

Lesson 3: x- and y-intercepts and Interpreting Relations

Friday, August 31, 2018 2:38 AM

Test Friday March 1

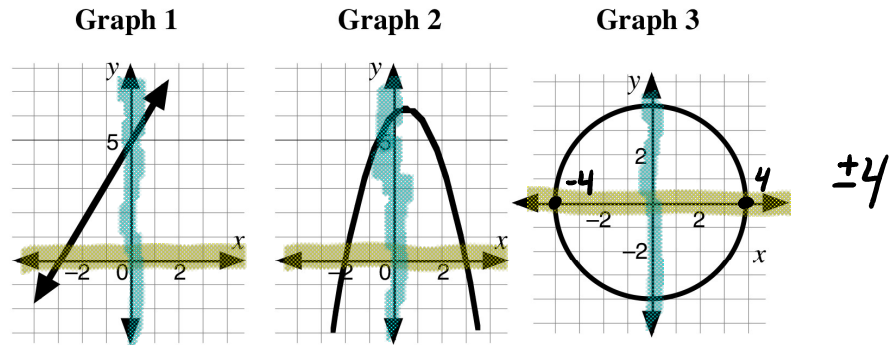
Relations Lesson #3: x- and y-intercepts and Interpreting Relations

Review

- a) A **relation** is a connection between two quantities. A relation can be represented graphically by a set of ordered pairs. (x,y)
- b) The first component of a set of ordered pairs is the x coordinate, also known as the input. Values of the input are values of the independent variable.
- c) The second component of a set of ordered pairs is the y coordinate, also known as the output. Values of the output are values of the dependent variable.

Exploring x- and y-intercepts

Consider the following graphs.



- a) List the coordinates of the point(s) where each graph crosses the x-axis.

- Graph 1 crosses the x-axis at $(-3, 0)$.
- Graph 2 crosses the x-axis at $(-2, 0)$ and $(3, 0)$.
- Graph 3 crosses the x-axis at $(-4, 0)$ and $(4, 0)$.

- b) What do all the points in a) have in common?

$$y = 0$$

- c) List the coordinates of the point(s) where each graph crosses the y-axis.

- Graph 1 crosses the y-axis at $(0, 5)$.
- Graph 2 crosses the y-axis at $(0, 6)$.
- Graph 3 crosses the y-axis at $(0, 4)$ and $(0, -4)$.

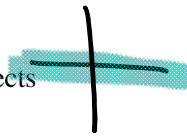
- d) What do all the points in c) have in common?

$$x = 0$$

x- and y- intercepts of a Graph

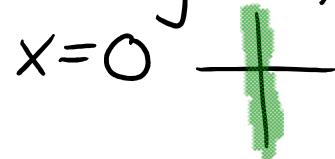
The **x-intercept** of a graph is the x-coordinate of the ordered pair where the graph intersects the x-axis. An x-intercept occurs at a point on the graph where the y-coordinate is zero. The x-intercept can be given as a value or as an ordered pair.

$x\text{-int, } y=0$



The **y-intercept** of a graph is the y-coordinate of the ordered pair where the graph intersects the y-axis. A y-intercept occurs at a point on the graph where the x-coordinate is zero. The y-intercept can be given as a value or as an ordered pair.

$y\text{-int, } x=0$



- Given the equation of the graph of a relation:
 - to determine the x-intercept, set $y=0$ and solve for x .
 - to determine the y-intercept, set $x=0$ and solve for y .

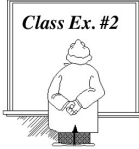
- The equation of a graph can be written in different forms, all of which are equivalent.

The equation of Graph 1 on the previous page is $y = \frac{5}{3}x + 5$, which can be written as $3y = 5x + 15$ or $5x - 3y + 15 = 0$. Equivalent forms of an equation will be studied in detail in a later unit. For the time being, use the instruction in note 1 to find the x- and y-intercepts of the graph of an equation given in any form.



The equation of Graph 1 on the previous page is $3y = 5x + 15$. Algebraically determine the values of the x-intercept and the y-intercept of Graph 1.

x-intercept $y=0$	y-intercept $x=0$
$3(0) = 5x + 15$ *solve for x $0 = 5x + 15$ $-15 = 5x$ $x = -3$	$3y = 5(0) + 15$ *solve for y $3y = 0 + 15$ $y = 5$



The equation of Graph 3 on the previous page is $x^2 + y^2 = 16$. Calculate the x-intercept and the y-intercept of the graph of $x^2 + y^2 = 16$. Give the answers as ordered pairs.

x-intercept $y=0$	y-intercept $x=0$
$x^2 + 0^2 = 16$ $x^2 = 16$ $x = \pm 4$	$0^2 + y^2 = 16$ $y^2 = 16$ $y = \pm 4$

*When you take the square root of a #, you must consider the (+) and (-) root
 ex. $\sqrt{4} = \pm 2$

Complete Assignment Questions #1 - #3

$\sqrt{16} = \sqrt{4 \times 4}$

ex. $\sqrt{4} = \pm 2$

$$\sqrt{16} = \sqrt{4 \times 4}$$

$$\sqrt{16} = \sqrt{-4 \times -4}$$



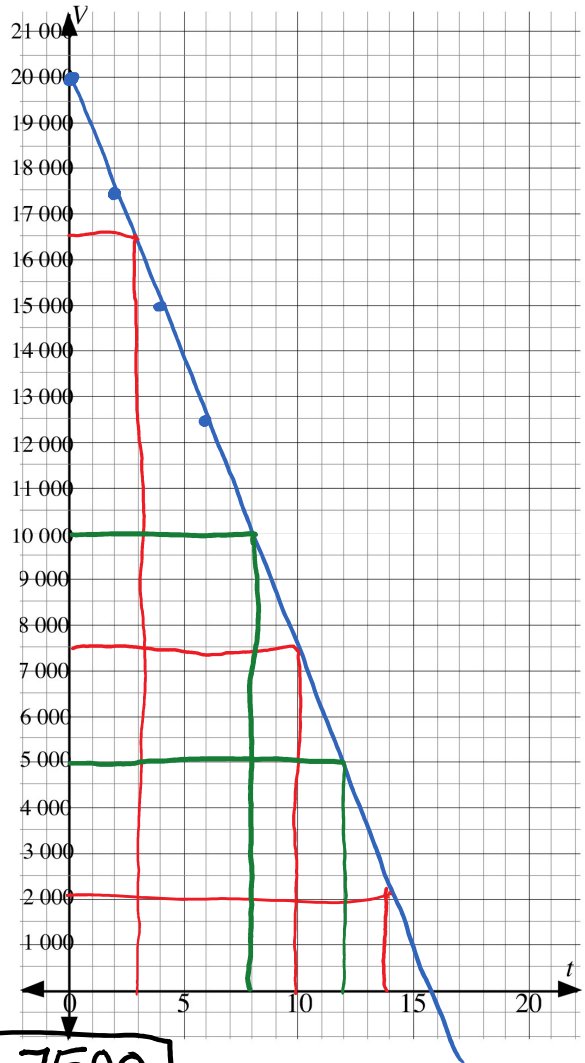
Lisa purchases a new car for \$20 000. The value of the car can be represented by the formula $V = 20\,000 - 1250t$, where V is the value of the car in dollars, and t is the age of the car in years.

a) Complete the table of values and plot the ordered pairs on the grid.

Input (t)	Output (V)	Ordered pair (t, V)
0	20000	(0, 20000)
2	17500	(2, 17500)
4	15000	(4, 15000)
6	12500	(6, 12500)

$V = 20000 - 1250(2) = 17500$
 $V = 20000 - 1250(4) = 15000$
 $V = 20000 - 1250(6) = 12500$

Connect the points with a straight line, and extend the line.



b) What does the ordered pair (0, 20 000) represent?

y-int
purchase price

c) Use the graph to estimate the t -intercept. What does the t -intercept represent?

$t = 16$
loses value

d) Use the graph to estimate the value of the car after
 i) 3 years ii) 10 years iii) 14 years.

16500 7500 2500

e) Use the formula to verify d) ii).

$t = 10$ $20000 - 1250(10)$
 $20000 - 12500 = 7500$

f) Use the graph to estimate when the car will be worth

i) \$5 000 12 years ii) half of the purchase price. 10000

g) Use the formula to verify f) ii).

$V = 10000$ $10000 = 20000 - 1250t$ solve for t
 $-10000 = -1250t$
 $\frac{-10000}{-1250} = \frac{-1250t}{-1250}$
 $t = 8$

h) Complete the following statement to describe the relation:

The original value of the car is 20000. It depreciates in value by 1250 per year and has no value after 16 years.

The original value of the car is 10000. It depreciates in value by 1000 per year and has no value after 10 years.

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x-int



In this lesson, using algebra determines the **exact** values for intercepts, etc. whereas using graphs gives an **estimate** for intercepts, etc. In lesson 5 we use the features of a graphing calculator to determine more accurate results from a graph.

In part **d)i)** we were asked to use the graph to find values lying between given points. This process is called **interpolation**. Extending the graph to predict values outside the plotted points is called **extrapolation**. Examples of extrapolation are **d)ii)** and **d)iii)**.

Complete Assignment Questions #4 - #9

Assignment #3 acegi, 4, 6, 7

1. Determine the value of the *y*-intercept of the graph of each equation.

a) $y = x - 5$

b) $y = 3x - 15$

c) $2y + 3x - 12 = 0$

d) $0.5x - 2.4y + 0.8 = 0$

e) $2y = x^2 - 60$

f) $y = 0.001x^2 - 0.001x + 12.44$

2. Determine the value of the *x*-intercept(s) of the graph of each equation.

a) $y = x - 2$

b) $y = 2x - 8$

c) $3y + 2x - 12 = 0$

d) $0.6x - 2y + 0.5 = 0$

e) $y = x^2 - 9$

f) $y = 12 - 3x$

3. Determine the *x*- and *y*-intercepts of each equation. Answer as ordered pairs.
a) $y = 4x + 7$ **b)** $y = 15 - 6x$ **c)** $4x - 2y + 16 = 0$

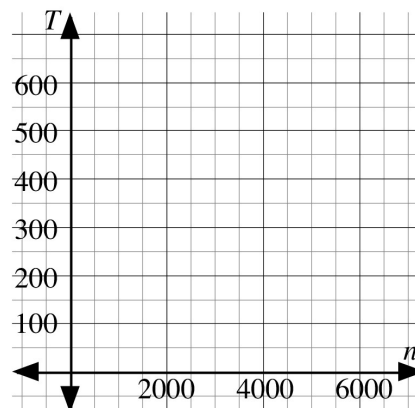
d) $y = \frac{x^2}{2} - 18$ **e)** $x^2 + y^2 = 25$ **f)** $y = 3x$

g) $y = x^2 + 4$ **h)** $9x^2 + y^2 = 81$ **i)** $9x^2 - y^2 = 81$

4. Triple A Car Rental charges \$100 per rental plus 10¢ per km. The total cost, T , in dollars of renting the car can be represented by the formula, $T = 100 + 0.10n$, where n is the number of km travelled.

- a) Complete the table of values, and plot the ordered pairs on the grid provided.

Number of km (n)	Total Rental Cost (T) dollars
0	
1000	
3500	
5000	

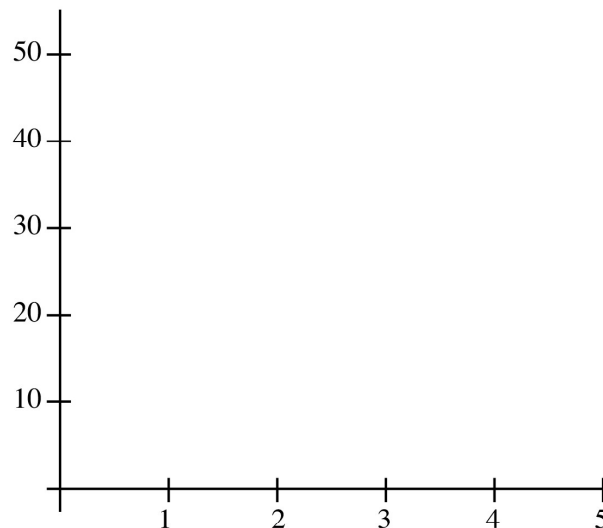


Connect the points with a straight line, and extend the line in both directions.

- b) What does the ordered pair $(0, 100)$ represent?
- c) Determine the n -intercept of the graph. Explain why it is not applicable to this problem.
- d) Interpolate from the graph to estimate the cost for a journey of:
 i) 2000 km ii) 4500 km
- e) Use the formula to verify the answers in d).
- f) If the total cost of rental is \$650, use the graph to estimate the number of km travelled.
- g) Verify the answer in f) using the formula.

5. An arrow is shot vertically into the air using a bow. The height, h metres, above the ground after t seconds, where $t \geq 0$ is approximated by the equation $h = -5t^2 + 20t + 25$.
- The maximum height of the arrow is reached after 2 seconds. Calculate the maximum height.
 - Complete the table of values, and plot the points on the grid. Join the points with a smooth curve, and label the graph.

time (seconds)	height (metres)
0	
1	
2	
3	
4	
5	

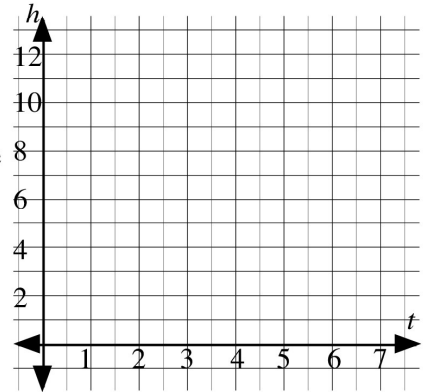


- Is this a linear or nonlinear relation?
- For how many seconds is the arrow in the air?
- What does the h -intercept represent in the context of the question?
- What does the t -intercept represent in the context of the question?
- Use the graph to estimate the height of the arrow after 1.5 seconds.
 - Use the equation to calculate the exact height of the arrow after 1.5 seconds.
- Does it make sense to extend the graph of the relation $h = -5t^2 + 20t + 25$ further in a downward direction to the left or right? Explain.

6. A candle manufacturer determined that its “Long-Last” candles melted according to the formula $h = -2t + 12$, where h is the height of the candle, in cm, after t hours.

a) Make a table of values and use this to construct the graph of $h = -2t + 12$.

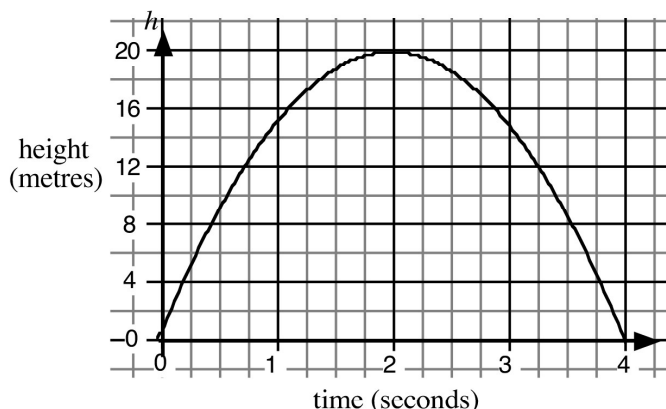
t				
h				



Use your graph to answer **b** – **e**.

- b) How tall is the candle before it begins to melt?
- c) How many hours will the candle last before it will completely burn out?
- d) How tall will the candle be after burning for 5 hours?
- e) How long will it take for the candle to burn down to a height of 7 cm?
- f) Verify the answers from **b**) - **e**) using the formula.

7. A football is kicked by a student. The graph of the relation between the height of the football above the ground and time is shown. The formula that represents the relation is given by $h = -4.9t^2 + 19.4t + 0.6$, where h is the height in metres above the ground and t is the time in seconds the football is in the air.



Use the graph to answer **a – c**:

- a) Estimate, to the nearest metre, the maximum height of the football above the ground.
- b) Estimate how long it takes for the football to reach the ground.
- c) Estimate the height, to the nearest metre, of the football when it is in the air for 3 seconds.
- d) Use the formula to calculate the exact answer to **c**).
- e) Calculate the h -intercept, and describe what it represents in the context of the question.

Multiple Choice

8. In which of the following relations does the graph of the relation have x - and y -intercepts with equal values?

- A. $y = x + 8$
- B. $2x + 2y = 7$
- C. $2x - 3y + 4 = 0$
- D. none of the above

Numerical Response

9. The graph of the relation $4x^2 + 9y^2 - 36 = 0$ has x -intercepts a and b , and y -intercepts c and d . The value of the product $abcd$ is _____.

(Record your answer in the numerical response box from left to right)

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Answer Key 1. a) -5 b) -15 c) 6 d) $\frac{1}{3}$ e) -30 f) 12.44

2. a) 2 b) 4 c) 6 d) $-\frac{5}{6}$ e) ± 3 f) 4

3. a) $x\text{-int} = \left(-\frac{7}{4}, 0\right)$, $y\text{-int} = (0, 7)$ b) $x\text{-int} = \left(\frac{5}{2}, 0\right)$ $y\text{-int} = (0, 15)$ c) $x\text{-int} = (-4, 0)$ $y\text{-int} = (0, 8)$

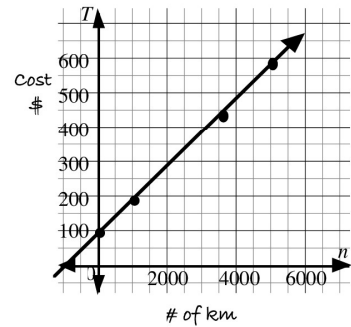
d) $x\text{-int} = (6, 0)$ and $(-6, 0)$, $y\text{-int} = (0, -18)$ e) $x\text{-int} = (5, 0)$ and $(-5, 0)$ $y\text{-int} = (0, 5)$ and $(0, -5)$

f) $x\text{-int} = (0, 0)$, $y\text{-int} = (0, 0)$ g) no $x\text{-int}$, $y\text{-int} = (0, 4)$

h) $x\text{-int} = (3, 0)$ and $(-3, 0)$ $y\text{-int} = (0, 9)$ and $(0, -9)$ i) $x\text{-int} = (3, 0)$ and $(-3, 0)$ no $y\text{-int}$

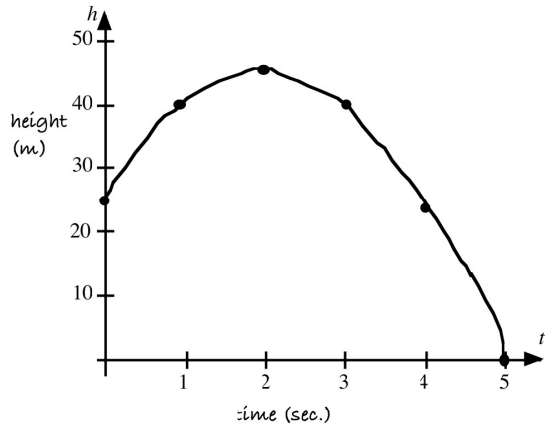
4. a) see table and graph
 b) Triple A Car Rental charges a fixed rate of \$100 before any distance is travelled
 c) $n\text{-int} = -1000$, distance in this scenario cannot be represented by a negative value
 d) i) \$300 ii) \$550
 f) 5500 km

Number of km (n)	Total Rental Cost (T) dollars
0	100
1000	200
3500	450
5000	600



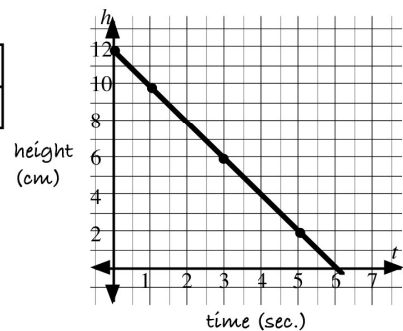
5. a) 45 m
 b) see table and graph
 c) non-linear
 d) 5
 e) The arrow was fired from a height of 25 m above the ground
 f) The number of seconds it takes to strike the ground.
 g) i) approximately 44 m ii) 43.75
 h) No to the left because time cannot be negative.
 No to the right because the ground stops the arrow from going further.

time (seconds)	height (metres)
0	25
1	40
2	45
3	40
4	25
5	0



6. a) see table and graph answers may vary
 b) 12 cm
 c) 6 hours
 d) 2 cm
 e) 2.5 hours

t	0	1	3	5
h	12	10	6	2



7. a) approx 20 m
 b) approx 4 seconds
 c) approx 15 m
 d) 14.7 m
 e) $h\text{-int}$ is 0.6 m.
 The football was punted 0.6 m above the ground.

8. B 9.

3	6		
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