of the work has been completed as a guide.

Shade set A



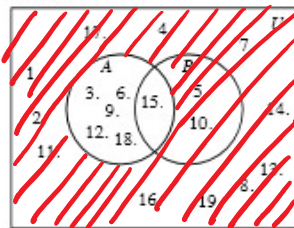
- A
- $n(A) = 6$
↑ number of elements

Shade set B



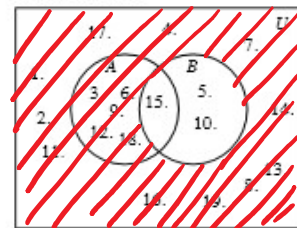
- B
- $n(B) = 3$

Shade set A'



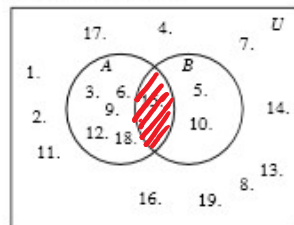
- A'
- $n(A') = 19 - 6 = 13$

Shade set B'



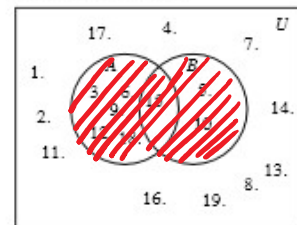
- B'
- $n(B') = 19 - 3 = 16$

Shade set $A \cap B$



- A and B
- $n(A \cap B) = 1$

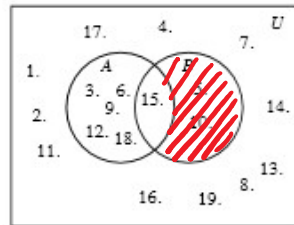
Shade set $A \cup B$



- A or B
- $n(A \cup B) = 8$

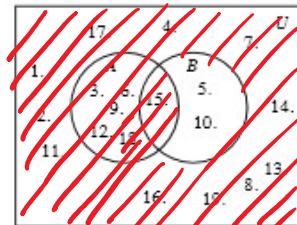
Copyright © by Absolute Value Publications. This book is **NOT** covered by the Copyright agreement.

Shade set $A' \cap B$



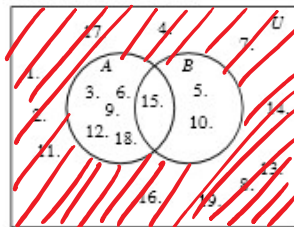
- not A and B
- $n(A' \cap B) = 2$

Shade set $A \cup B'$



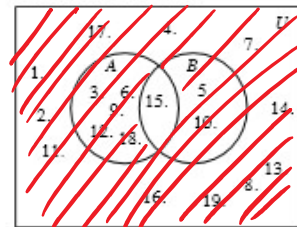
- A or not B
- $n(A \cup B') = 17$

Shade set $A' \cap B'$



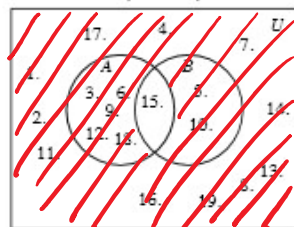
- not A and not B
- $n(A' \cap B') = 11$

Shade set $A' \cup B'$



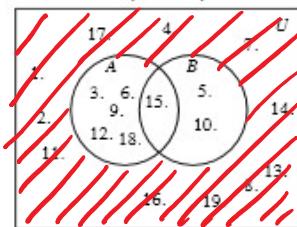
- not A or not B
- $n(A' \cup B') = 18$

Shade set $(A \cap B)'$



- not (A and B)
- $n((A \cap B)') = 18$

Shade set $(A \cup B)'$



- not (A or B)
- $n((A \cup B)') = 11$

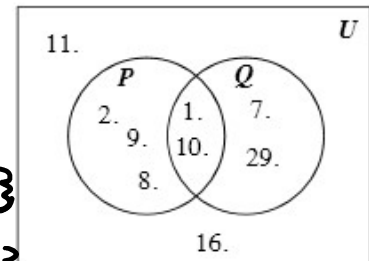
As demonstrated in the previous lesson, we can see from the above Venn diagrams that

- $(A \cap B)' = A' \cup B'$
- $(A \cup B)' = A' \cap B'$

22 Logical Reasoning and Set Theory Lesson #3: Venn Diagrams - Part One



Use the Venn diagram to list the elements of the following sets.



a) $P = \{1, 2, 8, 9, 10\}$ b) $Q = \{1, 7, 10, 29\}$

c) $P \text{ and } Q = \{1, 10\}$ d) $P \text{ or } Q = \{1, 2, 7, 8, 9, 10, 29\}$

e) $\text{not } P = \{7, 11, 16, 29\}$ f) $\text{not } Q = \{2, 8, 9, 11, 16\}$

g) $\text{not } P \text{ and not } Q = \{11, 16\}$ h) $\text{not } P \text{ or not } Q = \{2, 7, 8, 9, 11, 16, 29\}$

i) $\text{not } (P \text{ or } Q) = \{11, 16\}$ j) $\text{not } (P \text{ and } Q) = \{2, 7, 8, 9, 11, 16, 29\}$

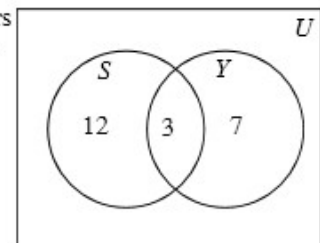
k) Which two pairs of sets are identical? g, i
 j, h



Venn diagrams can also be used to represent the number of elements in each set and not the individual elements, as the next class example illustrates. The numbers in the sets are not marked with dots, since they do not represent individual elements.



The diagram displays the number of students who are members of Students' Council (S) and the number of students who are on the Yearbook Committee (Y).



How many students are

a) on Students' Council? $12 + 3 = 15$

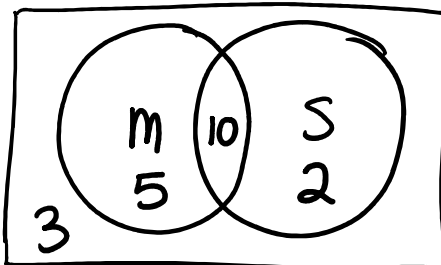
b) on both Students' Council and Yearbook Committee? 3

c) on the Yearbook Committee but not on Students' Council? 7

d) on Students' Council or Yearbook Committee? 22 $12 + 3 + 7$



In a homeroom of 20 students, 15 take Math, 12 take Social, and 10 take Math and Social. Show this information in a Venn diagram. How many students take neither Math nor Social?



$20 - 5 - 10 - 2 = 3$

$15 - 10$ take only math = 5

$12 - 10$ take only socials = 2

10 take both = 10

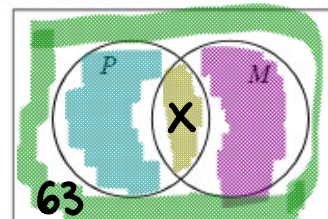
$20 - 17 = 3$ take neither. 17



In a survey of 400 households, 285 had personal video recorders (PVRs) and 320 had multi-function printers (MFPs). 63 households did not have PVRs or MFPs.

- Let P represent the set of households with a PVR.
- Let M represent the set of households with an MFP.
- Let x represent the number of households that have both a PVR and an MFP.

- a) Mark x and 63 in the appropriate parts of the Venn diagram.
- b) Write an expression for the number of households who have a PVR and not an MFP. Write this expression in the appropriate part of the Venn diagram.



- c) Write an expression for the number of households who have an MFP and not a PVR. Write this expression in the appropriate part of the Venn diagram.

$$285 - x$$

- d) Form an equation in x and determine the number of households in the survey who had both an MFP and PVR.

$$320 - x$$

$$\text{total \#} = 400$$

$$285 - x + x + 320 - x + 63 = 400$$

$$-x + 668 = 400$$

$$-x = -268$$

$$x = 268$$

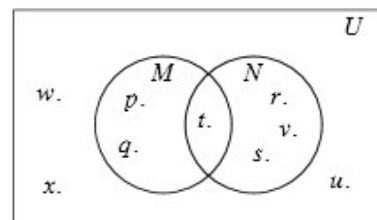
$$1, 5, 6, 9, 10, 11$$

Complete Assignment Questions #1 - #12

Assignment

1. Consider the Venn diagram shown.
List the elements of the following sets:

- a) $M = \{$
- b) M and N
- c) M or N
- d) not M
- e) not N
- f) not (M and N)



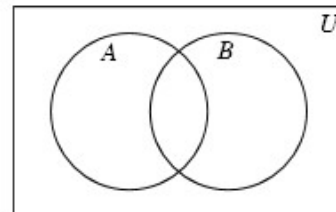
24 Logical Reasoning and Set Theory Lesson #3: Venn Diagrams - Part One

2. Using the Venn diagram in question #1, which sets are represented by:

- a) $\{r, s, t, v\}$ b) $\{p, q\}$ c) $\{u, w, x\}$

3. Consider the set of prime numbers less than 20.
Let $A = \{3, 5, 7, 11, 19\}$ and $B = \{2, 3, 7, 13\}$.

a) Complete the Venn diagram to illustrate this information.

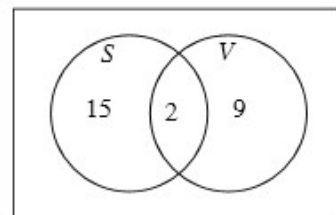


b) List the members of the following sets:

- i) A and B ii) A or B
iii) not A iv) not(A or B)

4. The diagram displays the number of girls who are members of the school soccer team (S) and the school volleyball team (V).

How many girls are:



- a) on both teams?
b) on the soccer team and not on the volleyball team?
c) on only one team?
d) on the soccer team or on the volleyball team?

5. Of the students in Grade 12 at a certain high school, 76 are enrolled in physical education, 24 are enrolled in music, and 10 are enrolled in both physical education and music. If there are 15 students in Grade 12 who are not enrolled in physical education or music, how many students are in Grade 12?

6. All the students in a class of 35 take Physics or Chemistry or both. 29 take Chemistry and 15 take Physics. How many take both?
7. In a school survey, it was found that 140 students had a cell phone or a tablet. If 86 students had a cell phone and 70 students had a tablet, how many students had both?

8. In each Venn diagram below, sketch two sets A and B satisfying the given condition.

a) $A \subset B$



b) $B \subset A$

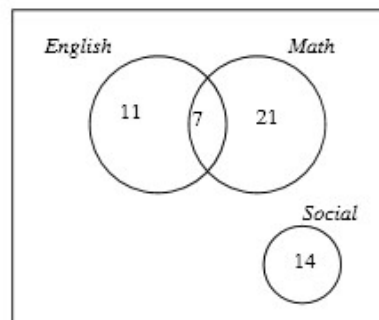


c) A and B are disjoint sets



9. The Venn diagram shows the number of students who did English homework, Math homework, or Social homework on the weekend. None of the students did homework for any other subject. How many students did

- a) English homework?
 b) only English homework?
 c) English homework and Social homework?
 d) English homework or Math homework?
 e) Math homework and not Social homework?
 f) homework for only one subject?

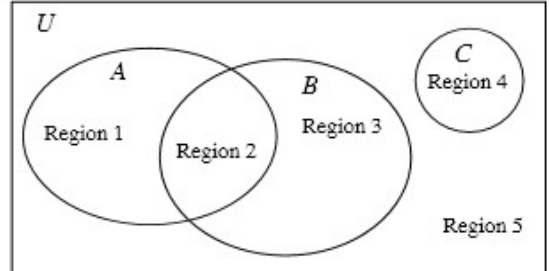


Multiple Choice

10. If $P = \{\text{quadrilaterals that have 4 equal sides}\}$ and $Q = \{\text{quadrilaterals that have 4 equal angles}\}$, then a trapezoid with only one pair of parallel sides is an element of which of the following sets?
- A. P and Q B. P and not Q
 C. not P and Q D. not P and not Q

Use the following information to answer the next two questions.

The Venn diagram illustrating sets A , B , and C has been divided into five non-intersecting regions.



11. The region(s) representing the set $(A \cup B)'$ is/are
- A. Region 5 B. Regions 4 and 5
 C. Regions 1, 3, and 5 D. Regions 1, 3, 4, and 5

Numerical Response

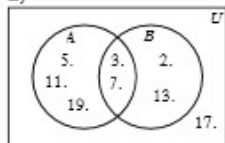
12. The set $A' \cap B$ can be represented by one region. This region number is _____.

(Record your answer in the numerical response box from left to right.)

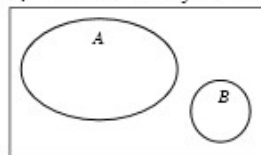
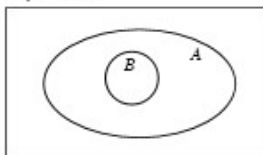
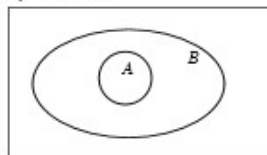
--	--	--	--

Answer Key

1. a) $\{p, q, t\}$ b) $\{t\}$ c) $\{p, q, r, s, t, v\}$
 d) $\{r, s, u, v, w, x\}$ e) $\{p, q, u, w, x\}$ f) $\{p, q, r, s, u, v, w, x\}$
 2. a) N b) M and not N c) not M and not N or not $(M \text{ or } N)$
 3. a) b) i) $\{3, 7\}$ ii) $\{2, 3, 5, 7, 11, 13, 19\}$ iii) $\{2, 13, 17\}$ iv) $\{17\}$



4. a) 2 b) 15 c) 24 d) 26 5. 105 6. 9 7. 16
 8. a) $A \subset B$ b) $B \subset A$ c) A and B are disjoint sets



9. a) 18 b) 11 c) 0 d) 39 e) 28 f) 46
 10. D 11. B 12.

3			
---	--	--	--