

## Lesson 6: Problem Solving Using Trigonometric Ratios

# Trigonometry Lesson #6: Problem Solving Using Trigonometric Ratios

## Overview

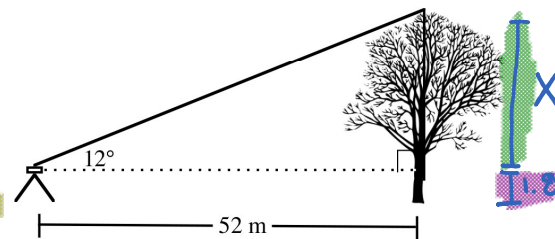
In this lesson we use our knowledge of trigonometry to solve problems including situations in which more than one calculation is required to determine the solution to a problem.

**The illustrations in this lesson are not to scale and are for sketch purposes only.**



Class Ex. #1

Karmen is a surveyor who is using a transit to determine the height of a tree. The transit is placed 52 m from the base of the tree and the angle to the top of the tree is measured to be  $12^\circ$ .



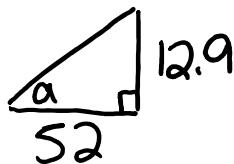
- a) If the transit stands 1.8 m high, calculate the height of the tree to the nearest tenth of a metre.

$$52 \tan 12 = \frac{x}{52}$$

$$52 \tan 12 = x$$

$$x = 11.1 + 1.8 = \text{height of the tree} = 12.9$$

- b) Determine the angle the base of the transit makes with the top of the tree. Answer to the nearest degree.



$$\tan a = \frac{12.9}{52}$$

$$a = \tan^{-1}(12.9 \div 52)$$

$$a = 14^\circ$$

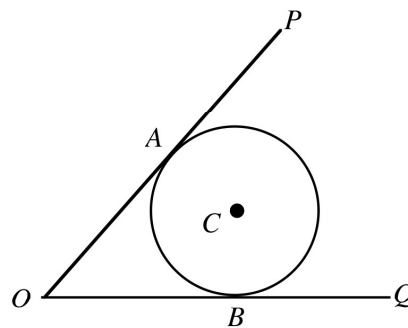


Class Ex. #2

A circle with centre  $C$  and radius 20 mm touches the arms of angle  $POQ$  at  $A$  and  $B$ .

Radii  $CA$  and  $CB$  are perpendicular to  $OP$  and  $OQ$  respectively and the diagram is symmetrical about the line through  $O$  and  $C$ .

If angle  $POQ = 50^\circ$  determine the length of  $OC$  to the nearest mm.



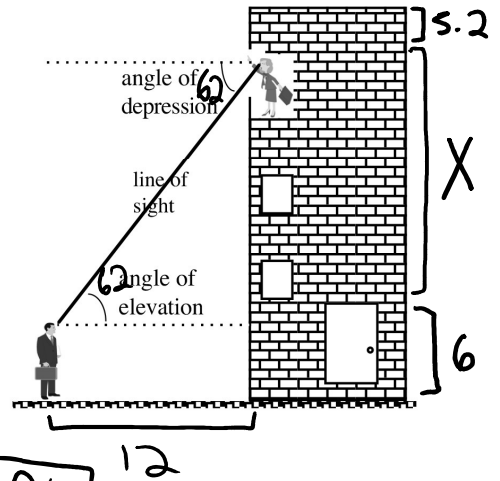
In many problems, the angle of elevation or depression is determined from the person's eye level and not from the ground.



Class Ex. #3

In the diagram the man on the ground is standing 12 feet from the wall. The angle of elevation from his eye level to the eye level of his wife in the building is  $62^\circ$ .

If the man's eye level is 6.0 feet above the ground and his wife's eye level is 5.2 feet from the top of the building, determine the height of the building to the nearest tenth of a foot.



$$12 \tan 62^\circ = \frac{X}{12}$$

$$12 \tan 62^\circ = X$$

$$X = 22.57$$

height of building =

$$5.2 + X + 6$$

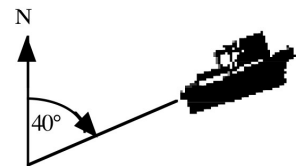
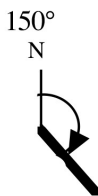
$$5.2 + 22.57 + 6 = 33.8 \text{ ft}$$

**Bearings**

A bearing is an angle measured clockwise from the North direction.

The boat is on a bearing of  $40^\circ$  (often written using three figures as  $040^\circ$ ).

Here are three other bearings.

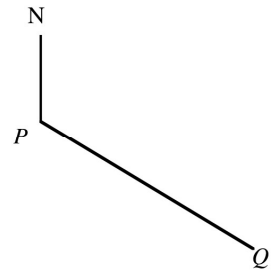


HW #1, 3, 4



A ship travels 6 km from a port,  $P$ , on a bearing of  $120^\circ$  to position  $Q$  as shown.

- a) The ship changes direction at  $Q$  and travels north until it reaches position  $R$  which is due east of  $P$ .  
How far east of  $P$  is the ship at position  $R$ ? Answer to the nearest 0.1 km.



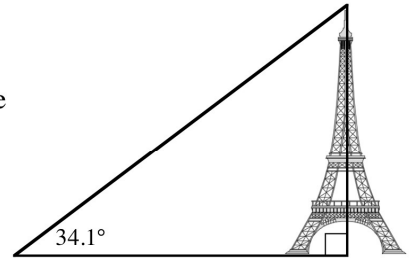
- b) The ship continues travelling north for a further 12 km to position  $S$ .  
Determine the three figure bearing of  $S$  from  $P$ .
- c) If the ship maintains an average speed of 9 km/h throughout the journey, determine to the nearest tenth of an hour, the time taken for the journey.

**Complete Assignment Questions #1 - #9**

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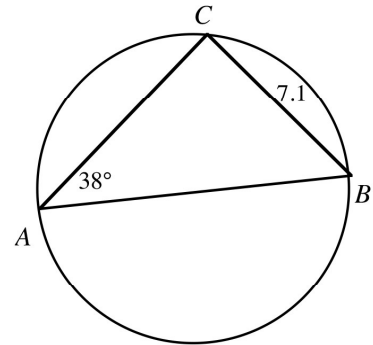
## Assignment

- On a sunny day, the Eiffel Tower standing 1063 ft high, casts a shadow on the ground. A tourist stands at the end of the shadow and measures the angle of elevation of the sun to be  $34.1^\circ$ . Determine, to the nearest foot, the length of the shadow.



- In the diagram,  $AB$  is a diameter of a circle.  $\angle CAB = 38^\circ$ ,  $CB = 7.1$  cm, and  $\angle ACB$  is a right angle.

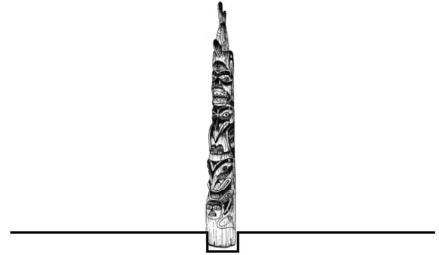
Determine the radius of the circle to the nearest tenth of a cm.



- A boat is 300 m away from the foot of a cliff. The angle of elevation from the boat to the top of the cliff is  $16^\circ$ .
  - Show this information on a diagram and determine the height of the cliff to the nearest metre.
  - If the boat then sails 75 m closer to the cliff, determine, to the nearest degree, the new angle of elevation of the cliff top from the boat.

4. A totem pole is being placed in a 2 metre deep hole in the ground. Two ropes attached to the top of the totem pole are used to pull the totem pole upright. The ropes are anchored into the ground on opposite sides of the pole until the hole is filled. Each rope is 19 m long and is anchored into the ground 12 m horizontally from the centre of the totem pole.

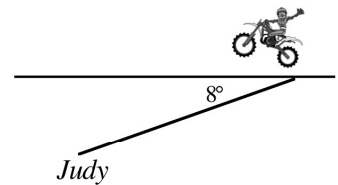
- a) Complete the sketch to illustrate this situation.
- b) Calculate, to the nearest tenth of a degree, the angle of elevation of the top of the totem pole from the point where the rope is anchored into the ground.



- c) Determine, to the nearest 0.1 m, the length of the totem pole before it was placed in the ground.

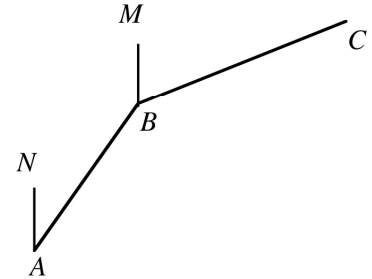
5. Judy is 60 m south of a main east-west highway. She looks to her right and sees a motor bike rider approaching at a steady speed along the highway. The diagram shows their positions relative to each other.

Twenty seconds later the motor bike rider is directly in front of Judy. Calculate the speed of the motor bike, to the nearest km/h.



6. A ship sails from a port  $A$  on a bearing of  $030^\circ$  for 22 miles to a marker  $B$  where it alters course and sails on a bearing of  $075^\circ$  for 27 miles to its destination  $C$ .  $AN$  and  $BM$  represent north lines.

- a) On the diagram mark the sizes of  $\angle BAN$  and  $\angle CBM$ .
- b) Determine, to the nearest 0.1 mile, how far  $C$  is east of  $A$ .



- c) Determine, to the nearest 0.1 mile, how far  $C$  is north of  $A$ .
- d) Determine, to the nearest 0.1 mile, the direct distance from  $A$  to  $C$ .
- e) Determine the three figure bearing of  $C$  from  $A$ .

Use the following information to answer questions #7 and #8.

A plane starts from a point  $P$  and flies 240 km on a bearing of  $120^\circ$  to  $Q$ .  
 At  $Q$  the plane makes a  $90^\circ$  turn and flies 100 km on a bearing of  $210^\circ$  to  $R$ .  
 At  $R$  the plane again changes direction and flies back to  $P$ .

**Multiple Choice**

7. The direct distance, in km, from  $R$  to  $P$  is
- A. 140
  - B. 218
  - C. 260
  - D. 340
8. The three figure bearing the pilot uses to fly from  $R$  to  $P$  is
- A. 023°
  - B. 037°
  - C. 323°
  - D. 337°

**Numerical Response**

9. A set of stairs has a vertical rise of 16 cm for every 27 cm horizontal run. The angle, to the nearest tenth of a degree, between the stairs and the floor is \_\_\_\_\_.

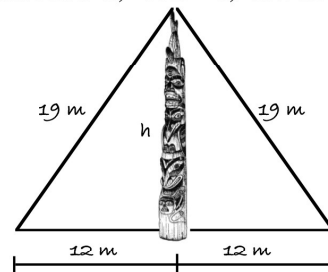
(Record your answer in the numerical response box from left to right)

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**Answer Key**

1. 1570 ft                      2. 5.8 cm
3. a) 86 m    b)  $21^\circ$                                               4. a) see diagram below    b)  $50.8^\circ$     c) 16.7 m
5. 77 km/h
6. a) angle  $BAN = 30^\circ$ , angle  $CBM = 75^\circ$ .    b) 37.1 miles  
 c) 26.0 miles                      d) 45.3 miles                      e)  $055^\circ$
7. C                      8. C                      9. 

3	0	.	7
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# Lesson 7: More Problem Solving Using Trigonometric Ratios

Friday, August 31, 2018 2:28 AM

# Trigonometry Lesson #7: More Problem Solving Using Trigonometric Ratios

## Overview

In this lesson we deal with problems involving two right triangles. In many cases we can determine the value of a quantity from one right triangle and then use that value in a second right triangle to determine the solution to a problem. When this situation arises, do not round intermediate answers. Only the final solution should be rounded. **The illustrations in this lesson are not to scale and are for sketch purposes only.**

## Problems in Two Dimensions



Class Ex. #1

In the diagram  $QS = 32$  mm, angle  $PQS = 50^\circ$  and angle  $RPS = 61^\circ$ . Use this information to determine the length of  $QR$  to the nearest mm.

for  $\triangle PSR$ , we don't have enough info to solve for  $RS$ . First solve for  $PS$

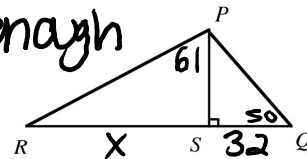
$$\cancel{32} \tan 50 = \frac{PS \cancel{32}}{\cancel{32}}$$

$$PS = 32$$

now solve  $RS(x)$

$$\cancel{32} \tan 61 = \frac{x \cancel{32}}{\cancel{32}}$$

$$x = 58$$



$$QR = x + 32$$

$$58 + 32$$

$$= 90 \text{ mm}$$



Class Ex. #2

Calculate the measure of  $\angle BEC$  to the nearest degree.

Use  $\triangle AEC$  to get  $EC$  to find angle  $x$

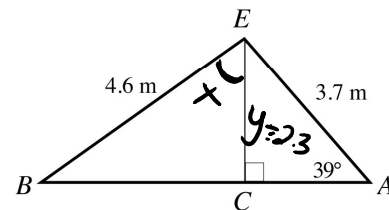
$$3.7 \sin 39 = \frac{y \cancel{3.7}}{\cancel{3.7}}$$

$$y = 2.3$$

$$\cos x = \frac{2.3}{4.6}$$

$$x = \cos^{-1}(2.3 \div 4.6)$$

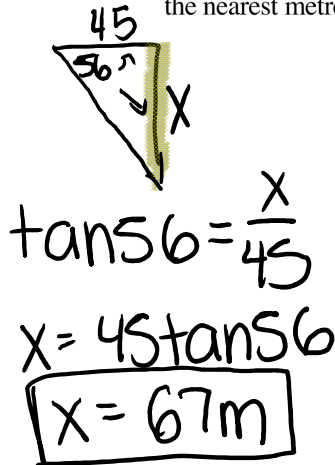
$$x = 60$$



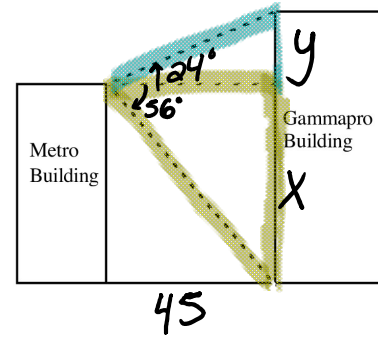
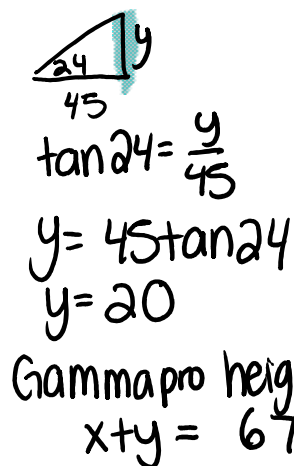
$$\angle BEC = 60^\circ$$



In January 2003, the tallest building in Rockyville was the Metro Building. Recently, a developer was commissioned by the Gammapro Oil Company to build a taller building next to the Metro Building. From the top of the Metro Building, the angle of elevation of the top of the Gammapro Building is  $24^\circ$  and the angle of depression to the foot of the Gammapro Building is  $56^\circ$ . If the buildings are 45 m apart, determine the height of each building to the nearest metre.



↑  
height  
of  
metro



Complete Assignment Questions #1 - #6

Problems in Three Dimensions

# 1, 3, 4

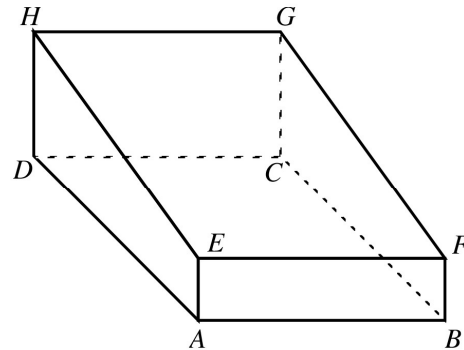
As an aid to understanding a problem in three dimensions, we sketch a two dimensional representation of the triangle we are working with.



The solid in the diagram was formed from a rectangular prism by removing a wedge.

$HD = GC = 5$  cm.  $EA = FB = 2$  cm.  
 $AD = 6$  cm and  $AB = 8$  cm.

- Name four rectangles in the diagram.
- Calculate the measure of  $\angle HEA$  to the nearest degree.



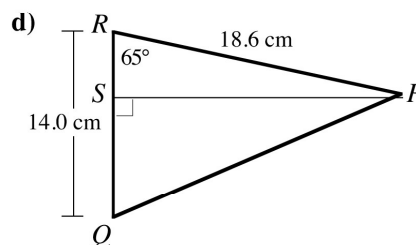
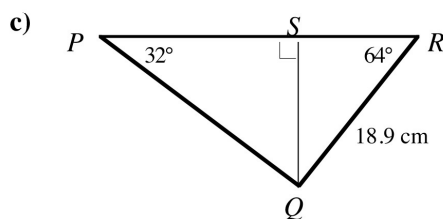
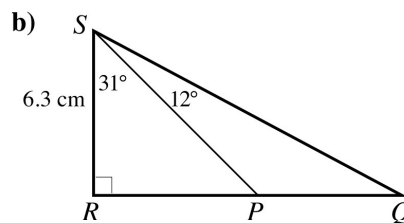
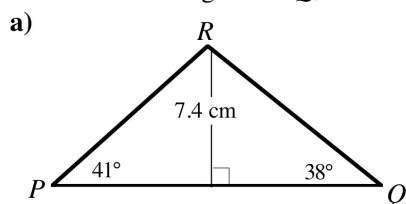
c) Calculate the measure of angle  $HFE$  to the nearest degree.

d) Calculate the measure of angle  $HBD$  to the nearest degree.

Complete Assignment Questions #7 - #10

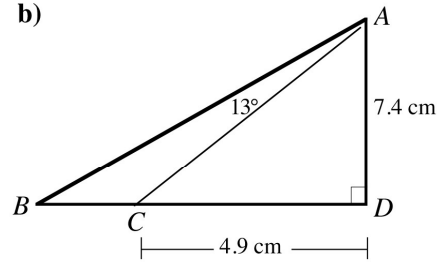
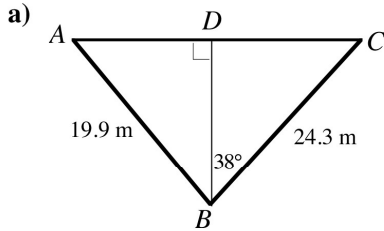
### Assignment

1. Determine the length of  $PQ$ , to the nearest 0.1 cm.



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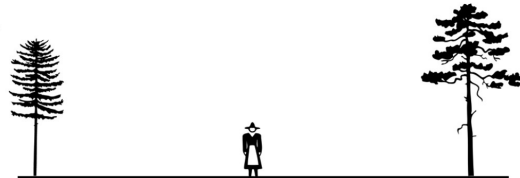
2. Determine the measure of angle  $ABC$ , to the nearest degree.



3. Two trees in a park are 14.80 metres apart. An observer, whose eyeline is 1.80 m above the ground, is standing halfway between the trees. The angles of elevation of the tops of the trees from the observer's eyeline are  $17^\circ$  and  $23^\circ$ .

a) Show this information on the diagram.

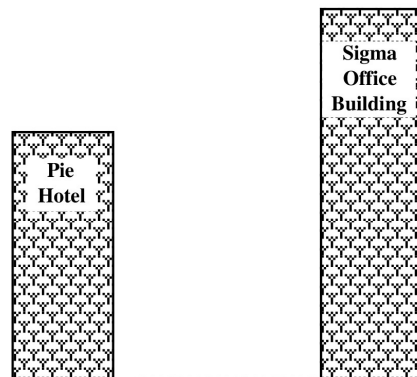
b) Determine the height of each tree to the nearest 0.01 m.



4. The angle of elevation from the top of the Pie Hotel to the top of the Sigma Office Building is  $17^\circ$ . The angle of depression from the top of the Pie Hotel to the foot of the Sigma Office Building is  $60^\circ$ . The height of the Pie Hotel is 150 m.

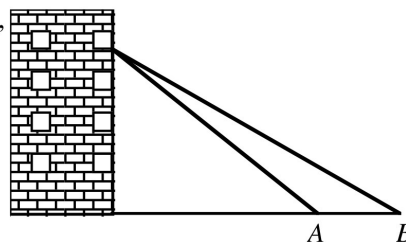
a) Complete the sketch to illustrate this situation.

b) Determine the height of the Sigma Office Building to the nearest tenth of a metre.



5. From the top of a cliff 110 m high, an observer sees two boats, one directly behind the other, heading for shore. The angle of depression from the observer to the boat furthest from the observer is  $48^\circ$  and the angle of depression to the nearest boat is  $57^\circ$ . Calculate the distance between the boats, to the nearest metre.

6. The diagram shows two marathon runners,  $A$  and  $B$ , heading towards the finish line of a race. From an apartment window 80 metres above the ground and 20 metres behind the finish line, Tony measures the angle of depression of the runners to be  $28^\circ$  and  $24^\circ$  respectively.



- a) Calculate the distance between the runners to the nearest metre.

- b)  $A$  is travelling at a constant speed of 4.5 m/s, while  $B$  is travelling at a constant speed of 5.1 m/s. Which runner will finish the race first?

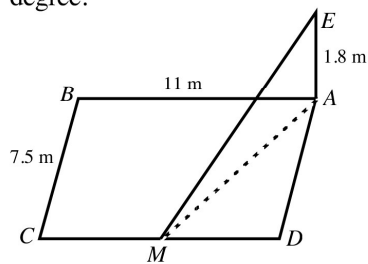
Use the following information to answer the next question.

The Indigenous Plains people of what is now Canada often lived in tipis until the mid-1800s. Strategically, these tipis were often located near rivers or other water sources.

In order to find the height of a tipi on the opposite bank of a river, Dancing Eagle made the measurements shown in the diagram.

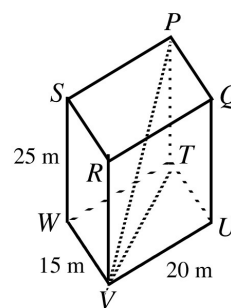
7. Calculate the height of the tipi to the nearest tenth of a metre.

8. In the diagram,  $ABCD$  represents a rectangular sandbox where kindergarten children play. A teacher stands at the corner of the area to supervise the children. At a certain time of day, the tip of the shadow cast by the teacher on the play area is exactly at  $M$ , the midpoint of  $CD$ . If the teacher is 1.8 m tall, calculate the measure of angle  $EMA$ , to the nearest degree.



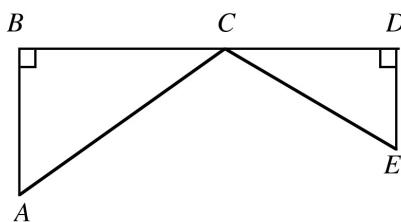


9. A rectangular prism has dimensions 15 m, 20 m, and 25 m as shown. The angle between diagonal  $VP$  and the plane  $TUVW$  is defined to be angle  $PVT$ . The tangent ratio of this angle is



- A.  $\frac{3}{4}$
- B. 1
- C.  $\frac{5}{4}$
- D.  $\frac{5}{3}$

- Numerical Response** 10. In the diagram,  $AB = 28$  m,  $DE = 21$  m,  $BD = 71$  m, and angle  $BAC = 55^\circ$ . To the nearest degree, the measure of angle  $CED$  is \_\_\_\_\_°.



(Record your answer in the numerical response box from left to right)

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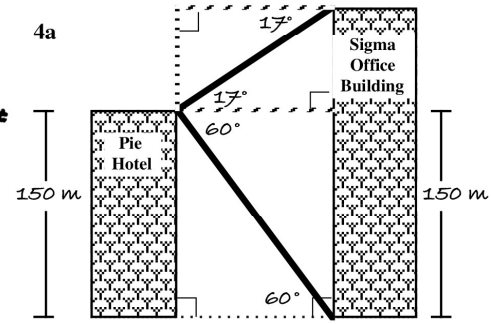
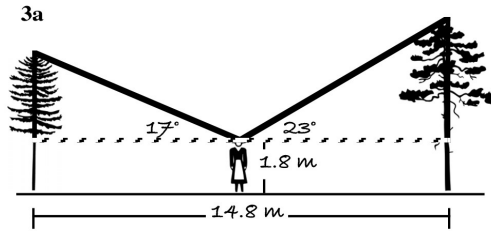
**Answer Key**

1. a) 18.0 cm      b) 2.1 cm      c) 32.1 cm      d) 17.9 cm

2. a)  $54^\circ$       b)  $43^\circ$

3. a) See the diagram below.      b) 4.06 m and 4.94 m

4. a) See below.      b) 176.5 m



5. 28 m      6. a) 29 m      b) A takes 29 sec, B takes 31 sec, so A finishes first.

7. 5.2 m

8.  $11^\circ$

9. B

10. 

5	6		
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