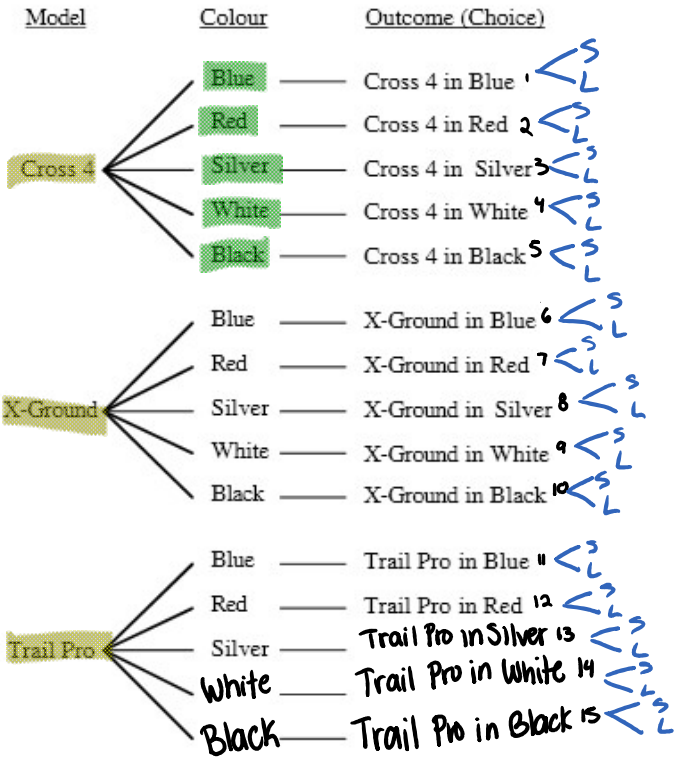
# Lesson 1: The Fundamental Counting Principle

# Permutations and Combinations Lesson #1:
## The Fundamental Counting Principle

***Investigating the Fundamental Counting Principle***

Lucy is shopping for a sports utility vehicle. The dealer says that he has 3 different models and that each model is available in 5 different colours.

**a)** Complete the tree diagram to determine how may different choices Lucy has.




Lucy has 15 choices.

**b)** Look at the number of choices there are for the model and the number of choices there are for the colour. How do you use these numbers to arrive at the answer in a)?

multiply $3 \times 5 = 15$

**c)** The dealer tells Lucy that each of the sports utility vehicles comes in two editions: SE (standard edition) and LE (leather edition).

**i)** Use the tree diagram in a) to determine how many choices she now has.

**ii)** Extend the method in b) to verify the answer.

$3 \times 5 \times 2 = 30$ choices

## *The Fundamental Counting Principle*

The answer to c) ii) on the previous page is determined by multiplying the number of models (3), by the number of colours (5), by the number of editions (2). This is an example of what is called the **fundamental counting principle.**

Consider a task made up of several stages. The **fundamental counting principle** states that if the number of choices for the first stage is $a$, the number of choices for the second stage is $b$, the number of choices for the third stage is $c$, etc., then the number of ways in which the task can be completed is $a \times b \times c \times \ldots$.

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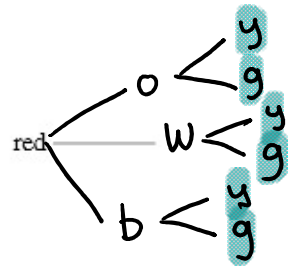
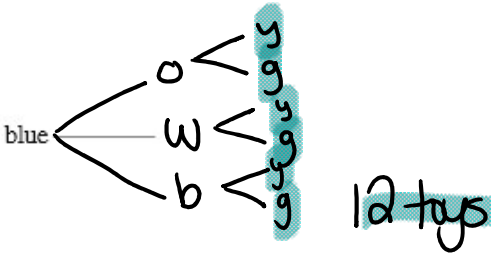
A toy manufacturer makes a wooden toy in three parts:

Part 1: the top part may be coloured red or blue.
Part 2: the middle part may be orange, white, or black.
Part 3: the bottom part may be yellow or green.

Determine how many different coloured toys can be produced using

**a)** a tree diagram




**b)** the fundamental counting principle

$2 \times 3 \times 2 = 12$


- In the previous example, the toy consisted of Part 1 and Part 2 **and** Part 3. The total number of possible toys was found by **multiplication**.
- In any example involving "and" the total number of arrangements will be found by multiplying.

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A math quiz consists of six multiple choice questions. Each question has four choices, A, B, C, or D. Use the fundamental counting principle to determine how many different sets of answers are possible.

$4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4096$

## *The Fundamental Counting Principle Involving Restrictions*

When solving problems involving restrictions, it is important that **the restriction is dealt with first** as in the example below.


**a)** Eric has been assigned the task of determining how many odd four digit numbers there are. He has been told that a number such as 5267 is a four digit number, whereas 0267 is classified as a three digit number.

**i)** The first step of his work is shown. Explain his reasoning for the number 5.

__ __ __ (5) The last digit must be odd. There are 5 possibilities → 1, 3, 5, 7, 9

**ii)** The second step of his work is shown. Explain his reasoning for the number 9.

(9) __ __ (5) A zero cannot be in 1st spot. There are 9 possibilities → 1, 2, 3, 4, 5, 6, 7, 8, 9

**iii)** Complete his work to determine how many odd four digit numbers there are.

(9) × (10) × (10) × (5) = 4500 options

**b)** Eric has now been assigned the task of determining how many odd four digit numbers there are which have no repeating digits.

**i)** Explain why the first step in a) is still valid, but the second step is not.

__ __ __ (5) ← the last # still needs to odd

(8) __ __ (5) ↖ can not zero, can not be the last # 10 − 2 = 8

**ii)** Determine how many odd four digit numbers have no repeating digits.

(8) × (8) × (7) × (5) = 2240 ↖ 10 − 2 = 8

7777 8017 ↖ 1, 2, 3, 4, 5, 6, X, X, 9


Car number plates in an African country consist of a letter other than I or O followed by three digits, the first of which cannot be zero, followed by any two letters which are not repeated. How many different car number plates can be produced?

(24) (9) (10) (10) (26) (25) = 14 040 000 license plate options

↑ 26 − 1 + 0 = 24

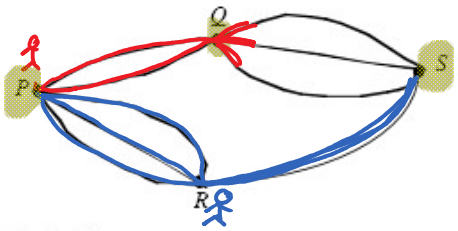
↑ 10 − zero = 9

* 26 letters in the alphabet
* 10 single digit #s

**Complete Assignment Questions #1 - #7**

**Class Ex. #5**

There are two routes from Pitland to Queensville, three routes from Queensville to St. Lukes, three routes from Pitland to Rutherford, and one route from Rutherford to St. Lukes.




**a)** How many routes are there from Pitland to St. Lukes passing through Queensville?

6 $2 \times 3 = 6$

**b)** How many routes are there from Pitland to St. Lukes passing through Rutherford?

$3 \times 1 = 3$

**c)** How many routes are there from Pitland to St. Lukes?

$6 + 3 = 9$ routes

**Note**

In part c) we could travel from $P$ to $S$ via $Q$ **or** from $P$ to $S$ via $R$. The total number of routes was found by **adding** the answer in a) to the answer in b).

In any example involving "or" the total number of arrangements will be found by adding.

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**Class Ex. #6**

The telephone numbers allocated to subscribers in a rural area consist of one of the following:

- the digits ~~345~~ 604 followed by any three further digits, *or*,
- the digit 2 followed by one of the digits 1 to 5 followed by any three further digits.

How many different telephone numbers are possible?

1 $604 \underline{10}\,\underline{10}\,\underline{10} = 1000$

OR

2 $2 \underline{5}\,\underline{10}\,\underline{10}\,\underline{10} = 5000$

1,2,3,4,5

$1000 + 5000 = 6000$

#1-3, 5-8 (more tomorrow)

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**Class Ex. #8**

Consider the number of distinguishable four letter arrangements that can be formed from the word **ENGLISH**. (Note: There is an implication that each of the letters in the word ENGLISH only occurs once and cannot be repeated).

Determine the number of distinguishable four letter arrangements that can be formed from the word ENGLISH, if (7)

**a)** any letter can be in any position

$\underline{7}\,\underline{6}\,\underline{5}\,\underline{4} = 840$

**b)** the first letter must be **E**

$\underline{E}\,\underline{6}\,\underline{5}\,\underline{4} = 120$

**c)** the first and last letters must be vowels

$\underline{2}\,\underline{5}\,\underline{4}\,\underline{1} = 40$

**d)** the "word" must contain **G**

$\underline{G}\,\underline{6}\,\underline{5}\,\underline{4} = 120$ +

$\underline{6}\,\underline{G}\,\underline{5}\,\underline{4} = 120$ +

$\underline{6}\,\underline{5}\,\underline{G}\,\underline{4} = 120$ +

$\underline{6}\,\underline{5}\,\underline{4}\,\underline{G} = 120$

480


Consider Class Ex. #3 in which Eric was asked to determine how many odd four digit numbers there are.

As an extension to his assignment, Eric has been asked to determine

**i)** how many even four digit numbers there are, and,

**ii)** how many **even** four digit numbers have no repeated digits.

**a)** Determine the answer to i).

9 · 10 · 10 · 5 = 4500

9: can not be 0, 10 − 1 = 9

5: 0, 2, 4, 6, 8

**b)** Eric started his answer to ii), as shown below, but he was not sure how to fill in the first space in his work. Explain why he needs to consider two separate cases to solve the problem.

__ __ __ 5

__: no 0 and can't repeat the first one

5: 0, 2, 4, 6, 8

- If the last digit is zero, there are 9 possibilities for the 1st
- If the last digit is 2, 4, 6 or 8, then there are 8 possibilities for the 1st.

**c)** Determine the answer to ii).

9 · 8 · 7 · 1 = 504

9: not 0

8: 10 − 2, we used 2 #'s already

1: 0

or

8 · 8 · 7 · 4 = 1792

8: − not 0, − not the last

4: 2, 4, 6, 8

1792 + 504 = 2296

#1-3, 5-8, 10-12

10:45

**Complete Assignment Questions #8 - #17**

# Assignment

**1.** A football team has the following kit :

jersey: red or black
pants: white, red, or black
socks: red or white

The team plays in a different uniform each week until it has to repeat a previous uniform.

Determine how many weeks the team can play before repeating a previous uniform by using

**a**) a tree diagram

**b**) the fundamental counting principle

**2.** How many ways are there of arranging 6 different books side by side on a shelf?

**3.** With the new renovations completed at Prestwick High School, there will be seven entrances. In how many different ways can a student coming for Math tutorials

**a**) enter and exit through any entrance?

**b**) enter the school and exit through a different entrance?

**c**) enter and exit through the same entrance?

**4.** The score at the end of the second period of a hockey game is: Flames 6 Oilers 3.

Jarome was attempting to determine how many different possibilities there are for the score at the end of the first period. He used the fundamental counting principle and multiplied 6 by 3 to get an answer of 18. Explain the error in his reasoning.

**5.** If each of the students in a class of 30 students is capable of winning any of the class prizes, how many ways are there of awarding

**a**) a first prize, a second prize, and a third prize in Mathematics?

**b**) a Mathematics prize, a Chemistry prize, and a Physics prize?

**6.** Three digit numbers are formed using only the digits 2, 3, 5, 6, 7, and 9.

**a)** If repetitions are not permitted, how many 3-digit numbers can be formed?

**b)** How many of these are

**i)** less than 400? **ii)** even? **iii)** odd? **iv)** multiples of 5?

**7.** A vehicle license plate consists of 3 letters followed by 3 digits. How many different license plates are possible if:

**a)** there are no restrictions on the letters or digits used?

**b)** no letters may be repeated?

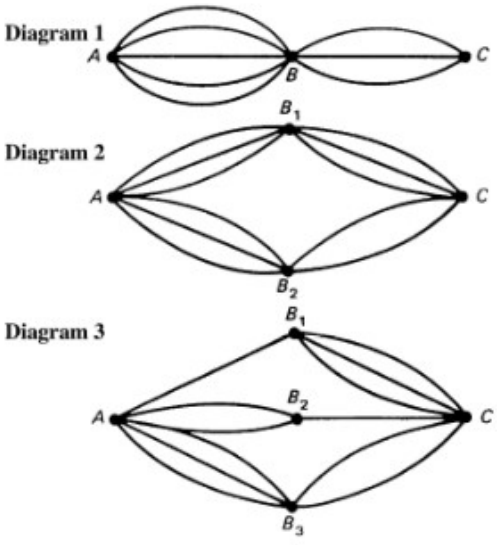
**c)** the first digit cannot be zero and no digits can be repeated?

**8.** How many ways are there of getting from A to C in each diagram, passing through each point at most once?

Answer to Diagram 1

Answer to Diagram 2

Answer to Diagram 3



**9.** Determine the number of four letter "words" that can be formed from the letters of the word **PRODUCE** if

**a)** each letter can only be used once

**b)** each letter can only be used once and the "word" must

**i)** contain only consonants

**ii)** begin and end with a consonant

**iii)** begin with a vowel

**iv)** contain the letter **P**

**v)** begin with **D** and end with a vowel

**10. a)** How many different three-digit numerals can be formed from the digits 1, 5, and 8 if the digits cannot be repeated?

**b)** How many different three-digit numerals can be formed using the digits 1, 3, 5, 7, and 9 if the digits may be repeated?

**c)** How many four-digit numerals can be formed from the digits 0, 2, and 3 if the digits may be repeated? (Note: 0223 is classified as the 3-digit numeral 223.)

**d)** How many different non-zero numerals are possible using some or all of the numerals 0, 1, 2, and 3 if the digits cannot be repeated?

**11.** Mr. and Mrs. McDonald want a family picture taken with their children, Hamish, Flora and James. In how many different ways can all five line up in a straight line for the picture if

**a)** there are no restrictions?

**b)** the parents must be at either end of the line?

**c)** baby James must be in the middle?

**d)** the children alternate with the adults?

*Use the following information to answer the next question.*

The word **PRODUCT** has been spelled using letter tiles. An illustration is shown.


**12.** Using only these tiles, determine the number of four letter arrangements if the arrangement

**a)** has any letter in any position

**b)** begins with **PR**

**c)** has two vowels in the middle

**d)** has two consonants in the middle

**13.** Ocean-going ships use coloured flags hung vertically for signalling.
By changing the order of the coloured flags, the ships can send out different signals.
If ships carry six different coloured flags, one flag of each colour, how many different signals are possible if

**a)** all six flags are used?

**b)** four flags are used?

**c)** at least two flags are used?

**14.** **a)** How many odd six digit numbers have no repeating digits?

**b)** Consider the question "How many even six digit numerals have no repeating digits?" Explain why we need to consider two separate cases to determine the answer.

**c)** How many even six digit numerals have no repeating digits?

**Multiple Choice** **15.** In the final of a 100-metre race there are 8 competitors. The number of possible ways in which the gold, silver, and bronze medals can be awarded is

**A.** 21 **B.** 24

**C.** 336 **D.** 512

**16.** How many even 5-digit whole numbers are there? Note that 31248 is acceptable, but 01248 is not.

**A.** 13 776

**B.** 15 120

**C.** 45 000

**D.** 50 000

Numerical Response

**17.** Sandra is taking an examination which consists of two parts, A and B, with the following instructions.

- Part A consists of three questions and the student must do two.
- Part B consists of four questions and the student must do two.
- Part A must be completed before starting Part B.
- At the end of the exam the student has to list the order in which she attempted the questions.

The number of different possible orders is _____ .

(Record your answer in the numerical response box from left to right.)

| | | | |
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| | | | |

### *Answer Key*

**1.** 12 **2.** 720 **3. a)** 49 **b)** 42 **c)** 7

**4.** He should have multiplied 7 by 4 to get 28. **5. a)** 24360 **b)** 27000

**6. a)** 120 **b) i)** 40 **ii)** 40 **iii)** 80 **iv)** 20

**7. a)** 17 576 000 **b)** 15 600 000 **c)** 11 389 248

**8. Diagram 1** → 15 **Diagram 2** → 15 **Diagram 3** → 11

**9. a)** 840 **b) i)** 24 **ii)** 240 **iii)** 360 **iv)** 480 **v)** 60

**10. a)** 6 **b)** 125 **c)** 54 **d)** 48

**11. a)** 120 **b)** 12 **c)** 24 **d)** 12 **12. a)** 840 **b)** 20 **c)** 40 **d)** 400

**13. a)** 720 **b)** 360 **c)** 1950

**14. a)** 67 200

**b)** If the last digit is zero, there are nine possible choices for the first digit.
If the last digit is 2, 4, 6, or 8, there are only eight choices for the first digit since zero cannot be used.

**c)** 68 880

**15.** C **16.** C **17.**

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