Lesson 2: Set Theory

Logical Reasoning and Set Theory Lesson #2: Set Theory

In the last lesson, we defined the use of "and", "or", and "not" in mathematics. In this lesson, we will apply the use of "and", "or", and "not" to sets. The concepts of universal set, subsets, and empty sets will also be introduced.

Sets and Set Notation

In mathematics, a **set** can be defined as a collection of distinct objects. For example, the numbers on the faces of a die form a set. Sets are usually denoted by capital letters or by a description inside curly brackets.

An object in a set is referred to as an **element**, e.g. the number 5 is an element of the set of numbers of a die.

There are three general ways of defining the contents of a set.

- Listing the elements of the set, e.g. → $A = \{1, 2, 3, 4, 5, 6\}$.
- A description of the set in words, e.g. → $A = \{\text{natural numbers less than } 7\}$.
- Using set builder notation to describe the set, e.g. → $A = \{x \mid x < 7, x \in N\}$.

Recall
- the curly brackets together is read as "the set of"
- $\mid$ represents, and is read as, "such that"
- $\in$ represents, and is read as, "is a member of".

The number of elements in set A is written as $n(A)$. In the above example, $n(A) = 6$.

Note that the number 2 belongs to set A, but the number 8 does not belong to set A.
To express this, we can write $2 \in A$ and $8 \notin A$.

The order of listing the elements in a set does not matter, e.g. $\{1, 2\} = \{2, 1\}$.

Consider the following two sets.

$$P = \{\text{whole numbers less than or equal to } 3\}$$
$$Q = \{\text{even whole numbers less than } 10\}$$

a) List the elements of each set.

b) Complete the following: **i)** $n(P) =$ **ii)** $n(Q) =$

c) Write set P using set builder notation.

d) Which of the following is set builder notation that describes set Q?

A. $Q = \{x \mid x < 10\ x \in W\}$ **B.** $Q = \{e \mid e = 2x, 0 \leq x < 10, x \in W\}$

C. $Q = \{e \mid e = 2x, 0 \leq x \leq 4, x \in W\}$ **D.** $Q = \{e \mid e = 2x, 0 \leq x \leq 5, x \in W\}$

Terminology Used in Set Theory

Betty is defining sets using non-negative single digit numbers.
From the list of non-negative single digit numbers, she defined the following sets.

$E = \{$even numbers$\}$ $E = \{$
$L = \{$whole numbers less than 7$\}$ $L = \{$
$O = \{$odd numbers less than 7$\}$ $O = \{$

List the elements of each set in the space above.

We will use these sets as an aid to understanding some terms used in set theory.

Universal Set

Within the context of any problem in set theory, there generally is some largest set that we have in mind. We refer to this set as the **universal set**. It is the set that contains all of the elements under consideration and relevant to the problem.

- State the universal set for the example above. Label this set U.

Subset

A **subset** of a set is a set that contains some or all or possibly none of the elements from a previously defined set. All sets we deal with in a problem must be subsets of the universal set and every set is a subset of itself.

- In set notation, the symbol $\subset$ is used to represent this relationship between sets.
  For example, set E is a subset of set U, since **all** the elements in set E are also in set U.
  This is denoted by $E \subset U$.
- Is set E a subset of set L? If so, write this relationship using the symbol $\subset$.
  If not, write this relationship using the symbol $\not\subset$.
- Set O is a subset of set L, which is a subset of set U.
  Write this relationship using appropriate symbols.

Empty Set

The empty set is a set that contains no elements . The empty set is a subset of every set.

- The notation for the empty set is $\varnothing$.
  For example, the set of all odd numbers in set E is $\varnothing$ because there are no odd numbers in set E.
- Provide another example of a set that is empty.

Complement of a Set

The complement of a set A is the set of all elements in the universal set that are not in set A.

- The complement of a set A is denoted by A' ← "not A"
- Write the complement of set E using the appropriate notation.

ex. Universal set is whole #'s less than 5

$U = \{0, 1, 2, 3, 4, 5\}$
$A = \{0, 2, 4\}$, A= whole #'s less than 5 that are even
$A' = \{1, 3, 5\}$

Consider the following two sets.

A = {natural numbers less than 20 that are divisible by 3}
B = {natural numbers less than 20 that are divisible by 5}

a) Explain why the set of natural numbers less than 20 is a suitable universal set.

b) List the elements of the following sets:

i) $A = \{3, 6, 9, 12, 15, 18\}$ **ii)** $B = \{5, 10, 15\}$

iii) $A' = \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19\}$

c) Write a description of the elements of the complement of set B in words.

d) Set C = {natural numbers less than 20 that are multiples of 6}. $C = \{6, 12, 18\}$
State whether the following are true or false.

i) $C \subset A$ **ii)** $C \not\subset B$ **iii)** $A' \subset C'$

"C is a subset of A" every element in C is from A

Complete Assignment Questions #1 - #7

Intersection of Sets and Union of Sets

We will use the example on the previous page to introduce further terminology used in set theory.

$E = \{0, 2, 4, 6, 8\}$ $L = \{0, 1, 2, 3, 4, 5, 6\}$ $O = \{1, 3, 5\}$

Intersection of Sets

The **intersection** of set A and set B is a set called **A and B** and is denoted by $A \cap B$. It is the set of elements that are members of both sets A and B.

- Write the intersection of sets E and L using appropriate terminology.

Union of Sets

The **union** of set A and set B is a set called **A or B** and is denoted by $A \cup B$. It is the set of elements that are members of either set A or set B, or both (inclusive use of "or").

- Use appropriate notation to write the union of set E and set L. Do not write the same element more than once in the set.

Disjoint Sets

Two sets are said to be **disjoint** if they have no elements in common, i.e. the intersection of the two sets is the empty set.

- e.g. $\{2, 3, 4\}$ and $\{5, 6, 7\}$ are disjoint because they have no element in common.
- Which two of the sets from set E, set L, and set O are disjoint?

Consider the sets A, B, C from Class Ex. #2, where U = {natural numbers less than 20}.

$A' = \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19\}$

A = {natural numbers less than 20 that are divisible by 3} = $\{3, 6, 9, 12, 15, 18\}$
B = {natural numbers less than 20 that are divisible by 5} = $\{5, 10, 15\}$
C = {natural numbers less than 20 that are multiples of 6} = $\{6, 12, 18\}$

a) List the elements of the following sets and describe each set in words.

i) $A \cap B$ $= \{15\}$ ...that are divisible by 3 and 5
"A and B"

ii) $A \cup B$ $= \{3, 5, 6, 9, 10, 12, 15, 18\}$ ...that are divisible by 3 or 5
"A or B"

iii) $A' \cap B$ $= \{5, 10\}$ ...that are not divisible by 3 and are divisible 5
"not A, and B"

b) Write a set equivalent to

i) $B \cap C$ disjointed $\emptyset$ **ii)** $A \cup C = A$ **iii)** $A \cap C = C$

c) If $D = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$, list the elements of the following sets

i) $A \cap (B \cup D)$ $\{6, 12, 15, 18\}$ **ii)** $(A \cap B) \cup D$ $\{2, 4, 6, 8, 10, 12, 14, 15, 16, 18\}$ * **iii)** $(A' \cap B')'$

Complete Assignment Questions #8 - #14

#1ABC, 2A, 3, 5, 6, 7, 8, 9, 10

Assignment

1. List the elements of each of the following sets.

a) P = {prime numbers less than 20}

b) M = {positive multiples of 4 that are less than 20}

c) F = {factors of 20}

d) $T = \{t \mid t = 4x, 1 \le x \le 4, x \in N\}$

e) W = {prime numbers between 24 and 28}

2. Describe the following sets in words.

a) $A = \{7, 14, 21, 28\}$

b) $B = \{1, 4, 9, 16, 25\}$

Use the following information to answer question #3.

Toni is the owner of a sandwich shop. Customers may put any of the following toppings on their sandwiches:

- Lettuce
- Tomatoes
- Cheese
- Pickles
- Onions
- Olives
- Roast Beef
- Salami

3. a) Let T = {sandwich toppings available at Toni's shop}.
Determine the value of $n(T)$ and explain its meaning.

b) List the elements of M = {meat toppings available at Toni's shop}.
Explain why M is a subset of T.

c) Determine whether each of the following is a subset of T.
Explain your reasoning.

i) {olives, pickles, tomatoes}

ii) {lettuce, cheese, bacon, salami}

iii) the empty set

iv) T

4. A, B, C, and D are the sets: $A = \{q\}$, $B = \{p, q, r\}$, $C = \{r, s\}$, and $D = \{p, q, s\}$.
Which of the following statements are true?

a) $q \in A$ **b)** $q \subset A$ **c)** $\{q\} \in A$ **d)** $\{q\} \subset A$ **e)** $q \notin C$ **f)** $\{q\} \not\subset D$

g) $A \subset B$ **h)** $B \subset A$ **i)** $A \in D$ **j)** $C \not\subset D$ **k)** $D \subset D$ **l)** $\varnothing \subset C$

5. If the universal set $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{1, 2, 4\}$, and $B = \{1, 3, 5\}$,
list the elements in A' and B'.

6. If the universal set $U = \{\text{Jo, Jane, Jack, Jacques, Jill}\}$ and $P = \{\text{Jane, Jacques}\}$, list the elements in P'.

7. If $A = \{\text{red, green, blue}\}$ and $A' = \{\text{black, white}\}$, list the elements in the universal set.

8. If $P = \{\text{red, orange, green}\}$ and $Q = \{\text{red, white, blue}\}$, list the elements of

a) $P \cap Q$ **b)** $P \cup Q$

Use the following information to answer the next question.

Sets $A = \{1, 2, 4, 8\}$
$B = \{2, 3, 5, 7\}$
$C = \{2, 4, 6, 8\}$
are subsets of the universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$

9. a) List the elements of the following sets.

i) $A \cap B$ **ii)** $A \cup B$ **iii)** $A \cap C$ **iv)** $A \cup C$

v) $B \cap C$ **vi)** $B \cup C$ **vii)** $A \cup B \cup C$ **viii)** $A \cap B \cap C$

b) List the elements of the following sets.

i) A' **ii)** B' **iii)** $A' \cap B'$

iv) $A' \cup B'$ **v)** $(A \cap B)'$ **vi)** $(A \cup B)'$

vii) $(A \cap B) \cup C$ **viii)** $(A \cup C) \cap B'$

c) Which pairs of sets in b) are equal?

10. Beverly is conducting a survey on the students in her class. She discovers that of the 18 boys in the class, 5 are left-handed, and of the 12 girls in the class, 9 are right-handed. No students are ambidextrous.

She defined the following sets for use in the survey:

$B = \{\text{boys}\}$ $G = \{\text{girls}\}$ $L = \{\text{left-handed}\}$ $R = \{\text{right-handed}\}$

a) State a suitable universal set for this example.

b) List two pairs of sets that are disjoint.

c) Complete the following:

i) $n(B) =$ **ii)** $n(L) =$ **iii)** $n(B \cap L) =$ **iv)** $n(B \cup L) =$

v) $n(B') =$ **vi)** $n(B' \cap R) =$ **vii)** $n((B \cap L)') =$ **viii)** $n(B' \cup L') =$

11. Consider the following sets. $U = \{x \mid x \leq 15, x \in N\}$
$O = \{\text{odd numbers less or equal to 15}\}$
$P = \{\text{prime numbers less than 15}\}$
$Q = \{\text{whole numbers larger than 15}\}$

a) List the members of the following sets.

i) O **ii)** P

iii) $O \cap P$ **iv)** $O \cup P$

b) Compare your answers to Class Ex. #4, a) to d), from Lesson 1, page 4. What do you notice?

c) Circle the correct alternative.

i) The intersection of two sets is an example of the connecting word "and" / "or".

ii) The union of two sets is an example of the connecting word "and" / "or".

d) State whether each of the following is true or false.

i) $P \subset O$ **ii)** $(O \cap P) \subset O \subset U$

iii) $(O \cup P) \subset (O \cap P)$ **iv)** $(O \cap P) \subset (O \cup P)$

Use the following information to answer questions #12, #13, and #14.

Lucy and Jerry run an online clothing business named LJ's.
An illustration of the set of items they offer is shown.

The coats, scarves, snow pants, boots, and sweaters are for the winter season.

U is the set of all items the online store offers.
W is the set of winter clothing the online store offers.
L is the set of jeans and pants the online store offers.
F is the set of footwear the online store offers.

12. List the elements of each of the following.

a) W'

b) $W \cup L$

c) $W \cap L$

d) $W' \cup L'$

e) $W' \cap L'$

Multiple Choice **13.** Which of the following is an incorrect statement?

A. $n(U) = 11$ **B.** $W \subset U$ **C.** $n(W \cap L') = 4$ **D.** $L \subset W$

14. The number of elements in $W \cup F$ can be represented by

A. $n(W \cup F) = n(W) - n(F)$
B. $n(W \cup F) = n(W) + n(F)$
C. $n(W \cup F) = n(W) + n(F) - n(W \cap F)$
D. $n(W \cup F) = n(W) + n(F) + n(W \cap F)$

Answer Key

1. **a)** $\{2, 3, 5, 7, 11, 13, 17, 19\}$ **b)** $\{4, 8, 12, 16\}$ **c)** $\{1, 2, 4, 5, 10, 20\}$
d) $\{4, 8, 12, 16\}$ **e)** $\varnothing$

2. Answers may vary.
a) {the first four positive multiples of 7} **b)** {the squares of the first five natural numbers}

3. **a)** $n(T) = 8$ and represents the number of different sandwich toppings available at Toni's Shop.
b) $M = \{$Roast Beef, Salami$\}$. M is a subset of T because every element of M is an element of T.
c) **i)** Yes. Each element is an element of T.
ii) No. The element "bacon" is not an element of T.
iii) Yes. The empty set has no elements and is a subset of every set.
iv) Yes. Every set is a subset of itself.

4. True → a), d), e), g), j), k), l) **5.** $A' = \{3, 5, 6\}$, $B' = \{2, 4, 6\}$

6. $P' = \{$Jo, Jack, Jill$\}$ **7.** {red, green, blue, black, white}

8. **a)** {red} **b)** {red, orange, green, white, blue}

9. **a)** **i)** $\{2\}$ **ii)** $\{1, 2, 3, 4, 5, 7, 8\}$ **iii)** $\{2, 4, 8\}$ **iv)** $\{1, 2, 4, 6, 8\}$
v) $\{2\}$ **vi)** $\{2, 3, 4, 5, 6, 7, 8\}$ **vii)** $\{1, 2, 3, 4, 5, 6, 7, 8\}$ **viii)** $\{2\}$
b) **i)** $\{3, 5, 6, 7\}$ **ii)** $\{1, 4, 6, 8\}$ **iii)** $\{6\}$ **iv)** $\{1, 3, 4, 5, 6, 7, 8\}$
v) $\{1, 3, 4, 5, 6, 7, 8\}$ **vi)** $\{6\}$ **vii)** $\{2, 4, 6, 8\}$ **viii)** $\{1, 4, 6, 8\}$
c) $A' \cup B' = (A \cap B)'$. $A' \cap B' = (A \cup B)'$

10. **a)** {Students in Beverly's class} **b)** B and G are disjoint. L and R are disjoint.
c) **i)** 18 **ii)** 8 **iii)** 5 **iv)** 21
v) 12 **vi)** 9 **vii)** 25 **viii)** 25

11. **a)** **i)** $\{1, 3, 5, 7, 9, 11, 13, 15\}\}$ **ii)** $\{2, 3, 5, 7, 11, 13\}$
iii) $\{3, 5, 7, 11, 13\}$ **iv)** $\{1, 2, 3, 5, 7, 9, 11, 13, 15\}$
b) The answers are the same.
c) **i)** "and" **ii)** "or"
d) **i)** false **ii)** true **iii)** false **iv)** true

12. **a)** $W' = \{$T-shirts, jeans, skirts, dresses, sneakers, sunglasses$\}$
b) $W \cup L = \{$coats, scarves, jeans, snow pants, boots, sweaters$\}$
c) $W \cap L = \{$snow pants$\}$
d) $W' \cup L' = \{$coats, scarves, T–shirts, jeans, skirts, dresses, boots, sneakers, sunglasses, sweaters$\}$
e) $W' \cap L' = \{$T-shirts, skirts, dresses, sneakers, sunglasses$\}$

13. D **14.** C

Logical Reasoning and Set Theory Lesson #3:
Venn Diagrams - Part One

In the last lesson, we used the connecting words "and", "or", and "not" in set theory. In this lesson we will continue to use these connecting words when we introduce **Venn Diagrams** as a visual aid in illustrating the relationships between sets.

Review

The following is taken from Class Ex. #2 of the last lesson.
Consider the following two sets, subsets of the universal set of natural numbers less than 20.

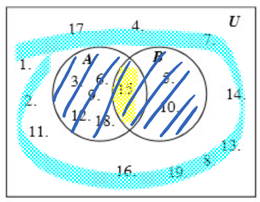
$A = \{$natural numbers less than 20 that are divisible by 3$\}$
$B = \{$natural numbers less than 20 that are divisible by 5$\}$

Using the appropriate notation, list the set of whole numbers less than 20 that are

a) divisible by 3 **b)** divisible by 5 **c)** divisible by 3 <u>and</u> 5 **d)** divisible by 3 <u>or</u> 5

$A = \{3,6,9,12,15,18\}$ $B = \{5,10,15\}$ $A \cap B = \{15\}$ $A \cup B = \{3,5,6,9,10,12,15,18\}$

Venn Diagrams



The relationships between set A, set B, and the universal set U can be illustrated visually in a **Venn Diagram**.

The elements of each set are marked with a dot to distinguish them from the number of elements in each set (see Class Ex. #4).

Notice the following points:

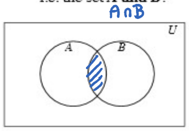
- The number divisible by 3 and 5 is located in the intersection (overlap) of the two circles.
- The numbers divisible by 3 <u>or</u> 5 are located within both circles.
- The numbers in the universal set that are not in set A or set B are located within the rectangle but outside the two circles.

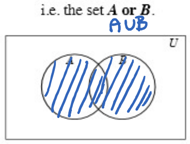
In each of the following Venn diagrams, shade the region representing the given set.

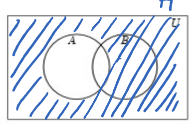
a) The **intersection** of the two sets, i.e. the set ***A* and *B***. $A \cap B$

b) The **union** of the two sets, i.e. the set ***A* or *B***. $A \cup B$

c) The **complement** of the set A, i.e. the set **not *A***. A'







Consider the following two sets from the review on the previous page.

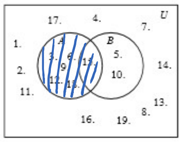
$A = \{$natural numbers less than 20 that are divisible by 3$\}$
$B = \{$natural numbers less than 20 that are divisible by 5$\}$

In each of the following Venn diagrams

- lightly shade the region representing the given set
- describe each set in terms of A and/or B using connecting words where appropriate
- determine the number of elements in the set.

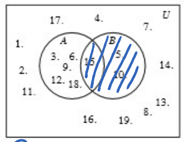
Some of the work has been completed as a guide.

Shade set A



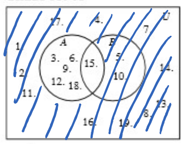
- A
- $n(A) = 6$

Shade set B



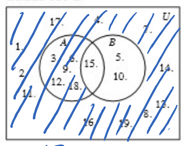
- B
- $n(B) = 3$

Shade set A'



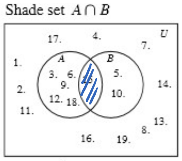
- A'
- $n(A') = 13$

Shade set B'



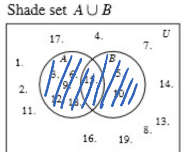
- not B
- $n(B') = 16$

Shade set $A \cap B$



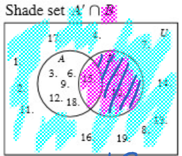
- A and B
- $n(A \cap B) = 1$

Shade set $A \cup B$



- A or B
- $n(A \cup B) = 8$

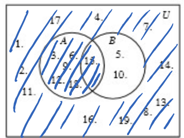
Shade set $A' \cap B$



· not A and B

· $n(A' \cap B) = 2$

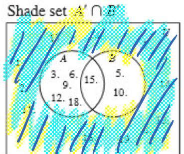
Shade set $A \cup B'$



· A or not B

$n(A \cup B') = 17$

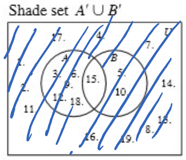
Shade set $A' \cap B'$



· not A and not B

· $n(A' \cap B') = 11$

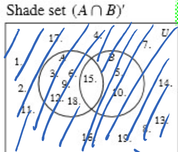
Shade set $A' \cup B'$



· not A or not B

· $n(A' \cup B') = 18$

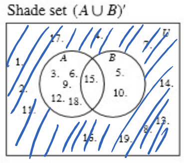
Shade set $(A \cap B)'$



· not (A and B)

· $n(A \cap B)' = 18$

Shade set $(A \cup B)'$



· not (A or B)

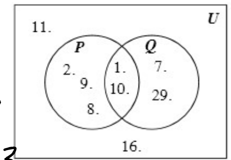
· $n(A \cup B)' = 11$

As demonstrated in the previous lesson, we can see from the above Venn diagrams that

- $(A \cap B)' = A' \cup B'$
- $(A \cup B)' = A' \cap B'$

Class Ex. #3

Use the Venn diagram to list the elements of the following sets.



a) P = {1,2,8,9,10} **b)** Q = {1,7,10,29}

c) P and Q = {1,10} **d)** P or Q = {1,2,7,8,9,10,29}

e) not P = {7,11,16,29} **f)** not Q = {2,8,9,11,16}

g) not P and not Q = {11,16} **h)** not P or not Q = {2,7,8,9,11,16,29}

i) not (P or Q) = {11,16} **j)** not (P and Q) = {2,7,8,9,11,16,29}

k) Which two pairs of sets are identical?

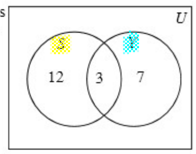
hj gi

Note

Venn diagrams can also be used to represent the number of elements in each set and not the individual elements, as the next class example illustrates. The numbers in the sets are not marked with dots, since they do not represent individual elements.

Class Ex. #4

The diagram displays the number of students who are members of Students' Council (S) and the number of students who are on the Yearbook Committee (Y).



How many students are

a) on Students' Council? 12+3=15 S

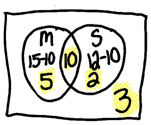
b) on both Students' Council and Yearbook Committee? 3 $S \cap Y$

c) on the Yearbook Committee but not on Students' Council? 7 $Y \cap S'$

d) on Students' Council or Yearbook Committee? 12+3+7=22 $S \cup Y$

Class Ex. #5

In a homeroom of 20 students, 15 take Math, 12 take Social, and 10 take Math and Social. Show this information in a Venn diagram. How many students take neither Math nor Social?

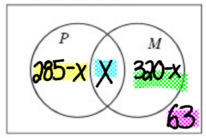


15-10 take only math 5
12-10 take only socials 2
take both 10 +
17

20-17 = $\boxed{3 \text{ take neither}}$

In a survey of 400 households, 285 had personal video recorders (PVRs) and 320 had multi-function printers (MFPs). 63 households did not have PVRs or MFPs.

- Let P represent the set of households with a PVR.
- Let M represent the set of households with an MFP.
- Let x represent the number of households that have both a PVR and an MFP.



a) Mark x and 63 in the appropriate parts of the Venn diagram.

b) Write an expression for the number of households who have a PVR and not an MFP. Write this expression in the appropriate part of the Venn diagram.

$285 - x$

c) Write an expression for the number of households who have an MFP and not a PVR. Write this expression in the appropriate part of the Venn diagram.

$320 - x$

d) Form an equation in x and determine the number of households in the survey who had both an MFP and PVR.

total: 400 $\quad 285 - x + x + 320 - x + 63 = 400$

$285 + 320 + 63 - x + x - x = 400$

$668 - x = 400$ solve for x

$668 - 668 - x = 400 - 668$

$-x = -268$

$\frac{-x}{-1} = \frac{-268}{-1}$

$\boxed{x = 268}$

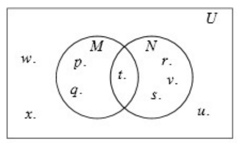
268 households have both

Complete Assignment Questions #1 - #12

Assignment

#1-9

1. Consider the Venn diagram shown.
List the elements of the following sets:



a) $M = \{$

b) M and N

c) M or N

d) not M

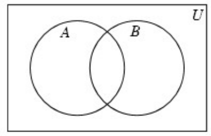
e) not N

f) not (M and N)

2. Using the Venn diagram in question #1, which sets are represented by:

a) $\{r, s, t, v\}$ **b)** $\{p, q\}$ **c)** $\{u, w, x\}$

3. Consider the set of prime numbers less than 20.
Let $A = \{3, 5, 7, 11, 19\}$ and $B = \{2, 3, 7, 13\}$.



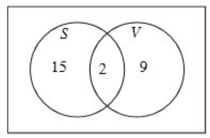
a) Complete the Venn diagram to illustrate this information.

b) List the members of the following sets:

i) A and B **ii)** A or B

iii) not A **iv)** not(A or B)

4. The diagram displays the number of girls who are members of the school soccer team (S) and the school volleyball team (V).



How many girls are:

a) on both teams?

b) on the soccer team and not on the volleyball team?

c) on only one team?

d) on the soccer team or on the volleyball team?

5. Of the students in Grade 12 at a certain high school, 76 are enrolled in physical education, 24 are enrolled in music, and 10 are enrolled in both physical education and music. If there are 15 students in Grade 12 who are not enrolled in physical education or music, how many students are in Grade 12?

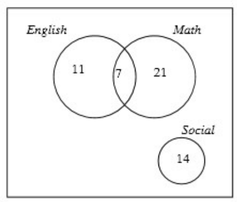
6. All the students in a class of 35 take Physics or Chemistry or both. 29 take Chemistry and 15 take Physics. How many take both?

7. In a school survey, it was found that 140 students had a cell phone or a tablet. If 86 students had a cell phone and 70 students had a tablet, how many students had both?

8. In each Venn diagram below, sketch two sets A and B satisfying the given condition.

a) $A \subset B$ **b)** $B \subset A$ **c)** A and B are disjoint sets

9. The Venn diagram shows the number of students who did English homework, Math homework, or Social homework on the weekend. None of the students did homework for any other subject. How many students did



a) English homework?

b) only English homework?

c) English homework and Social homework?

d) English homework or Math homework?

e) Math homework and not Social homework?

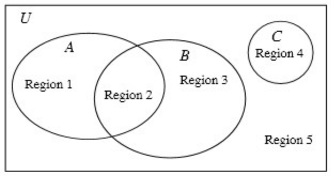
f) homework for only one subject?

Multiple Choice **10.** If P = {quadrilaterals that have 4 equal sides} and Q = {quadrilaterals that have 4 equal angles}, then a trapezoid with only one pair of parallel sides is an element of which of the following sets?

A. P and Q **B.** P and not Q
C. not P and Q **D.** not P and not Q

Use the following information to answer the next two questions.

The Venn diagram illustrating sets A, B, and C has been divided into five non-intersecting regions.



11. The region(s) representing the set $(A \cup B)'$ is/are

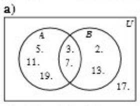
A. Region 5 **B.** Regions 4 and 5
C. Regions 1, 3, and 5 **D.** Regions 1, 3, 4, and 5

Numerical Response **12.** The set $A' \cap B$ can be represented by one region. This region number is _____ .

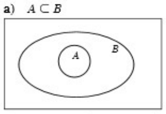
(Record your answer in the numerical response box from left to right.)

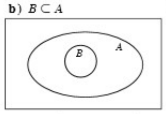
Answer Key

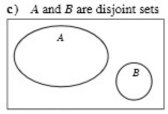
1. **a)** $\{p, q, t\}$ **b)** $\{t\}$ **c)** $\{p, q, r, s, t, v\}$
d) $\{r, s, u, v, w, x\}$ **e)** $\{p, q, u, w, x\}$ **f)** $\{p, q, r, s, u, v, w, x\}$
2. **a)** N **b)** M and not N **c)** not M and not N or not (M or N)
3. **a)** **b)** **i)** {3, 7} **ii)** {2, 3, 5, 7, 11, 13, 19} **iii)** {2, 13, 17} **iv)** {17}



4. **a)** 2 **b)** 15 **c)** 24 **d)** 26 **5.** 105 **6.** 9 **7.** 16
8. **a)** $A \subset B$ **b)** $B \subset A$ **c)** A and B are disjoint sets





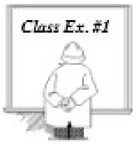


9. **a)** 18 **b)** 11 **c)** 0 **d)** 39 **e)** 28 **f)** 46
10. D **11.** B **12.**

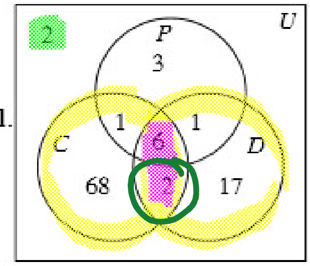
3			

Lesson 4: Venn Diagrams - Part Two

Logical Reasoning and Set Theory Lesson #4:
Venn Diagrams - Part Two



The Venn diagram displays the results of a survey of 100 students in a high school regarding movies they watched in the last two weeks.



P represents the number of students who watched **Perfect Rebel**.
C represents the number of students who watched **Chill**.
D represents the number of students who watched **Dazed**.

a) How many students watched only "Perfect Rebel"? 3

b) How many students watched "Perfect Rebel"? 3+1+6+1 = 11

c) How many students watched "Chill" and "Dazed"? 6+2=8

d) How many students watched "Chill" or "Dazed"? 68+1+6+2+1+17 = 95

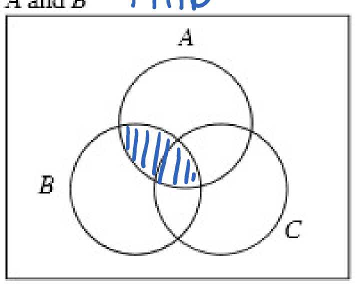
e) How many students watched all three films? 6

f) What does the number 2 outside the three circles represent?
2 people didn't watch any movies

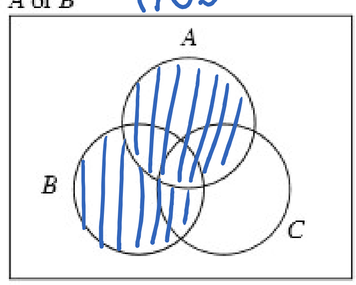
g) What does the number 2 inside the circles represent?
2 people who watched Chill and Dazed but not Perfect

In each of the following Venn diagrams, shade the region indicated.

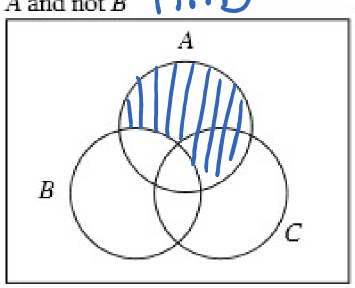
a) *A* and *B* $A \cap B$



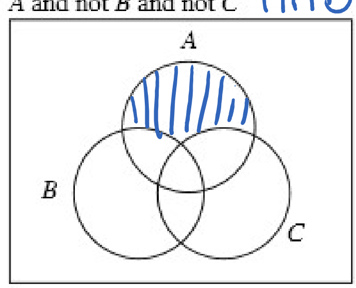
b) *A* or *B* $A \cup B$



c) *A* and not *B* $A \cap B'$



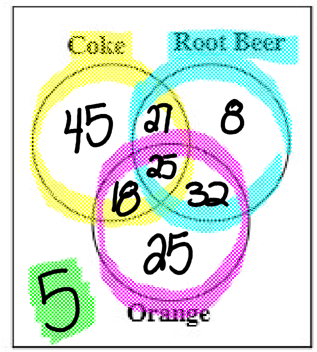
d) *A* and not *B* and not *C* $A \cap B' \cap C'$



Class Ex. #3

In Big Hill High School, 185 Grade 12 students were surveyed to determine which soft drinks they liked to drink.

- ✓25 drank all three
- ✓52 drank root beer and coke 25+27
- ✓43 drank coke and orange
- ✓57 drank root beer and orange
- 92 drank root beer
- 115 drank coke
- 100 drank orange



Sabine started to organize the results in a Venn diagram, starting from the inside and working outwards. She began by placing the number 25 in the centre of the three circles.

a) Explain why she placed the number 27 in the region immediately above the 25.

52-25=27

52 for C and RB, but 25 are already there who also drink O

b) Use similar reasoning to place numbers in the regions to the left and right of the 25.

43-25=18 57-25=32

orange + coke only orange + RB only

c) Continue the process until numbers have been placed in all the regions inside the circles.

Root Beer only
92-27-25-32
= 8

coke only
115-27-25-18
=45

orange only
100-18-25-32
= 25

d) Determine the sum of all the numbers inside the circles. If this sum is not equal to 185, determine the number that must be placed in the region outside the circles.

115+8+32+25=180 185-180=5

e) Determine how many students

i) drank only coke **ii)** did not like to drink any of the three drinks

iii) drank coke or root beer

45+27+25+18+32+8=155

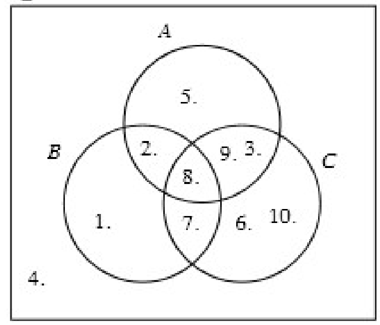
or 185-25-5=155

Complete Assignment Questions #1 - #9 #1-5

Quiz Friday
Test Tuesday.

Assignment

1. Use the Venn diagram to list the elements of the following sets.



a) A

b) A and B

c) A and B and C

d) B or C

e) A or B or C

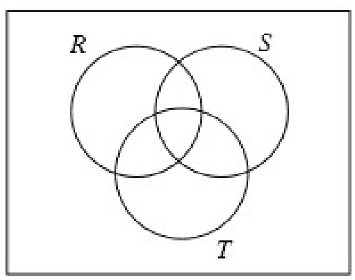
f) not C

g) A and not B

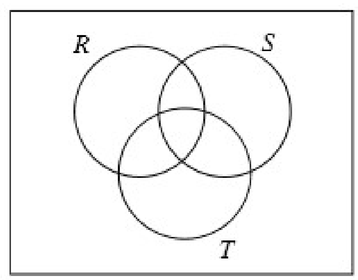
h) C and not A and not B

2. In each of the following Venn diagrams, shade the region indicated.

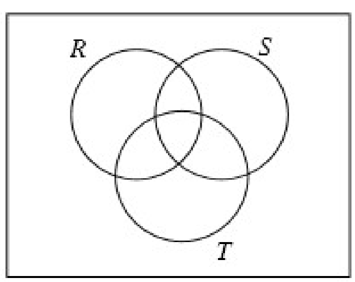
a) R or T



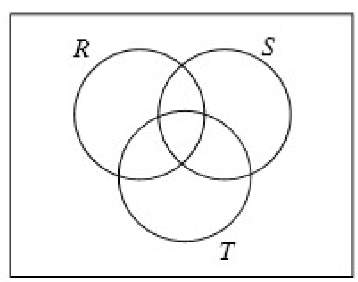
b) R and not S



c) R and S and not T

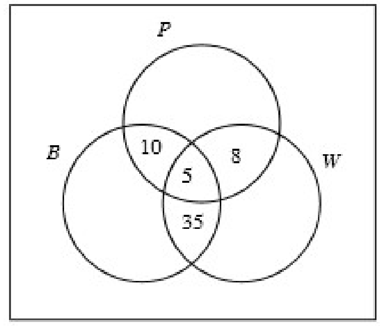


d) not R and not S and not T



3. The partially completed Venn diagram displays the results of a fast food survey of 145 teenagers.

- $n(P) = 75$ represents the number of teenagers who liked pizza.
- $n(B) = 60$ represents the number of teenagers who liked burgers.
- $n(W) = 68$ represents the number of teenagers who liked wraps.



a) Complete the Venn diagram.

b) How many teenagers liked

i) pizza and burgers and wraps?

ii) burgers and not wraps?

iii) burgers and pizza?

iv) only burgers and pizza?

v) burgers or pizza?

vi) none of the three types of fast food?

4. To cater for a school party, all of the 115 students involved brought at least one of the following items: sandwiches (S), chips (C), or lemonade (L).

5 brought sandwiches, chips, and lemonade
24 brought chips and lemonade
27 brought sandwiches and lemonade
17 brought sandwiches and chips
54 brought sandwiches
70 brought lemonade

How many students brought only chips?

5. The students from Mr. Hennesey's Grade 12 class were surveyed. 19 students take Math, 10 students take Math and Physics, 14 students take only Chemistry, 12 students take Chemistry and Math, and 3 students take all three subjects. 2 students do not take any of these subjects and all Physics students take Math.

a) Show this information in a Venn diagram.

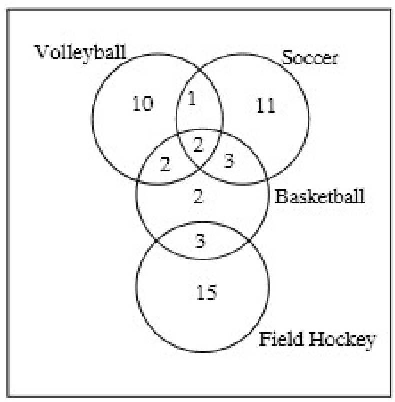
b) How many students

i) are there in Mr. Hennesey's homeroom? **ii)** take only Math?

iii) take Chemistry and not Math? **iv)** take Chemistry or Physics?

Use the following information to answer questions #6 and #7.

The Venn diagram illustrates the number of girls who played on various school sports teams.



Multiple Choice

6. The number of girls who played soccer and volleyball was

A. 1 **B.** 3 **C.** 8 **D.** 29

7. The number of girls who played on exactly two teams was

A. 6 **B.** 8 **C.** 9 **D.** 11

Use the following information to answer questions #8 and #9.

Of the 21 teachers in a high school who teach Biology, Chemistry, or Physics, or some combination of these, no one teaches both Biology and Physics. 8 teach Biology, of whom 5 do not also teach Chemistry. 7 teach Physics, and 3 teach both Chemistry and Physics.

Numerical Response

8. The number of teachers who teach Chemistry or Physics is _____ .
(Record your answer in the numerical response box from left to right.)

9. The number of teachers who teach only one of the subjects is _____ .
(Record your answer in the numerical response box from left to right.)

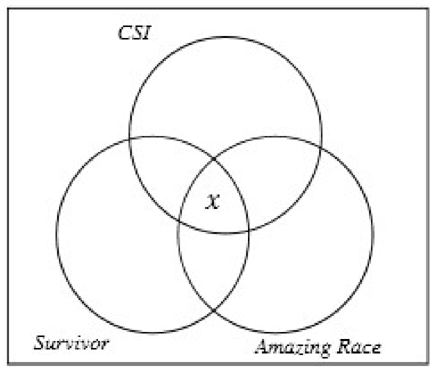
Group Work

150 Grade 12 students were asked which of the following 3 television programs they watch regularly - "CSI", "Survivor", and "Amazing Race".

102 students watched "CSI"
70 watched "Survivor"
40 watched "Amazing Race"
25 students watched both "CSI" and "Survivor"
27 watched "CSI" and "Amazing Race"
30 watched "Survivor" and "Amazing Race"
Every student watched at least 1 program

Let x = the number of students who watch all three programs.

Complete each section of the Venn diagram in terms of x, starting from the inside out.
Form an equation in x that can be solved to give the number of students who watched all three programs and determine the number of students who watched all three programs.

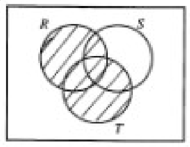


Answer Key

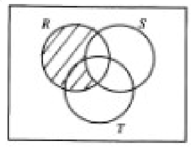
1. **a)** {2, 3, 5, 8, 9} **b)** {2, 8} **c)** {8} **d)** {1, 2, 3, 6, 7, 8, 9, 10}
e) {1, 2, 3, 5, 6, 7, 8, 9, 10} **f)** {1, 2, 4, 5} **g)** {3, 5, 9} **h)** {6, 10}

2.

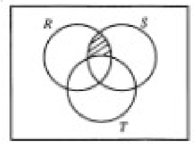
a)



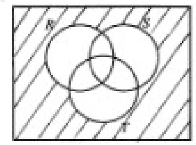
b)



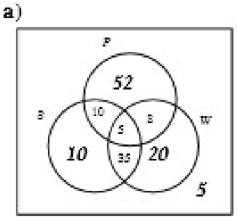
c)



d)



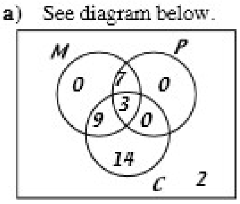
3. **a)**



b) **i)** 5 **ii)** 20 **iii)** 15
iv) 10 **v)** 120 **vi)** 5

4. 18

5. **a)** See diagram below. **b)** **i)** 35 **ii)** 0 **iii)** 14 **iv)** 33



6. B **7.** C

8.

1	6		

9.

1	5		

Enrichment Group Work 20 students