

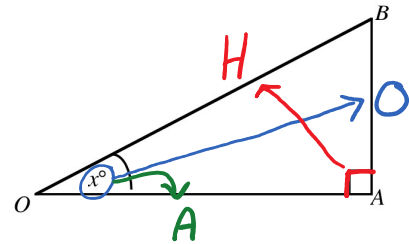
4.1

# Trigonometry Lesson #1: Review of Right Triangle Trigonometry

## Ratios of Sides in a Right Triangle

Consider the right triangle  $AOB$  shown.

Let angle  $AOB = x^\circ$ .



Each of the sides of the triangle is given a special name relative to the angle of  $x^\circ$ .

The longest side,  $OB$ , is called the HYPOTENUSE (hyp).

The side opposite the angle of  $x^\circ$ ,  $AB$ , is called the OPPOSITE (opp). *(across from the angle)*

The remaining side of the triangle,  $OA$ , is called the ADJACENT (adj). *beside the angle*

Recall the definitions of the three primary trigonometric ratios:

$$\sin x^\circ = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{AB}{OB}$$

$$\cos x^\circ = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{OA}{OB}$$

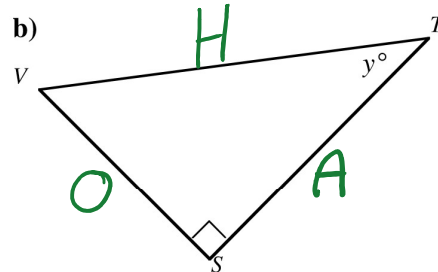
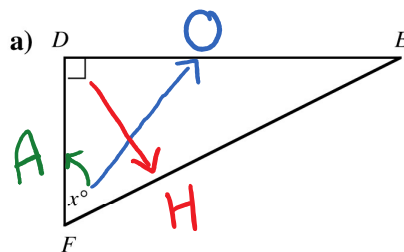
$$\tan x^\circ = \frac{\text{opposite}}{\text{adjacent}} = \frac{AB}{OA}$$

The rules for determining the sine ratio, cosine ratio, and tangent ratio for an angle in a right triangle can be memorized by using the acronym SOH CAH TOA.

SOH CAH TOA



Mark on each of these triangles the hypotenuse (hyp), the opposite (opp), and the adjacent (adj) relative to the angles of  $x^\circ$  and  $y^\circ$ .



Complete the following.

$$\sin x^\circ = \frac{DF}{DE} \quad \cos x^\circ = \frac{FE}{DE} \quad \tan x^\circ = \frac{DF}{FE} \quad \sin y^\circ = \frac{VS}{VT} \quad \cos y^\circ = \frac{ST}{VT} \quad \tan y^\circ = \frac{VS}{ST}$$

SOH CAH TOA

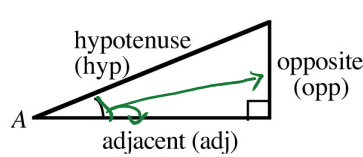
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In solving problems in right triangle trigonometry we need to be given a right angle, a side length and one other angle or side.

Note the following points emphasized in the right triangle diagrams below.

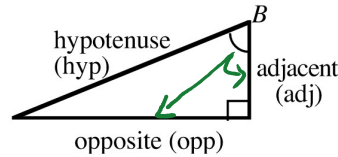
- The opposite and adjacent sides can switch depending on the angle being used.
- The Pythagorean Theorem can be used if two sides of the triangle are known and the third side is required.



$$\sin A = \frac{\text{opp}}{\text{hyp}}$$

$$\cos A = \frac{\text{adj}}{\text{hyp}}$$

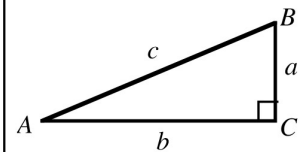
$$\tan A = \frac{\text{opp}}{\text{adj}}$$



$$\sin B = \frac{\text{opp}}{\text{hyp}}$$

$$\cos B = \frac{\text{adj}}{\text{hyp}}$$

$$\tan B = \frac{\text{opp}}{\text{adj}}$$



$$a^2 + b^2 = c^2$$

2 little sides added

$$c^2 - b^2 = a^2$$

long side minus a short side



Class Ex. #2

Consider the following triangle.

- a) Use the Pythagorean Theorem to calculate the length of  $ST$ .

$$25^2 - 7^2 = a^2$$

$$576 = a^2$$

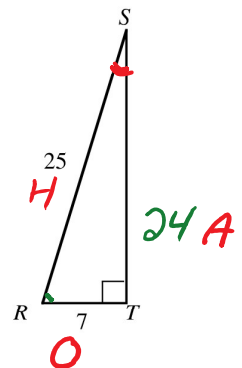
$$a = 24$$

- b) State, as rational numbers, the values of the following trigonometric ratios.

$$\sin R = \frac{24}{25} \quad \cos R = \frac{7}{25} \quad \tan R = \frac{24}{7}$$

$$\sin S = \frac{7}{25} \quad \cos S = \frac{24}{25} \quad \tan S = \frac{7}{24}$$

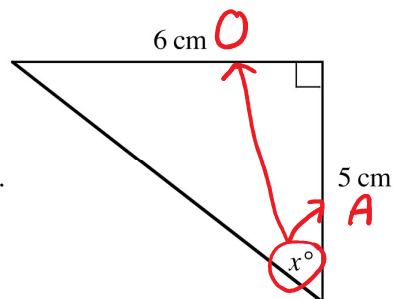
- c) Comment on any relationships you see from your answers in b).



**Calculating Angles in Right Triangles**

Complete the following example to review how to calculate an angle measure using SOHCAHTOA.

In the diagram we are required to determine the measure of the angle marked  $x^\circ$  to the nearest degree.



Relative to the angle  $x^\circ$ , the OPPOSITE side is 6 and the ADJACENT side is 5 so we use the TANGENT ratio.

We write  $\tan x^\circ = \frac{6}{5} = 1.2$ .

TA

If  $\tan x^\circ = 1.2$ , the measure of the angle  $x^\circ$  can be determined by using the inverse tangent function  $\tan^{-1}$ .

If  $\tan x^\circ = 1.2$ , then  $\tan^{-1}(1.2) = x$ .

On a calculator, access the inverse tangent function by pressing 2nd tan.

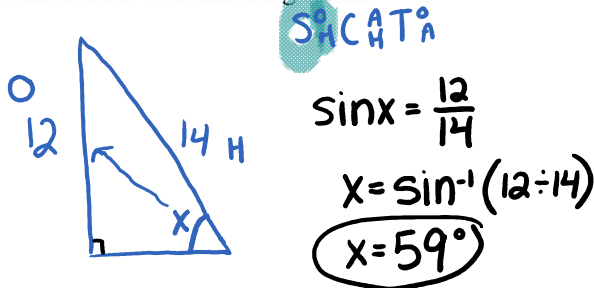
Use your calculator to complete the solution.

$\tan x^\circ = 1.2$  so  $x^\circ = 11^\circ$  (to the nearest degree).



A telephone pole is 12 metres high and is supported by a wire, 14 metres long, fixed to the top of the pole and to the ground.

Draw a sketch to illustrate the information and calculate, to the nearest degree, the angle between the wire and the ground.



**Calculating Sides in Right Triangles**

Complete the following examples to review how to calculate a side using SOHCAHTOA.

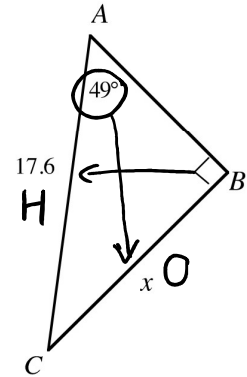
- a) In the diagram we are required to determine the measure of the side  $BC$ .

Relative to the angle of  $49^\circ$ , the given side is the HYPOTENUSE and the required side is the OPPOSITE so we use the SINE ratio.

We write  $\sin 49^\circ = \frac{x}{17.6}$  **SOH**

Cross multiply to get  $17.6 \sin 49^\circ = x$

To one decimal place,  $x = \underline{13.3}$  so  $BC = \underline{13.3}$ .



- b) In the diagram we are required to determine the measure of the side  $QR$ .

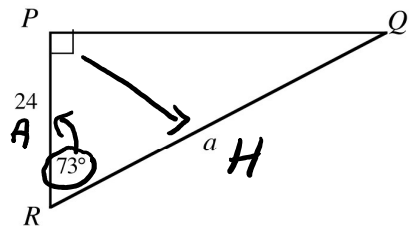
Relative to the angle of  $73^\circ$ , the given side is the ADJACENT and the required side is the HYPOTENUSE so we use the COSINE ratio.

We write  $\cos 73^\circ = \frac{24}{a}$  **CAH**

Cross multiply to get  $a \cos 73^\circ = 24$

Divide both sides by  $\cos 73^\circ$  to get  $a = \frac{24}{\cos 73^\circ}$

To one decimal place,  $a = \underline{82.1}$  so  $QR = \underline{82.1}$ .



Use trigonometric ratios to determine the lengths of  $AB$  and  $AC$  in the given triangle. Answer correct to the nearest foot.

**SOHCAHTOA**

$$\tan 74^\circ = \frac{x}{109}$$

$$109 \tan 74^\circ = x$$

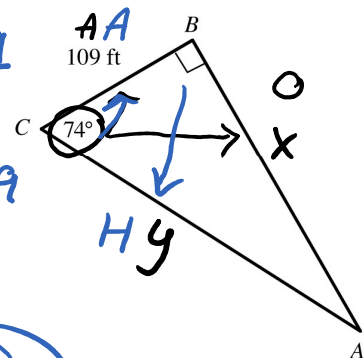
$$x = 380$$

$$\cos 74^\circ = \frac{109}{y}$$

$$y \cos 74^\circ = 109$$

$$y = \frac{109}{\cos 74^\circ}$$

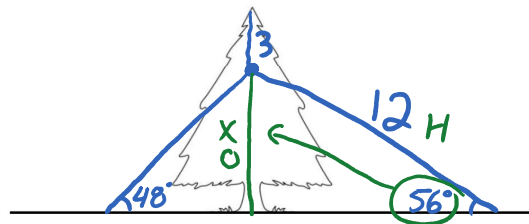
$$y = 395$$





A large tree is to be transported to a new location. The tree is held vertical by means of two guy wires of unequal length on opposite sides of the tree. One of the wires makes an angle of  $48^\circ$  with the ground. The other wire is 12 m long and makes an angle of  $56^\circ$  with the ground. Both wires are attached 3 m down from the top of the tree.

a) Illustrate this scenario below.



b) Determine the height of the tree to one decimal place.

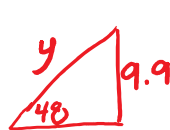
$$\sin 56 = \frac{x}{12}$$

$$12 \sin 56 = x$$

$$x = 9.9$$

$$9.9 + 3 = 12.9 \text{ m tree height}$$

c) Determine the length of the other wire to the nearest tenth of a metre.



$$\sin 48 = \frac{9.9}{y}$$

$$y = \frac{9.9}{\sin 48} = 13.3 \quad \#1-6$$

d) Determine, to the nearest tenth of a metre, the horizontal distance at ground level between the two guy wires.

#1-6

- Unit test Thursday
- flex tutorial Thurs.
- HW unit 2 due!

e) The guy wire in c) breaks and a new wire of the same length is attached 2 m down from the top of the tree. Calculate, to the nearest degree, the angle which this guy wire makes with the ground.

**Complete Assignment Questions #1 - #8**

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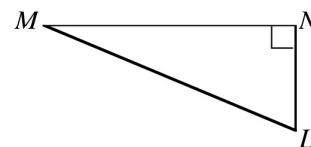
## Assignment

1. Consider  $\triangle LMN$ .

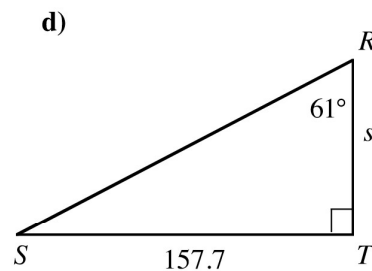
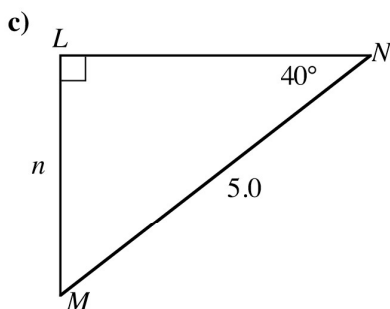
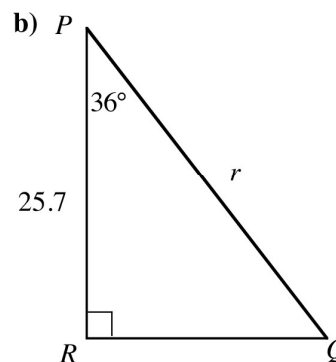
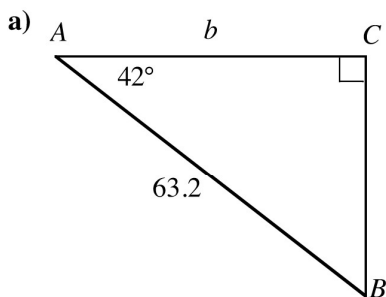
Write the ratio of sides for each of the following.

i)  $\sin L =$       ii)  $\cos L =$       iii)  $\tan L =$

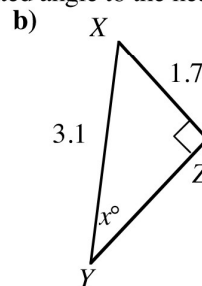
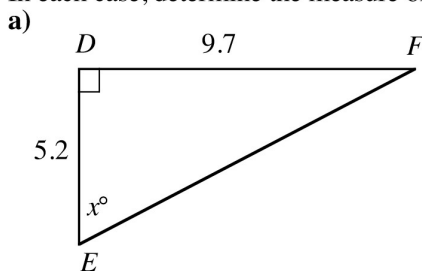
iv)  $\sin M =$       v)  $\cos M =$       vi)  $\tan M =$



2. In each case, determine the length of the indicated side to the nearest tenth.

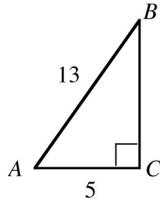


3. In each case, determine the measure of the indicated angle to the nearest degree.

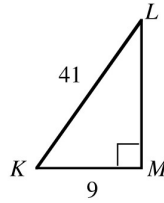


4. Determine, to 2 decimal places, the value of each trigonometric ratio.

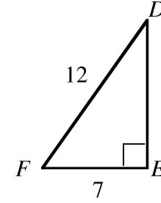
a)  $\cos A$



b)  $\sin K$



c)  $\tan F$

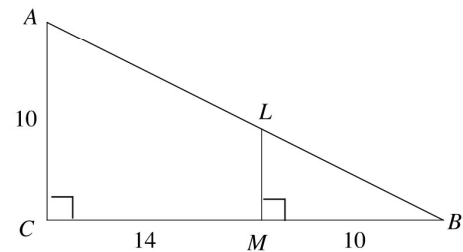


5. Determine the exact value of the following.

a)  $\tan \angle ABC$

b)  $LM$

c)  $\sin \angle BAC$



6. Christine places a ladder against the side of a house so that the top of the ladder makes an angle of  $52^\circ$  with the side of the house. The bottom of the ladder is 1.20 m from the house.

a) Calculate, to the nearest hundredth of a metre, the vertical distance from the top of the ladder to the ground.

b) Calculate, to the nearest hundredth of a metre, the length of the ladder.



**Multiple Choice**

7. In a right triangle  $PQR$ ,  $PQ = 50$  units,  $PR = 48$  units and  $RQ = 14$  units. The value of  $\sin Q$  and  $\cos Q$  are respectively

- A.  $\frac{7}{25}$  and  $\frac{24}{25}$
- B.  $\frac{24}{25}$  and  $\frac{7}{25}$
- C.  $\frac{24}{7}$  and  $\frac{7}{24}$
- D.  $\frac{24}{7}$  and  $\frac{7}{25}$

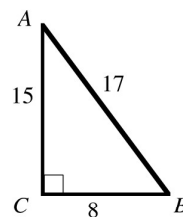
Use the following information to answer the next question.

Three statements are made for the right triangle  $ABC$ .

Statement 1:  $\tan A = \frac{\sin A}{\cos A}$

Statement 2:  $1 + (\tan A)^2 = \frac{1}{(\cos A)^2}$

Statement 3:  $(\sin B)^2 + (\cos B)^2 = 1$



8. How many of the statements are true?
- A. Zero
  - B. One
  - C. Two
  - D. Three

**Answer Key**

1. i)  $\frac{MN}{LM}$     ii)  $\frac{LN}{LM}$     iii)  $\frac{MN}{LN}$     iv)  $\frac{LN}{LM}$     v)  $\frac{MN}{LM}$     vi)  $\frac{LN}{MN}$
2. a) 47.0    b) 31.8    c) 3.2    d) 87.4    3. a)  $62^\circ$     b)  $33^\circ$
4. a) 0.38    b) 0.98    c) 1.39    5. a)  $\frac{5}{12}$     b)  $\frac{25}{6}$  or 4.25    c)  $\frac{12}{13}$
6. a) 0.94 m    b) 1.52 m    7. B    8. D

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