

Lesson 1: Trigonometric Ratios

Friday, August 31, 2018 2:21 AM

Trigonometry Lesson #1: Trigonometric Ratios

Overview of Unit

Trigonometry (from the Greek trigonon = three angles and metro = measure) is a branch of mathematics dealing with angles, triangles and trigonometric functions such as sine, cosine and tangent. In this unit we study relationships between sides and angles in right triangles.

Investigation #1

The diagram shows a series of similar right triangles, $\triangle OA_1B_1$, $\triangle OA_2B_2$, etc.

Complete the work below using a ruler to measure the indicated sides to 1 decimal place.

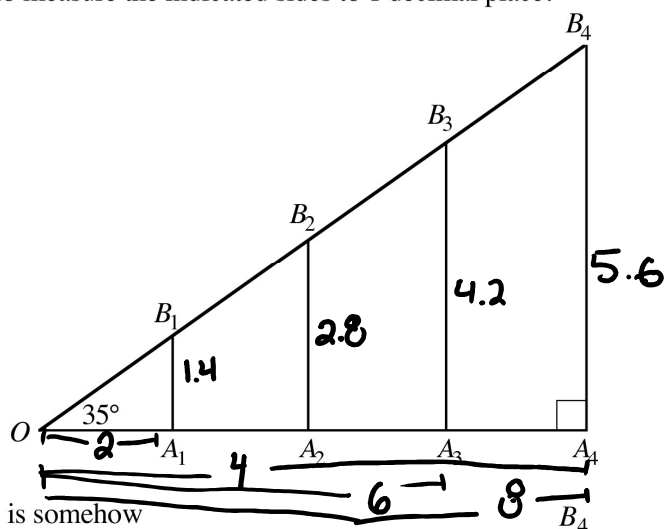
Calculate each ratio to 1 decimal place.

$$\frac{A_1B_1}{OA_1} = \frac{1.4}{2} = 0.7$$

$$\frac{A_2B_2}{OA_2} = \frac{2.8}{4} = 0.7$$

$$\frac{A_3B_3}{OA_3} = \frac{4.2}{6} = 0.7$$

$$\frac{A_4B_4}{OA_4} = \frac{5.6}{8} = 0.7$$



We can conclude that the ratio 0.7 is somehow connected to the angle of 35° .

Investigation #2

$$\frac{A_1B_1}{OA_1} = \frac{2.4}{1.5} = 1.6$$

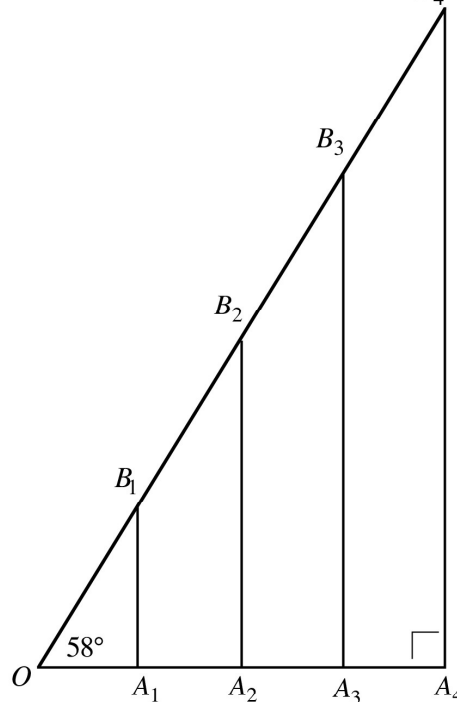
$$\frac{A_2B_2}{OA_2} = \frac{3}{3} =$$

$$\frac{A_3B_3}{OA_3} =$$

$$\frac{A_4B_4}{OA_4} =$$

We can conclude that the ratio 1.6 is somehow connected to the angle of 58° .

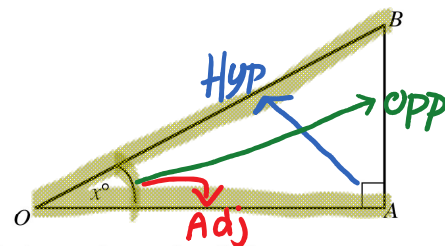
We will investigate these ratios further in the rest of the lesson.



Ratios of Sides in a Right Triangle

Consider the right triangle AOB shown.

Let angle $AOB = x^\circ$.



Each of the sides of the triangle is given a special name relative to the angle of x° .

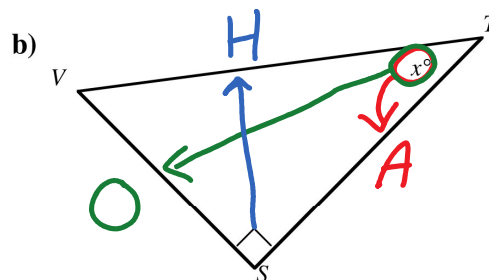
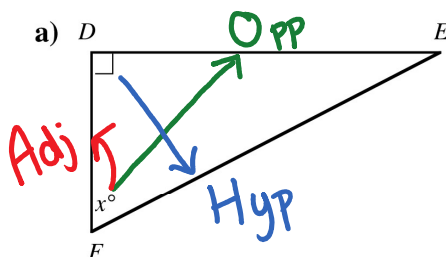
The longest side, OB , is called the **HYPOTENUSE** (hyp). *across from right angle 90°*

The side opposite the angle of x° , AB , is called the **OPPOSITE** (opp). *across from given angle*

The third side of the triangle, OA , is called the **ADJACENT** (adj). *beside the given angle*



Mark on each of these triangles the hypotenuse (hyp), the opposite (opp), and the adjacent (adj) relative to the angle of x° .



- The ratio $\frac{AB}{OA}$ in the diagram at the top of this page is the same ratio we used in Investigations #1 and #2.
- This ratio can be written as $\frac{\text{opposite}}{\text{adjacent}}$ or $\frac{\text{opp}}{\text{hyp}}$ and is known as the tangent ratio.
- There are ~~five other ratios~~ possible using two of the sides of triangle AOB . All six ratios are listed below.

$$\frac{AB}{OB} = \frac{\text{opposite}}{\text{hypotenuse}} = \text{the sine ratio}$$

$$\frac{OA}{OB} = \frac{\text{adjacent}}{\text{hypotenuse}} = \text{the cosine ratio}$$

$$\frac{AB}{OA} = \frac{\text{opposite}}{\text{adjacent}} = \text{the tangent ratio}$$

The three ratios above left are the primary trigonometric ratios and will be studied in this course.

$$\frac{OB}{AB} = \frac{\text{hypotenuse}}{\text{opposite}} = \text{the cosecant ratio}$$

$$\frac{OB}{OA} = \frac{\text{hypotenuse}}{\text{adjacent}} = \text{the secant ratio}$$

$$\frac{OA}{AB} = \frac{\text{adjacent}}{\text{opposite}} = \text{the cotangent ratio}$$

The three ratios above right are the reciprocal trigonometric ratios and will be studied in higher level math courses.

SOH CAH TOA
 S: sine, O: opposite, H: hypotenuse
 C: cosine, A: adjacent, H: hypotenuse
 T: tangent, A: adjacent, O: opposite

$$\sin = \frac{O}{H}$$

SOH CAH TOA

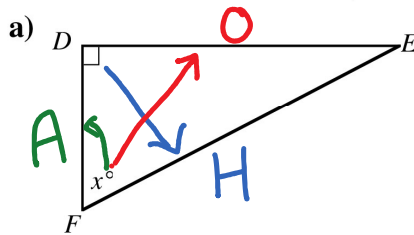
$$\cos = \frac{A}{H}$$

$$\tan = \frac{O}{A}$$

The rules for determining the sine ratio, the cosine ratio and tangent ratio for an angle in a right triangle can be memorized by using the acronym SOH CAH TOA.



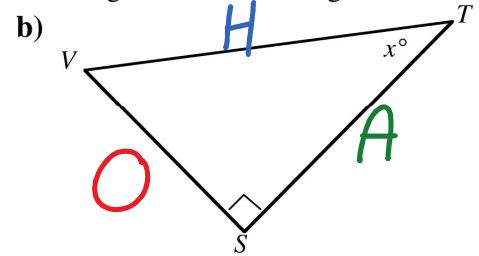
The diagrams shown below are from Class Example #1. Use these diagrams to complete the work below to list the sine ratio, the cosine ratio, and the tangent ratio for the angle of x° .



sine ratio for the angle of $x^\circ = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{DE}{FE}$
SOH

cosine ratio for the angle of $x^\circ = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{DF}{FE}$
CAH

tangent ratio for the angle of $x^\circ = \frac{\text{opposite}}{\text{adjacent}} = \frac{DE}{FD}$
TOA



sine ratio for the angle of $x^\circ = \frac{O}{H} = \frac{VS}{VT}$
SOH

cosine ratio for the angle of $x^\circ = \frac{A}{H} = \frac{ST}{VT}$
CAH

tangent ratio for the angle of $x^\circ = \frac{O}{A} = \frac{VS}{ST}$
TOA

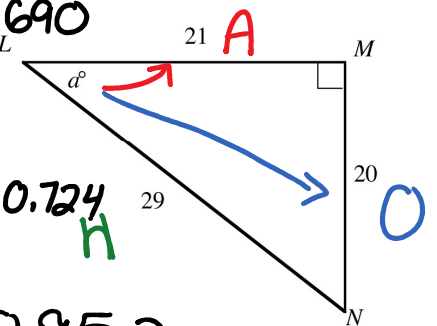


Complete the work below to express the three primary trigonometric ratios, relative to angle a° , as rational numbers and in decimal form to the nearest thousandth, **3 decimal places**.

sine ratio relative to angle $a^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{20}{29} = 0.690$
SOH

cosine ratio relative to angle $a^\circ = \frac{\text{Adj}}{\text{Hyp}} = \frac{21}{29} = 0.724$
CAH

tangent ratio relative to angle $a^\circ = \frac{\text{Opp}}{\text{Adj}} = \frac{20}{21} = 0.952$
TOA



Complete Assignment Questions #1 - #3

Using sin, cos, tan



Consider right triangle PQR shown.

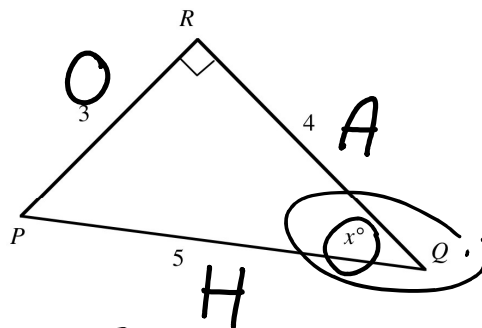
The sine ratio for the angle of x° is $\frac{PR}{PQ} = \frac{3}{5}$.

In short we write $\sin x^\circ = \frac{3}{5}$.

• Similarly $\cos x^\circ = \frac{A}{CAH} = \frac{4}{5}$ and $\tan x^\circ = \frac{TOA}{CAH} = \frac{3}{4}$



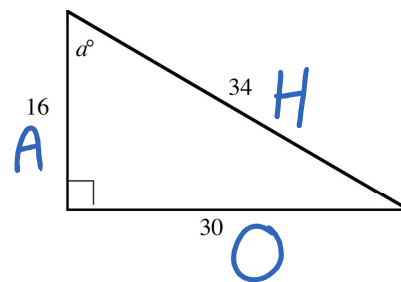
Sometimes the trigonometric ratios for an angle are given in terms of the letter at which the angle is located. For example, in $\triangle PQR$ above we could write $\sin Q = \frac{3}{5}$.



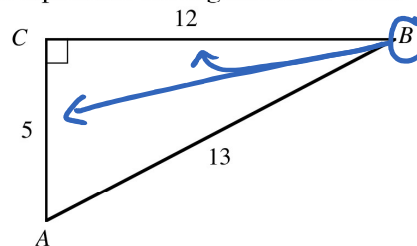
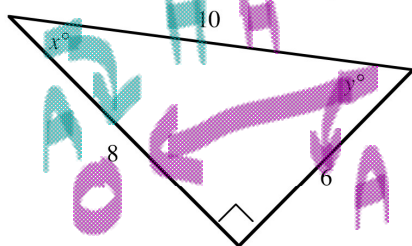
Complete the following, writing the ratio in simplest rational form (fraction).

a) $\sin a^\circ = \frac{30}{34} = \frac{15}{17}$ b) $\cos a^\circ = \frac{16}{34} = \frac{8}{17}$

c) $\tan a^\circ = \frac{30}{16} = \frac{15}{8}$



Write the rational number, in simplest form, which represents the trigonometric ratio.



i) $\sin x^\circ = \frac{6}{10} = \frac{3}{5}$ ii) $\tan y^\circ = \frac{8}{6} = \frac{4}{3}$ iii) $\cos A = \frac{5}{13}$

iv) $\tan B = \frac{5}{12}$

v) $\sin y^\circ = \frac{8}{10} = \frac{4}{5}$ vi) $\cos x^\circ = \frac{8}{10} = \frac{4}{5}$ vii) $\cos B =$

viii) $\sin A =$

Complete Assignment Questions #4 - #11

HW Quiz next day

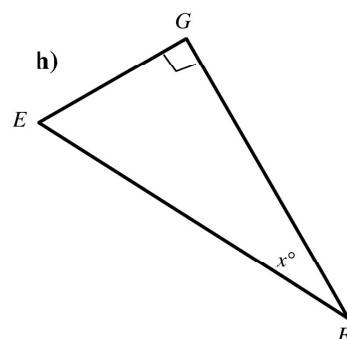
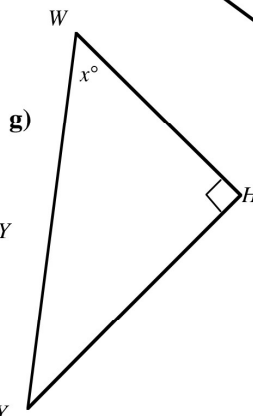
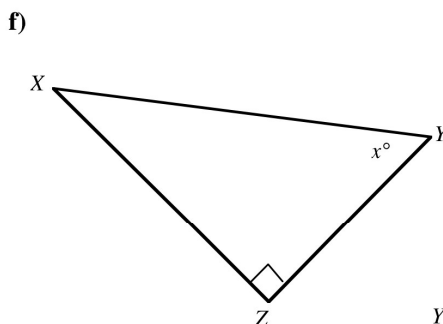
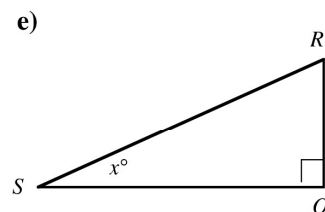
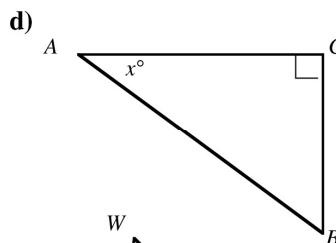
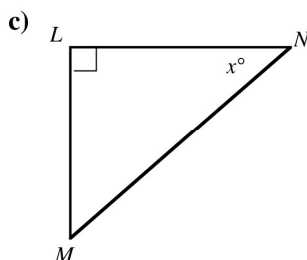
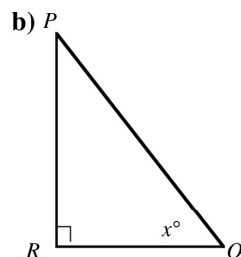
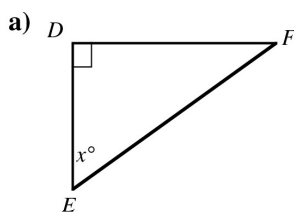
HW Quiz next day
#1 a c e g, 2 b, 3, 4, 6 a c e, 7

Assignment

1. Consider the eight triangles below. For each triangle, complete the tables for the angle x° .

Triangle	Opposite Side	Adjacent Side	Hypotenuse
a)		DE	EF
b)			
c)			
d)			
e)			
f)			
g)			
h)			

Triangle	sine ratio	cosine ratio	tangent ratio
a)		$\frac{DE}{EF}$	
b)			
c)			
d)			
e)			
f)			
g)			
h)			



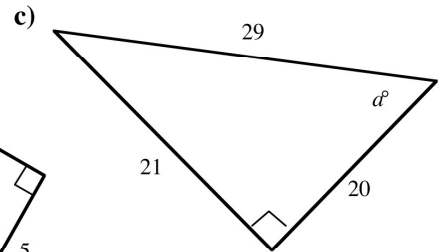
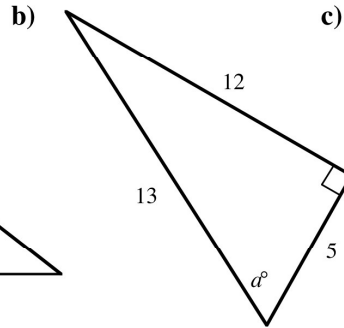
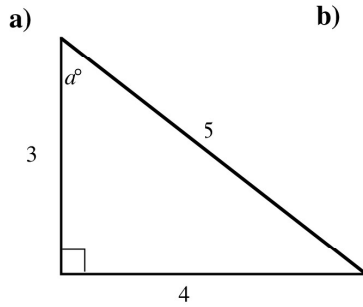
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2. On each triangle mark the (hyp), (opp) and (adj) relative to angle a° and determine the trigonometric ratios associated for angle a° .

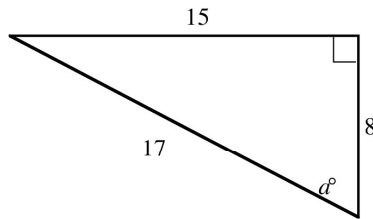
Complete the table.

Triangle	sine ratio	cosine ratio	tangent ratio
a)	$\frac{4}{5}$		
b)			
c)			

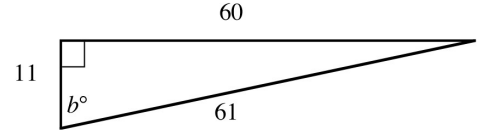


3. Determine, as a fraction in simplest form, the value of the trigonometric ratio indicated.

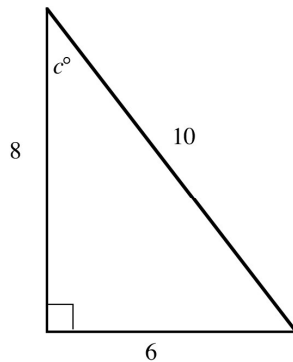
- a) cosine ratio for the angle a°



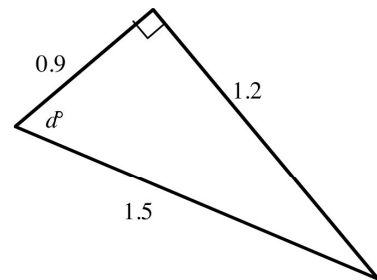
- b) sine ratio relative to angle b°



- c) tangent ratio relative to angle c°

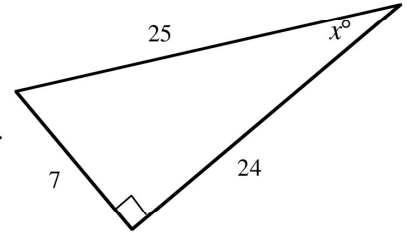


- d) sine ratio for the angle d°

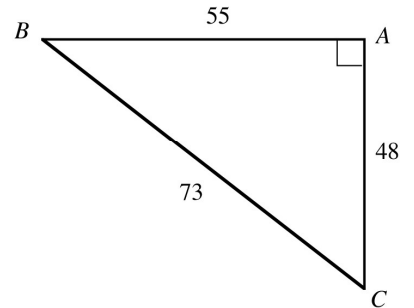


4. On the triangle, mark the hyp, opp, and adj for the angle x° and determine the values of $\sin x^\circ$, $\cos x^\circ$, and $\tan x^\circ$.

Write each answer as a decimal to the nearest hundredth.

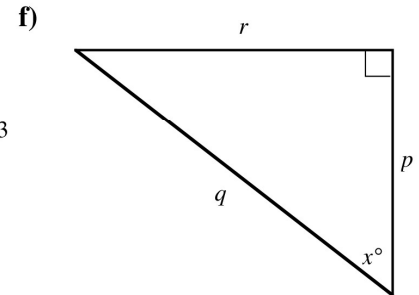
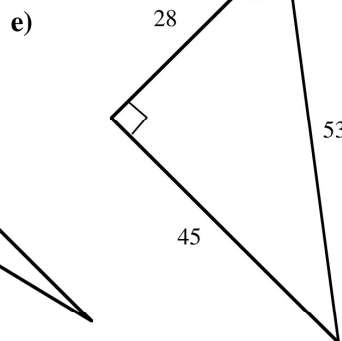
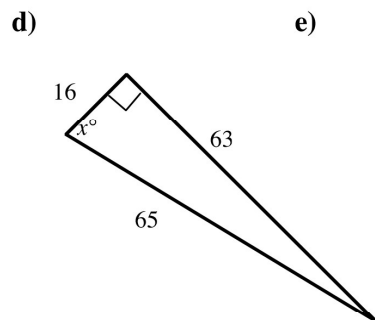
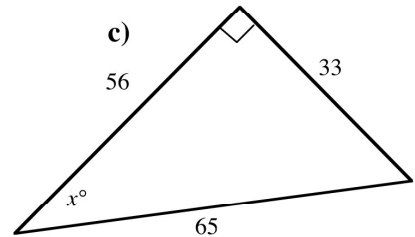
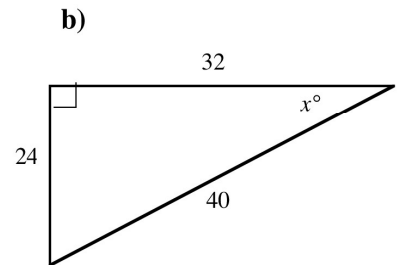
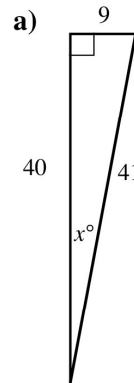


5. In the triangle, determine the exact values of $\sin B$, $\cos B$, and $\tan B$.



6. In each triangle, determine the values of $\sin x^\circ$, $\cos x^\circ$, and $\tan x^\circ$ in simplest rational form.

Triangle	$\sin x^\circ$	$\cos x^\circ$	$\tan x^\circ$
a)			
b)			
c)			
d)			
e)			
f)			



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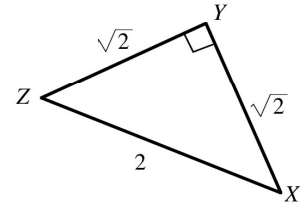
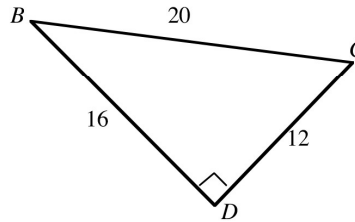
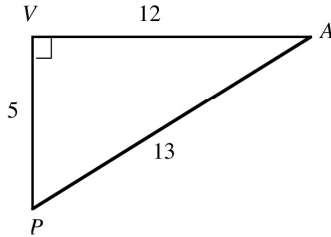
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7. In each case, write the rational number which represents the trigonometric ratio.

a) $\sin A =$

b) $\cos B =$

c) $\tan X =$



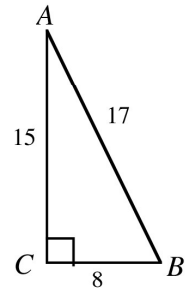
Multiple Choice

8. In right triangle ABC , $AB = 52$ units, $AC = 48$ units and $BC = 20$ units. The value of $\cos B$ and $\sin B$ are respectively

- A. $\frac{5}{13}$ and $\frac{12}{13}$
- B. $\frac{12}{13}$ and $\frac{5}{13}$
- C. $\frac{5}{12}$ and $\frac{12}{5}$
- D. $\frac{5}{13}$ and $\frac{12}{5}$

9. For the right angled triangle ABC , only one of the following ratios is correct. The correct ratio is

- A. $\sin A = \frac{8}{15}$
- B. $\cos A = \frac{8}{17}$
- C. $\tan B = \frac{8}{15}$
- D. $\sin B = \frac{15}{17}$



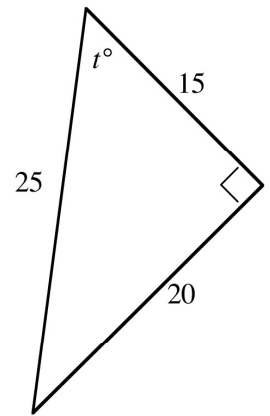
10. In a right triangle $\tan x^\circ = \frac{7}{5}$. A student claims this indicates that in the right triangle the side opposite to the angle x° is 7 units and the side adjacent to the angle x° is 5 units.

The student's claim

- A. is always true
- B. is always false
- C. may be true or false.
- D. depends on the value of x°

Numerical Response

11. In the diagram, to the nearest tenth, the value of $\frac{\sin t^\circ}{\cos t^\circ}$ is _____.



(Record your answer in the numerical response box from left to right)

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Answer Key

1.

Triangle	Opposite Side	Adjacent Side	Hypotenuse
a)	DF	DE	EF
b)	PR	QR	PQ
c)	LM	LN	MN
d)	BC	AC	AB
e)	OR	OS	RS
f)	XZ	YZ	XY
g)	HY	HW	WY
h)	EG	FG	EF

Triangle	sine ratio	cosine ratio	tangent ratio
a)	$\frac{DF}{EF}$	$\frac{DE}{EF}$	$\frac{DF}{DE}$
b)	$\frac{PR}{PQ}$	$\frac{QR}{PQ}$	$\frac{PR}{QR}$
c)	$\frac{LM}{MN}$	$\frac{LN}{MN}$	$\frac{LM}{LN}$
d)	$\frac{BC}{AB}$	$\frac{AC}{AB}$	$\frac{BC}{AC}$
e)	$\frac{OR}{RS}$	$\frac{OS}{RS}$	$\frac{OR}{OS}$
f)	$\frac{XZ}{XY}$	$\frac{YZ}{XY}$	$\frac{XZ}{YZ}$
g)	$\frac{HY}{WY}$	$\frac{HW}{WY}$	$\frac{HY}{HW}$
h)	$\frac{EG}{EF}$	$\frac{FG}{EF}$	$\frac{EG}{FG}$

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2.

Triangle	sine ratio	cosine ratio	tangent ratio
a)	$\frac{4}{5}$	$\frac{3}{5}$	$\frac{4}{3}$
b)	$\frac{12}{13}$	$\frac{5}{13}$	$\frac{12}{5}$
c)	$\frac{21}{29}$	$\frac{20}{29}$	$\frac{21}{20}$

3. a) $\frac{8}{17}$ b) $\frac{60}{61}$ c) $\frac{3}{4}$ d) $\frac{4}{5}$

4. $\sin x^\circ = 0.28$ $\cos x^\circ = 0.96$ $\tan x^\circ = 0.29$

5. $\sin B = \frac{48}{73}$ $\cos B = \frac{55}{73}$ $\tan B = \frac{48}{55}$

6.

Triangle	$\sin x^\circ$	$\cos x^\circ$	$\tan x^\circ$
a)	$\frac{9}{41}$	$\frac{40}{41}$	$\frac{9}{40}$
b)	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{3}{4}$
c)	$\frac{33}{65}$	$\frac{56}{65}$	$\frac{33}{56}$
d)	$\frac{63}{65}$	$\frac{16}{65}$	$\frac{63}{16}$
e)	$\frac{45}{53}$	$\frac{28}{53}$	$\frac{45}{28}$
f)	$\frac{r}{q}$	$\frac{p}{q}$	$\frac{r}{p}$

7. a) $\frac{5}{13}$ b) $\frac{4}{5}$ c) 1

8. A 9. D 10. C

11.

1	.	3	
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