

Lesson 2: Trigonometric Ratios on a Calculator and Patterns in Trigonometric Ratios

Friday, August 31, 2018 2:27 AM

Trigonometry Lesson #2: Trigonometric Ratios on a Calculator and Patterns in Trigonometric Ratios

Review

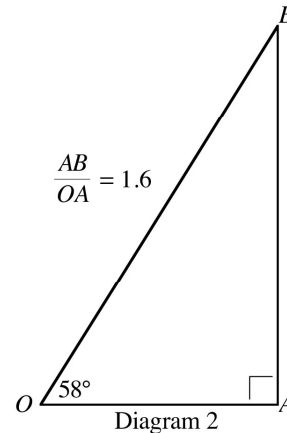
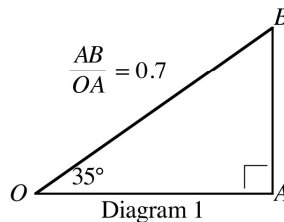
Recall the following from the previous lesson.

SOH CAH TOA

$$\text{sine ratio} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{cosine ratio} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{tangent ratio} = \frac{\text{opposite}}{\text{adjacent}}$$



Using a Calculator to Determine Trigonometric Ratios

From the investigations in the previous lesson we can now say that the ratio 0.7 is the approximate tangent ratio for an angle of 35° and that the ratio 1.6 is the approximate tangent ratio for an angle of 58° .

We can write the ratios in the form $\tan 35^\circ = 0.7$ and $\tan 58^\circ = 1.6$ (to 1 decimal place).

More accurate approximations can be found using a calculator.

To four decimal places, $\tan 35^\circ = 0.7002$ and $\tan 58^\circ = 1.6003$.



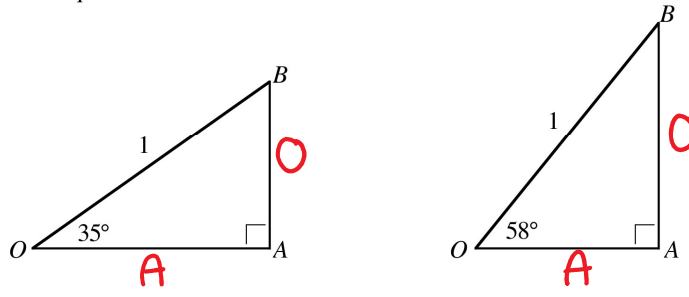
a) Use a calculator to determine the value of each trigonometric ratio to four decimal places.

i) $\sin 35^\circ = 0.5736$ **ii)** $\cos 35^\circ = 0.9192$

iii) $\sin 58^\circ = 0.8480$ **iv)** $\cos 58^\circ = 0.5299$

b) With respect to the diagrams at the top of the page, which ratio of sides is represented by each of the ratios in **a)**?

The diagrams below show triangles similar to the ones on the previous page in which the two hypotenuses are equal to 1 unit.



As the angle AOB increases from 35° to 58° explain, using SOHCAHTOA, why the sine ratio increases in value but the cosine ratio decreases in value.

Determine what happens to the tangent ratio.

$$\sin 35 = \frac{\text{opp}}{\text{hyp}} = \frac{AB}{1} = AB \quad \sin 58 = \frac{\text{opp}}{\text{hyp}} = AB$$

sine ratio is larger with the larger angle

$$\cos 35 = \frac{\text{adj}}{\text{hyp}} = \frac{OA}{1} = OA \quad \cos 58 = \frac{\text{adj}}{\text{hyp}} = OA$$

cosine ratio is smaller with larger angle (adj)

small angle = bigger adjacent, smaller opposite



Without using a calculator, state which of the following ratios has the greater value.

a) $\sin 14^\circ$, $\sin 20^\circ$

b) $\cos 7^\circ$, $\cos 62^\circ$

c) $\tan 15^\circ$, $\tan 75^\circ$

d) $\sin 80^\circ$, $\cos 80^\circ$, $\tan 80^\circ$

The trigonometric ratios for all angles between 0° and 90° can be approximated using a calculator.



Use a calculator to determine the value of each trigonometric ratio to four decimal places.

a) $\sin 81^\circ = 0.9877$ b) $\tan 14^\circ = 0.2493$ c) $\cos 44^\circ = 0.7193$

*calculator needs to be in degree mode.

Determining the Angle Given a Trigonometric Ratio

We have discovered that $\tan 35^\circ = 0.7$ and $\tan 58^\circ = 1.6$, but is there an angle for which the tangent ratio is 1.2?

Use a "guess and check" technique to discover the measure of this angle to 1 decimal place.

This method is time consuming and the calculator provides us with a quicker method by using the inverse trigonometric function.

If $\tan x^\circ = 1.2$, then the measure of the angle x° can be determined by using the inverse tangent function \tan^{-1} .

If $\tan x^\circ = 1.2$, then $\tan^{-1}(1.2) = x^\circ$ = 50

On a calculator, access the inverse tangent function, \tan^{-1} , by pressing

2nd	tan
-----	-----

.

2nd	tan	1	.	2)	ENTER
-----	-----	---	---	---	---	-------

 determines the value of x .



Part i: Write the keying sequence which will determine the acute measure of the angle a° if $\sin a^\circ = 0.3584$

$\boxed{2nd} \boxed{\sin} \boxed{0.3584} = 21$ / $\boxed{0.3584} \boxed{\sin^{-1}} = 21$

Part ii: In each case, determine the measure of the acute angle to the nearest degree.

a) $\sin a^\circ = 0.3584$

$a^\circ = \sin^{-1}(0.3584)$

b) $\cos b^\circ = 0.5389$ $b^\circ = 57^\circ$

c) $\tan P = 3.2106$ $\angle P = 73^\circ$

d) $\sin C = \frac{1}{2}$ $\angle C = 30^\circ$
 \uparrow
 0.5

Complete Assignment Questions #1 - #4

Copyright © by Absolute Value Publications. This book is NOT covered by the Cancopy agreement.

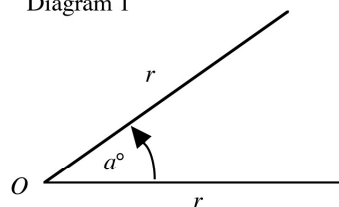
(1-4) every 2nd letter

Trigonometric Ratios for Angles of 0° and 90°
--

The trigonometric ratios for angles of 0° and 90° can be determined using the idea of rotation angles.

A rotation angle of a° is formed when a line of length r is rotated counterclockwise about a point O , as shown in Diagram 1.

Diagram 1



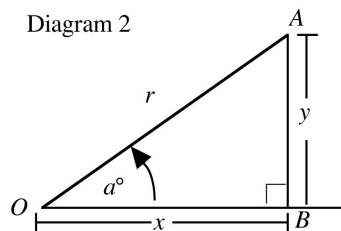
Right triangle AOB , where AB is perpendicular to OA , is formed from Diagram 1 as shown in Diagram 2.

$$\sin a^\circ = \frac{y}{r}$$

Similarly $\cos a^\circ =$

and $\tan a^\circ =$

Diagram 2



As the rotation angle, a° , is made smaller, then the opposite side of the triangle, y , decreases in value and the adjacent side, x , increases in value.

If $a = 0$, then the opposite side $y = 0$ and the adjacent side $x = r$.

$$\text{So } \sin 0^\circ = \frac{y}{r} = \quad \cos 0^\circ = \frac{x}{r} = \quad \tan 0^\circ = \frac{y}{x} =$$

As the rotation angle, a° , is made larger, then the opposite side of the triangle, y , increases in value and the adjacent side, x , decreases in value.

If $a = 90^\circ$, then the adjacent side $x = 0$ and the opposite side $y = r$.

$$\text{So } \sin 90^\circ = \frac{y}{r} = \quad \cos 90^\circ = \frac{x}{r} = \quad \tan 90^\circ = \frac{y}{x} =$$

Patterns in Trigonometric Ratios

Class Ex. #6



Complete the following table, rounding if necessary to 2 decimal places.

Angle x°	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
$\sin x^\circ$										

Complete the following statement:

As the angle x° increases from 0° to 90° , $\sin x^\circ$ increases from _____ to _____.

Class Ex. #7



Draw a right triangle LMN in which $\angle L = 90^\circ$ and $\angle N = x^\circ$.

a) Write an expression for the measure of $\angle M$ in terms of x° .

b) Explain, using ratios, why $\sin x^\circ = \cos(90 - x)^\circ$.

c) Complete the following, using an angle between 0° and 90° .

i) $\sin 20^\circ = \cos$ _____

ii) $\sin 8^\circ = \cos$ _____

Complete Assignment Questions #5 - #14

Copyright © by Absolute Value Publications. This book is **NOT** covered by the Cancopy agreement.

Assignment

1. Use a calculator to determine the value of each trigonometric ratio to four decimal places where necessary.

a) $\sin 68^\circ =$	b) $\tan 30^\circ =$	c) $\cos 19^\circ =$
d) $\cos 22^\circ =$	e) $\tan 85^\circ =$	f) $\sin 7^\circ =$
g) $\tan 60^\circ =$	h) $\sin 18^\circ =$	i) $\cos 72^\circ =$
j) $\cos 45^\circ =$	k) $\sin 45^\circ =$	l) $\tan 45^\circ =$

2. Use a calculator to determine the value of each trigonometric ratio to four decimal places.

a) $\sin 37.9^\circ =$	b) $\tan 4.8^\circ =$	c) $\cos 83.2^\circ =$
d) $\cos 5.7^\circ =$	e) $\tan 63.6^\circ =$	f) $\sin 17.4^\circ =$

3. In each case, use a calculator to determine the measure of the acute angle to the nearest degree.

a) $\sin a^\circ = 0.7612$, $a^\circ =$ \quad°	b) $\cos b^\circ = 0.5398$, $b^\circ =$ \quad°
c) $\tan P = 1.2173$, $\angle P =$ \quad°	d) $\sin C = \frac{1}{3}$, $\angle C =$ \quad°
e) $\tan r^\circ = \sqrt{3}$, $r^\circ =$ \quad°	f) $\cos x^\circ = \frac{\sqrt{2}}{2}$, $x^\circ =$ \quad°
g) $\sin A = \frac{\sqrt{2}}{2}$, $\angle A =$ \quad°	h) $\tan X = \frac{1}{\sqrt{3}}$, $\angle X =$ \quad°

4. In each case, use a calculator to determine the indicated acute angle to the nearest tenth of a degree.

a) $\sin A = 0.6789$	b) $\cos X = 0.1234$	c) $\tan P = 0.55$
d) $\sin a^\circ = 0.09$	e) $\cos e^\circ = \frac{7}{24}$	f) $\tan y^\circ = \frac{4}{3}$
g) $\cos K = 0.3$	h) $\tan M = 50$	i) $\sin R = \frac{5}{6}$

5. Explain, using diagrams, why the sine ratio for an angle of 20° is smaller than the sine ratio for an angle of 40° .
6. Explain, using diagrams, why the cosine ratio for an angle of 30° is greater than the cosine ratio for an angle of 70° .
7. Explain, using diagrams, why the tangent ratio for an angle of 15° is smaller than the tangent ratio for an angle of 50° .
8. Without using a calculator, state which of the following ratios has the greater value.
- | | |
|-----------------------------------|--|
| a) $\sin 58^\circ, \sin 32^\circ$ | b) $\cos 67^\circ, \cos 35^\circ$ |
| c) $\tan 56^\circ, \tan 86^\circ$ | d) $\sin 29^\circ, \cos 29^\circ$ |
| e) $\sin 74^\circ, \cos 74^\circ$ | f) $\sin 60^\circ, \cos 60^\circ, \tan 60^\circ$ |

Copyright © by Absolute Value Publications. This book is **NOT** covered by the Cancopy agreement.

74 Trigonometry Lesson #2: *Trigonometric Ratios on a Calculator*

9. Complete the following table, rounding if necessary to 2 decimal places

Angle x°	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
$\cos x^\circ$										

Complete the following statement:

As the angle x° increases from 0° to 90° , $\cos x^\circ$ _____ from _____ to _____.

10. a) Complete the following table, rounding if necessary to 2 decimal places

Angle x°	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
$\tan x^\circ$										

b) Complete: $\tan 89^\circ =$ _____ $\tan 89.9^\circ =$ _____ $\tan 89.99^\circ =$ _____

c) Complete the following statement:

As the angle x° increases from 0° to 90° , $\tan x^\circ$ _____ from _____ to _____

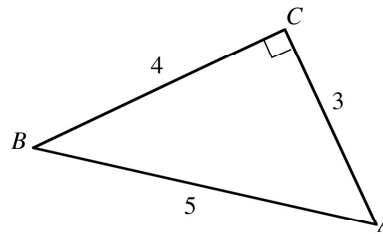
11. Consider right triangle ABC shown.

a) Referring to triangle ABC , write the rational number which represents the indicated trigonometric ratio.

$\sin A =$ _____ $\sin B =$ _____

$\cos A =$ _____ $\cos B =$ _____

$\tan A =$ _____ $\tan B =$ _____



b) Which pairs of trigonometric ratios have the same value?

c) How do the values of $\tan A$ and $\tan B$ relate to each other?

12. a) Draw a right triangle PQR in which $\angle P = 90^\circ$ and $\angle Q = x^\circ$.

b) Write an expression for the measure of $\angle R$ in terms of x .

c) Explain why $\cos x^\circ = \sin(90 - x)^\circ$

d) Complete the following using an angle less than 90° .

i) $\cos 76^\circ = \sin$ _____ ii) $\cos 13^\circ = \sin$ _____

Multiple Choice

13. Triangle LMN is right angled at L with $\tan M = a$.
Three statements are made about the trigonometric ratios in triangle LMN .

Statement 1: $\cos M = \sin N$

Statement 2: $\cos N = \sin M$

Statement 3: $\tan N = \frac{1}{a}$

How many of these statements are true?

- A. 0 B. 1
C. 2 D. 3

Numerical Response

14. To the nearest tenth, the value of $\frac{\cos 30^\circ + \sin 30^\circ}{\tan 30^\circ}$ is _____.

(Record your answer in the numerical response box from left to right)

--	--	--	--

Answer Key

1. a) 0.9272 b) 0.5774 c) 0.9455 d) 0.9272 e) 11.4301 f) 0.1219
 g) 1.7321 h) 0.3090 i) 0.3090 j) 0.7071 k) 0.7071 l) 1

2. a) 0.6143 b) 0.0840 c) 0.1184 d) 0.9951 e) 2.0145 f) 0.2990

3. a) 50° b) 57° c) 51° d) 19° e) 60° f) 45° g) 45° h) 30°

4. a) 42.8° b) 82.9° c) 28.8° d) 5.2° e) 73.0° f) 53.1° g) 72.5° h) 88.9° i) 56.4°

5. If we keep the hypotenuse the same length, the opposite side is smaller for an angle of 20° than for an angle of 40°. Since the sine ratio is $\frac{\text{opp}}{\text{hyp}}$, the sine ratio for an angle of 20° is smaller than for an angle of 40°.

6. If we keep the hypotenuse the same length, the adjacent side is greater for an angle of 30° than for an angle of 70°. Since the cosine ratio is $\frac{\text{adj}}{\text{hyp}}$, the cosine ratio for an angle of 30° is greater than for an angle of 70°.

7. If we keep the hypotenuse the same length, the opposite side is smaller for an angle of 15° than for an angle of 50°, and the adjacent side is greater for an angle of 15° than for an angle of 50°. Since the tangent ratio is $\frac{\text{opp}}{\text{adj}}$, the tangent ratio for an angle of 15° is smaller than for an angle of 50°.

8. a) $\sin 58^\circ$ b) $\cos 35^\circ$ c) $\tan 86^\circ$ d) $\cos 29^\circ$ e) $\sin 74^\circ$ f) $\tan 60^\circ$

9.

Angle x°	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
$\cos x^\circ$	1	0.98	0.94	0.87	0.77	0.64	0.5	0.34	0.17	0

As the angle x° increases from 0° to 90°, $\cos x^\circ$ decreases from 1 to 0.

- 10 a)

Angle x°	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
$\tan x^\circ$	0	0.18	0.36	0.58	0.84	1.19	1.73	2.75	5.67	not defined

- b) $\tan 89^\circ = 57.29$ $\tan 89.9^\circ = 572.96$ $\tan 89.99^\circ = 5729.58$

- c) As the angle x° increases from 0° to 90°, $\tan x^\circ$ increases from 0 to infinity.

11. a) $\sin A = \frac{4}{5}$, $\sin B = \frac{3}{5}$, $\cos A = \frac{3}{5}$, $\cos B = \frac{4}{5}$, $\tan A = \frac{4}{3}$, $\tan B = \frac{3}{4}$.

- b) $\sin A = \cos B$, $\cos A = \sin B$ c) they are reciprocals

12. b) $(90 - x)^\circ$ c) $\cos x^\circ = \frac{PQ}{QR}$ $\sin(90 - x)^\circ = \frac{PQ}{QR}$ so $\cos x^\circ = \sin(90 - x)^\circ$

- d) i) $\cos 76^\circ = \sin 14^\circ$ ii) $\cos 13^\circ = \sin 77^\circ$

13. D

14.

2	.	4	
---	---	---	--