7.1

Quadratic Functions Lesson #1:
Review and Preview

Relations

Much of mathematics involves the search for patterns and relationships between sets of data. Many real life applications of mathematics investigate the relationship between two quantities.

For example:

- The cost, C (cents per km), of driving a car is related to the speed, s (km per hour), at which it is driven.
- The price, P (dollars), of a watermelon is related to its weight, W (kilograms).

In mathematics, a comparison between two sets of elements is called a **relation**.

In previous courses, we represented the relationship between two quantities in many ways, including:

- in words
- an equation
- a table of values
- a graph
- a set of ordered pairs
- function notation (where relevant)

These representations will be reviewed in this lesson.

Consider the following relationships between x and y.

1. The relation defined by the equation $y = 3x + 5$

2. The relation defined by the equation $y = x^2 + 4$

a) In what ways are the equations **i)** alike **ii)** different?

i) both are written as "y=" equations, both have an x, both add a constant

ii) one is degree 1 (x) and other is degree 2 (x^2)

b) Use the equations to complete the table of values.

$y = 3x + 5$

x	−3	−1	1	3	5
y	-4	2	8	14	20

+4 +4

$3(-3)+5$

*sub in x value, solve for y

$y = x^2 + 4$

x	−4	−2	0	2	3	4
y	20	8	4	8	13	20

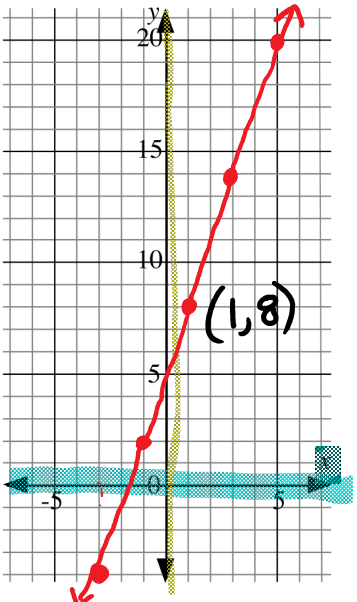
$(-4)^2+4$

Solve for y

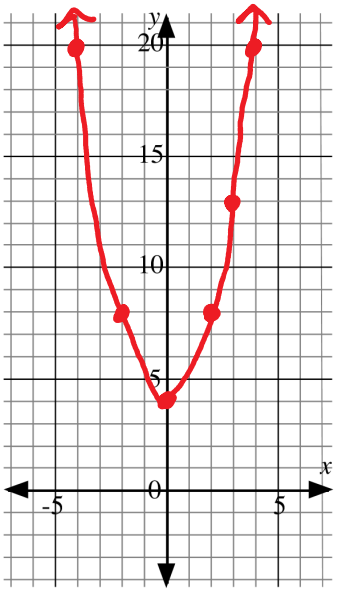
c) Use the table of values to plot the ordered pairs on the grid.

$y = 3x + 5$

x	y
-3	-4
-1	2
1	8
3	14
5	20



$y = x^2 + 4$



x	y
-4	20
-2	8
0	4
2	8
3	13
4	20

d) Join the points on the graph of $y = 3x + 5$ with a straight line and extend the graph.
For $y = x^2 + 4$, join the points with a smooth curve and extend the graph.

e) In what ways are the graphs **i)** alike **ii)** different?

i) continuous
ii) straight line, "U" shaped

f) The **domain** of a relation is the set of all possible values which can be used for the **input** of the **independent variable** (x).

i) State the domain of the graph of $y = 3x + 5$. $x \in \mathbb{R}$ "x includes all real numbers"

ii) State the domain of the graph of $y = x^2 + 4$. $x \in \mathbb{R}$

g) The **range** of a relation is the set of all possible values of the **output** of the **dependent variable** (y).

i) State the range of the graph of $y = 3x + 5$. $y \in \mathbb{R}$

ii) State the range of the graph of $y = x^2 + 4$. $y \geq 4$

h) A functional relation, or **function**, is a special type of relation in which each element of the domain is related to exactly one element of the range. If any element of the domain is related to more than one element of the range, then the relation is not a function.

Do the graphs in part c) represent functions? What visual test can be used?

Both are functions, they pass the vertical line test

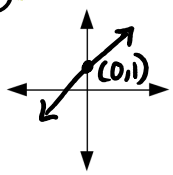
Investigating the Graphs of Linear and Quadratic Functions

A **linear function** is a function whose graph is **a line**.

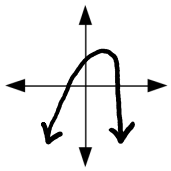
A **quadratic function** is a function whose graph is "∪" shaped (opens up), or "∩" shaped (opens down). Such a graph is called **a parabola**.

a) The equations of the graphs of some functions are given. In each case, use a graphing calculator to sketch the graph of the function on the grid provided. Do not list any x- or y-intercepts.

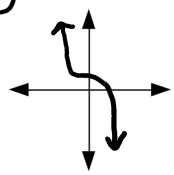
① $y = 3x + 1$



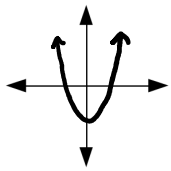
② $y = -2x^2 + 5x + 3$



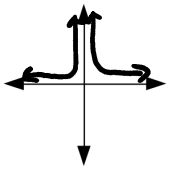
③ $y = -x^3 + 1$



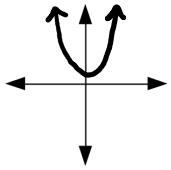
$y = x^2 - 3$



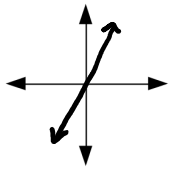
$y = \frac{1}{x^2}$



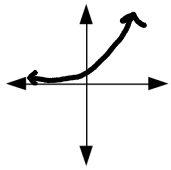
$y = x^2 + x + 1$



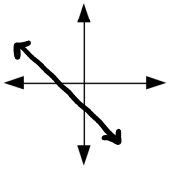
$y = 4x$



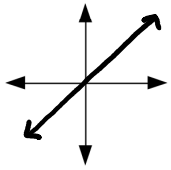
$y = 2^x$



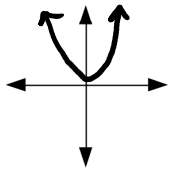
$y = \frac{2}{3}x - 2$



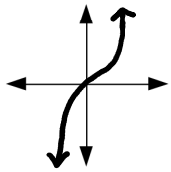
$y = x$



$y = x^2$



$y = x^3 - x^2$



b) List the equations of the graphs in the appropriate row.

LINEAR: ✓

QUADRATIC: ∪

NEITHER:

c) How can you tell from the equation of a function whether the function is a quadratic function or not?

must have degree 2 as the highest degree and not in the denominator

Quadratic Functions

A **quadratic function** is a function which can be written in the form

$$f(x) = ax^2 + bx + c, \text{ where } a, b, c \in R, \text{ and } a \neq 0,$$

or in equation form as

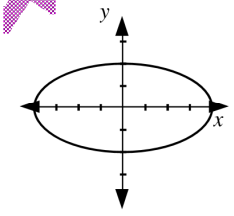
$$y = ax^2 + bx + c, \text{ where } a, b, c \in R, \text{ and } a \neq 0.$$

Without the use of technology, identify which of the following are quadratic functions.

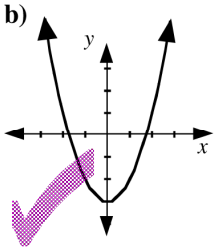
a) $y = 2x^2 - 9$ Quadratic **b)** $y = x^2 - x^5$ not **c)** $f(x) = 3 - 2x - \frac{1}{5}x^2$ quadratic

Which of the following graphs appear to represent quadratic functions of the form $f(x) = ax^2 + bx + c$? b,d

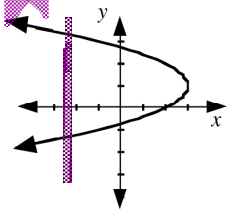
a)



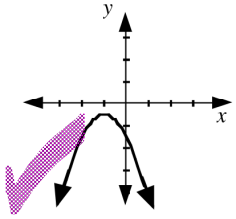
b)



c)



d)



#1, 4, 6, 8, 9, 10

Complete Assignment Questions #1 - #11

Assignment

1. Consider the function $y = x^2 + 2x + 1$.

x	–5	–3	–1	1	3
y					

a) Complete the table of values opposite.

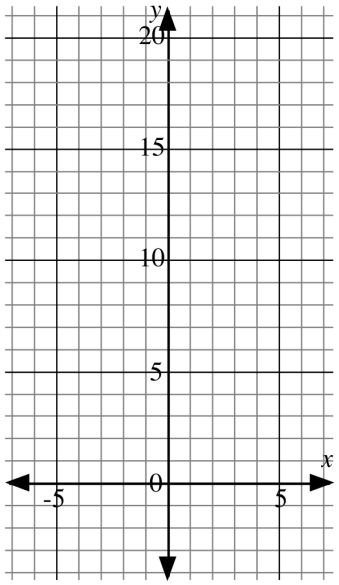
b) Use the table of values to plot the ordered pairs on the grid.

c) Join the points on the graph with a smooth curve and extend the graph.

d) State whether the graph has a maximum or minimum point.

e) State the domain and range.

f) Use a graphing calculator, with an appropriate window setting, to verify the accuracy of your graph in part c).



2. Consider the function $f(x) = 10 - 4x - x^2$.

x	–6	–4	–2	0	2
y					

a) Complete the table of values opposite.

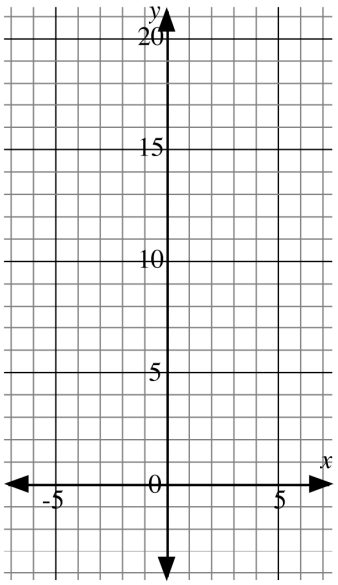
b) Use the table of values to plot the ordered pairs on the grid.

c) Join the points on the graph with a smooth curve and extend the graph.

d) State whether the graph has a maximum or minimum point.

e) State the domain and range.

f) Use a graphing calculator, with an appropriate window setting, to verify the accuracy of your graph in part c).



3. How can you tell by looking at the equation of a function whether the function is a quadratic function or not?

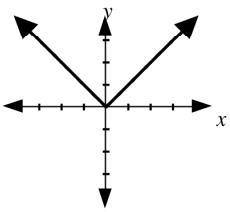
4. Without the use of technology, identify which of the following are quadratic functions.

a) $y = x - 16$ **b)** $y = x^2 - 16$ **c)** $y = x^2 - 16x$ **d)** $y = x^2 - 16x^3$

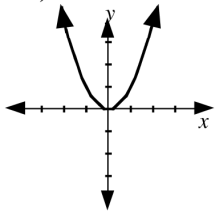
5. How can you tell by looking at the graph of a function whether the function is a quadratic function or not?

6. Which of the following graphs appear to represent quadratic functions of the form $f(x) = ax^2 + bx + c$?

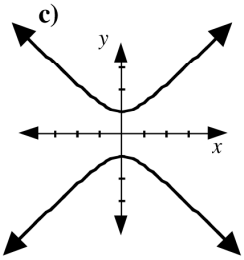
a)



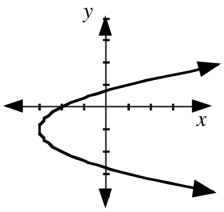
b)



c)



d)



7. David says that the equation $y = ax^2 + bx + c$ will represent a quadratic function for all values of a, b, and c. Is David correct? Explain.

Multiple Choice **8.** The height, h metres, of an object above the ground after t seconds, is represented by a quadratic function $h(t)$. This function could be

A. $h(t) = 10 - t$ **B.** $h(t) = t^3 + 10$

C. $h(t) = 10 - t^2$ **D.** $h(t) = 10 - t + t^2 - t^4$

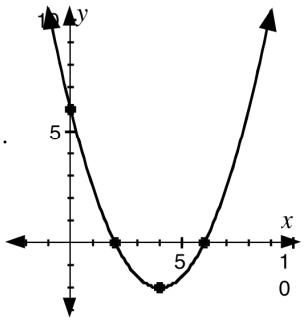
9. Which of the following functions is quadratic?

A. $f(x) = \frac{1}{x^2} - 7$ **B.** $f(x) = x^{-2} + 3x + 1$

C. $f(x) = 2$ **D.** $f(x) = (x - 2)(x + 3)$

Use the following information to answer questions #9 and #10.

- The graph of a quadratic function is shown.
- The four points marked have integer coordinates.



10. The domain and range, respectively, of the function are

A. $x \in R$ and $y \in R$
B. $x \geq -2$ and $y \in R$
C. $x \in R$ and $y \geq -2$
D. $2 \leq x \leq 6$ and $y \geq -2$

Numerical Response

11. The sum of the x and y-intercepts is _____ .

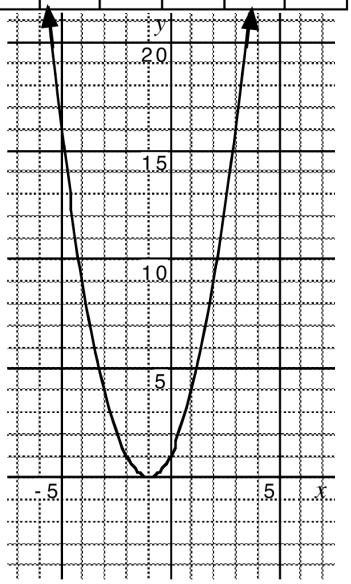
(Record your answer in the numerical response box from left to right.)

Answer Key

1. a)

x	–5	–3	–1	1	3
y	16	4	0	4	16

b)



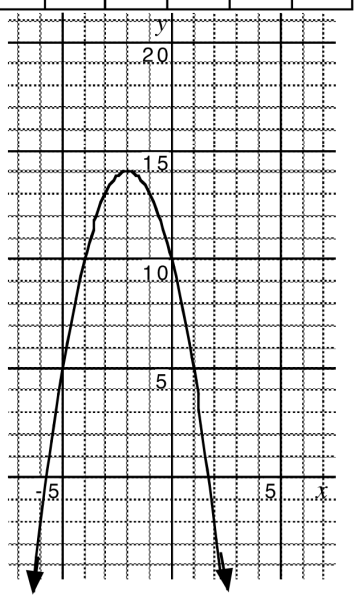
d) minimum

e) Domain $x \in R$
Range $\{y \mid y \geq 0, y \in R\}$

2. a)

x	–6	–4	–2	0	2
y	-2	10	14	10	-2

b)



d) maximum

e) Domain $x \in R$,
Range $\{y \mid y \leq 14, y \in R\}$

3. The function has degree 2 and is of the form $y = ax^2 + bx + c,\ a \neq 0$.

4. b) and c)

5. The graph is a parabola.

6. b) only

7. David is not correct. a cannot equal zero.

8. C

9. D

10. C

11.

1	4		