# Relations Lesson #2:
## Relationships Between Two Quantities

### *Relations*

Much of mathematics involves the search for patterns and relationships between sets of data. Many real life applications of mathematics investigate the relationship between two quantities.

For example:

- the value of a computer is related to its age
- the price of a watermelon is related to its weight
- the time taken for a person to walk to school is related to the walking distance.

In mathematics, a comparison between two sets of elements is called a **relation**.


List one more example of a relation.

### *Representing the Relationship Between Two Quantities*

In the next two units we will consider seven ways in which the relationship between two quantities can be represented.

- in words
- a table of values
- a set of ordered pairs (x,y)
- a mapping (or arrow) diagram
- an equation
- a graph
- function notation (some relations can be represented in this way as in the next unit)

We will use the relation below as an example.

### *Investigating a Relation*

Consider the following relation:

> **" The cost, $C$ (cents per km), of driving a car is related to the speed, $s$ (km/h), at which it is driven."**

We will use this relation to introduce some ideas which will be developed throughout the course of this unit. Our task is to represent this relation in some form.

The example illustrates a relationship between two variables, $C$ and $s$.
In the statement of the relation, the cost depends on the speed.

$C$ is called the **dependent variable** and $s$ is called the **independent variable**.

When representing a relation, we often regard the values of the independent variable as the input and the values of the dependent variable as the output.

Before considering how to represent this relation we need some data: we need input values and output values.

The input values make up the domain of the relation, and the output values make up the range of the relation. These concepts will be discussed in more detail later.

Obviously we would not attempt to collect data for <u>every</u> possible input value (i.e. for every possible speed at which the car could be driven). Suppose that we choose as input values speeds of $20, 30, 40, \ldots .120$ km/h. and that the output values are as given in the diagrams below.

**Table of Values**

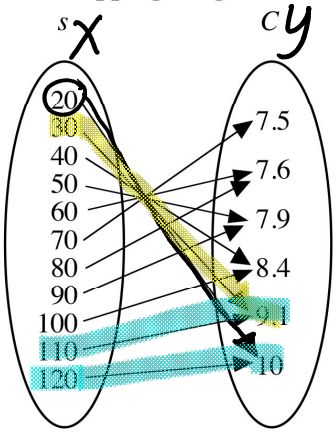
| Input ($s$) | Output ($C$) |
|---|---|
| 20 | 10 |
| 30 | 9.1 |
| 40 | 8.4 |
| 50 | 7.9 |
| 60 | 7.6 |
| 70 | 7.5 |
| 80 | 7.6 |
| 90 | 7.9 |
| 100 | 8.4 |
| 110 | 9.1 |
| 120 | 10 |

**Ordered Pairs**

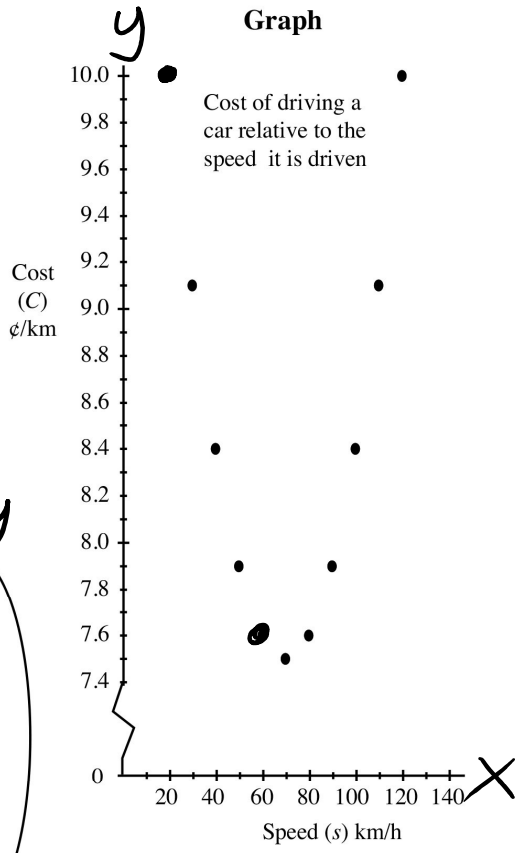
$(s, C)$

$(20, 10)$
$(30, 9.1)$
$(40, 8.4)$
$(50, 7.9)$
$(60, 7.6)$
$(70, 7.5)$
$(80, 7.6)$
$(90, 7.9)$
$(100, 8.4)$
$(110, 9.1)$
$(120, 10)$

**Mapping Diagram**




**Graph**




The diagrams above also illustrate how the information collected can be represented

- as ordered pairs in a table of values
- as a mapping diagram
- graphically

We have only chosen some of the possible input values, however it is obvious that an output value could have been determined for any input value greater than zero and up to the maximum speed of the car. It makes sense then to connect the points on the graph in some way.

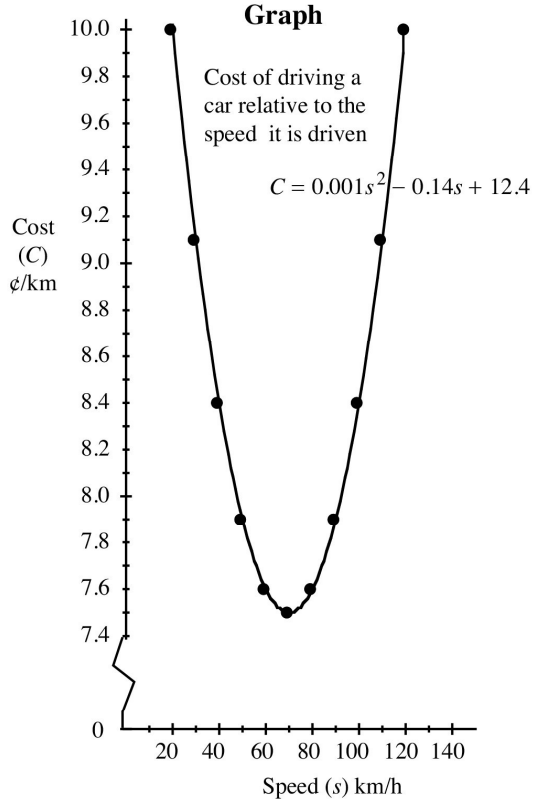
Later we will learn how this can be done and how an equation can be determined that best represents the data.

The graph and equation for the relation are given below. Note that the equation is only valid for certain input values which make up the domain of the relation. For example, the equation would not be valid for $s = 5000$!

**Equation**

$C = 0.001s^2 - 0.14s + 12.4$

**Graph**



## *Independent and Dependent Variables in a Relation*

The values of the independent variable represent the inputs, and the corresponding values of the dependent variable are the outputs.

- In an ordered pair, the values of the first coordinate are those of the independent variable and the values of the second coordinate are values of the dependent variable.
- In a table of values, the independent variable is usually given first - either to the left or above the values of the dependent variable.
- In a mapping diagram, the arrows point from the independent variable to the dependent variable.
- On a graph, the independent variable is on the horizontal axis, often the $x$-axis, and the dependent variable is on the vertical axis, often the $y$-axis.
- In an equation, we usually try to isolate the dependent variable to the left side.

The illustration below uses the equation $y = 3x - 5$ as an example to illustrate the independent and dependent variables of an equation.

$$y = 3x - 5$$

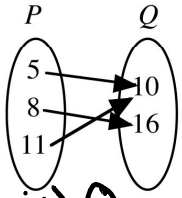
| | |
|---|---|
| • The dependent variable. | • The independent variable. |
| • Values of the dependent variable represent the outputs of the relation. | • Values of the independent variable represent the inputs of the relation. |
| • Values of the dependent variable are represented by the second coordinate of an ordered pair and are on the vertical axis. | • Values of the independent variable are represented by the first coordinate of an ordered pair and are on the horizontal axis. |


The diagrams show relations expressed in different ways. In each case

**i)** state the independent variable **ii)** state the dependent variable

**iii)** list the inputs **iv)** list the outputs

**a)**




i) P ii) Q
iii) 5, 8, 11
iv) 10, 16

**b)**

| $V$ | $A$ |
|---|---|
| 4 | 15 |
| 10 | 12 |
| 25 | 15 |

i) V ii) A
iii) 4, 10, 25
iv) 15, 12

**c)**

$(B, c)$: $(\underline{3}, \overline{7})$, $(\underline{4}, \overline{11})$, $(\underline{5}, \overline{15})$, $(\underline{6}, \overline{19})$

i) B ii) C
iii) 3, 4, 5, 6
iv) 7, 11, 15, 19


Relations are expressed in different ways below. In each case

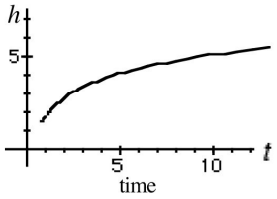
**i)** state the independent variable **ii)** state the dependent variable

**a)** $C = 2\pi r$

i) r
ii) C

radius causes the circumference to change.
C depends on r

**b)**




i) t
ii) h

**c)** The amount of sap, $s$, obtained from a maple tree is dependent on the time, $t$, a container is left attached to the maple tree.

i) t ii) s

time is typically independent

**Complete Assignment Questions #1 - #4**

## *Investigating Relationships by Plotting Ordered Pairs*

In this section we will consider relations defined by an equation, and sketch a graph by plotting ordered pairs using the following steps:

- Make a table of inputs by choosing replacements for the independent variable.
- For each of the input values, calculate the corresponding value (the output) of the dependent variable.
- Plot the ordered pairs on a Cartesian plane.


Consider the relation described by the equation $y = 2x - 5$.

**a)** Complete the first five rows of the following table of values which shows some of the possible input values.

| Input ($x$) | Output ($y$) | Ordered pair $(x, y)$ |
|---|---|---|
| −2 | −9 | (−2, −9) |
| −1 | −7 | (−1, −7) |
| 0 | −5 | (0, −5) |
| 1 | −3 | (1, −3) |
| 2 | −1 | (2, −1) |
| 3 | 1 | (3, 1) |
| 4 | 3 | (4, 3) |

$y = 2(-2) - 5$
$= -9$

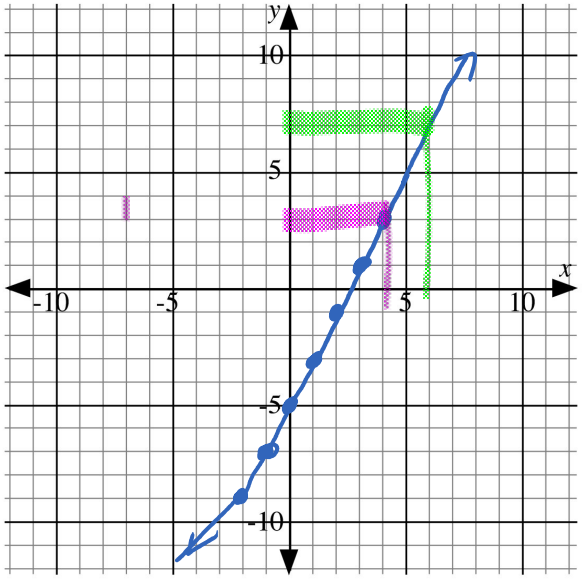
$y = 2(-1) - 5$
$= -7$

$y = 2(0) - 5$
$= -5$

$y = 2(1) - 5$
$= -3$

**b)** Plot the ordered pairs in **a)** on the grid provided.

**c)** Connect the points on the grid, and extend the line in both directions with arrows at both ends.




**d)** Use the graph to determine the value of $y$ when $x = 6$. $y = 7$

**e)** Use the equation to determine the value of $y$ when $x = 6$, and verify the answer in **d)**.

$y = 2(6) - 5$
$= 12 - 5$
$= 7$

**f)** Write the value of $y$ when $x = 6$ in the table of values using the first blank space in **a)**.

**g)** Use the graph to determine the value of $x$ when $y = 3$. Put this information in the last row in **a)**. $x = 4$

**h)** Complete the following statement:

This relation is called a linear relation because the graph of the relation is a straight line.


Consider the relation described by the equation $y = x^2 - 6$.

**a)** Complete the table of values to the right which shows some of the possible input values.

$y = (4)^2 - 6 = 16 - 6 = 10$

$y = (3)^2 - 6 = 9 - 6 = 3$

$y = (2)^2 - 6 = 4 - 6 = -2$

$y = (1)^2 - 6 = 1 - 6 = -5$

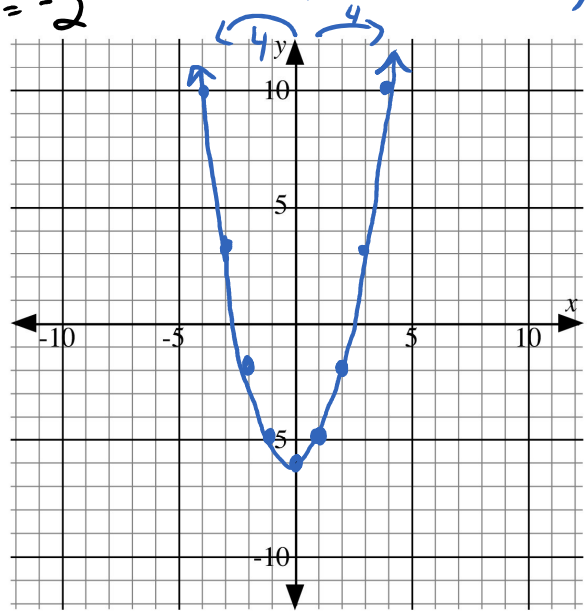
$0 - 6 = -6$

$(-1)^2 - 6 = 1 - 6 = -5$

$(-2)^2 - 6 = 4 - 6 = -2$

| Input ($x$) | Output ($y$) | Ordered pair ($x, y$) |
|---|---|---|
| 4 | 10 | (4,10) |
| 3 | 3 | (3,3) |
| 2 | -2 | (2,-2) |
| 1 | -5 | (1,-5) |
| 0 | -6 | (0,-6) |
| –1 | -5 | (-1,-5) |
| –2 | -2 | (-2,-2) |
| –3 | 3 | (-3,3) |
| -4 | 10 | (-4,10) |

**b)** Plot the ordered pairs in **a)** on the grid provided.




**c)** Use the symmetry of the graph or table to predict the value of $y$ when $x = -4$.

symmetry

$y = 10$

**d)** Use the equation to determine the value of $y$ when $x = -4$, and verify the answer in **c)**.

$y = (-4)^2 - 6$

$= 16 - 6$

$= 10$

**e)** Write the value of $y$ when $x = -4$ in the table of values using the first blank space in **a)**.

**f)** Connect the points on the grid with a smooth curve.

**g)** Why do you think this type of relation is called a **nonlinear relation**?

not a straight line

**Complete Assignment Questions #5 - #9**