Lesson 4: Difference of Squares

Friday, August 31, 2018 2:37 AM

Factoring Polynomial Expressions Lesson #4:
Difference of Squares

Investigation

a) Complete the following using the trinomial factoring method from the previous lessons.

	Sum (+)	Product (×)	Integers	Polynomial	Factored Form
i)	–6	–16	−8, 2	$x^2 - 6x - 16$	$(x-8)(x+2)$
ii)	–15	–16	−16, 1	$x^2 - 15x - 16$	$(x-16)(x+1)$
iii)	0	–16	−4, 4	$x^2 + 0x - 16 = x^2 - 16$	$(x-4)(x+4)$
iv)	0	–64	−8, 8	$x^2 + 0x - 64 = x^2 - 64$	$(x-8)(x+8)$
v)	0	–25	−5, 5	$x^2 - 25$	$(x-5)(x+5)$

b) The third row in a) shows that the factored form of $x^2 - 16$ is $(x-4)(x+4)$. Use the pattern from the last three rows to factor the following.

i) $x^2 - 9 =$ ($x^2 - 3^2$) $(x-3)(x+3)$

ii) $x^2 - 49 =$ ($x^2 - 7^2$) $(x-7)(x+7)$

iii) $x^2 - 36 =$ ($x^2 - 6^2$) $(x-6)(x+6)$

iv) $x^2 - 1 =$ ($x^2 - 1^2$) $(x-1)(x+1)$

v) $a^2 - 100 =$ ($a^2 - 10^2$) $(a-10)(a+10)$

c) Extend the procedure from above to factor $m^2 - n^2$. Verify your answer by expanding the factored form.

$(m+n)(m-n)$

Difference of squares

d) Consider the expansion $(x - y)(x + y) = x^2 + bx + c$.

i) Explain why the value of b is zero.

ii) Express c in terms of y.

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Difference of Squares

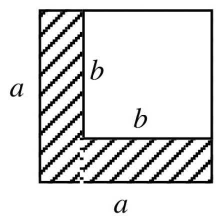
The examples on the previous page are trinomials of the form $x^2 + bx + c$, where $b = 0$ and c is the negative of a square number.

This results in a difference of squares such as $x^2 - 25$, $x^2 - 100$, etc.

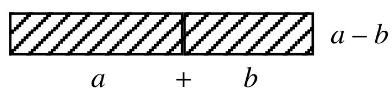
To factor a difference of squares we can use the identity:

$$a^2 - b^2 = (a - b)(a + b)$$

The identity $a^2 - b^2 = (a - b)(a + b)$ can be illustrated in the following diagram.



Shaded area = $a^2 - b^2$



Shaded area = $(a - b)(a + b)$

The shaded area on the left is cut along the dotted line and rearranged to form the diagram on the right.
The shaded area on the left is represented by $a^2 - b^2$ *and the shaded area on the right is represented by* $(a - b)(a + b)$.

Factor the following polynomials using the difference of squares method.

a) $a^2 - 4$

$a^2 - 2^2$

$(a+2)(a-2)$

b) $t^2 - 144$

$= t^2 - 12^2$

$= (t-12)(t+12)$

*two squares & subtract inbetween

c) $x^2 - y^2$

$= (x-y)(x+y)$

d) $p^2 - 7^2$

$= (p-7)(p+7)$

Note that it is not possible to factor a sum of squares like $x^2 + 4$, i.e. $x^2 + 0x + 4$.
It is not possible to find two integers whose product is positive and whose sum is zero.

In the identity $a^2 - b^2 = (a - b)(a + b)$ we can replace a and/or b by numbers, variables, monomials and even polynomials.

For example, $4x^2 - 25$ can be written as $(2x)^2 - (5)^2$ and can be factored using the above identity with $a = 2x$ and $b = 5$.

$$4x^2 - 25 = (\quad)(\quad)$$

$9m^2 - 4n^2$ can be written as $(3m)^2 - (2n)^2$, and can be factored using the above identity with $a = 3m$ and $b = 2n$.

$$9m^2 - 4n^2 = (\quad)(\quad)$$

The factoring above can be verified by expanding the product of the factors.

Class Ex. #2

Factor, if possible, using the difference of squares method.

a) $16t^2 - 49$ **b)** $81a^2 - 1$ **c)** $100 - y^2$

d) $36p^2 - 25q^2$ **e)** $4x^2 + 25$ **f)** $64 - 9a^2b^2$

Class Ex. #3

The floor of an international doubles squash court is rectangular with an area of $25a^2 - b^2$ square feet.

a) Write expressions for the length and width of the floor.

b) The perimeter of the floor is 140 feet. Determine the length and width of the floor if the length is 1.8 times the width.

Difference of Squares involving a Common Factor

The first step in factoring any polynomial expression should be to determine if we can remove a common factor.

Factor the following polynomials by first removing the greatest common factor.

a) $2a^2 - 50$ GCF: 2 **b)** $3x^2 - 12y^2$ **c)** $144p^2q^2 - 4$ **d)** $3x^3 - 27x$ GCF: 3x

$2(a^2-25)$ $=2(a-5)(a+5)$

$3(x^2-4y^2)$ $=3(x-2y)(x+2y)$

$(12pq)^2-2^2$ $(12pq-2)(12pq+2)$

$\frac{3x^3}{3x} - \frac{27x}{3x}$ $=3x(x^2-9)$ $=3x(x-3)(x+3)$

Complete Assignment Questions #1 - #14

Assignment

1c, 5ab, (6,7) every 2nd letter

1. Complete the following by determining the missing factor.

a) $x^2 - 36 = (x - 6)(\quad)$ **b)** $c^2 - 121 = (c + 11)(\quad)$ **c)** $j^2 - k^2 = (j - k)(\quad)$

2. Factor the following polynomials using a difference of squares.

a) $x^2 - 49$ **b)** $x^2 - 1$ **c)** $x^2 - 15^2$ **d)** $x^2 - 400$

3. Explain how factoring a difference of squares in one variable can be regarded as a special case of factoring trinomials by inspection.

4. Factor where possible.

a) $m^2 - n^2$ **b)** $c^2 - 7^2$ **c)** $1 - k^2$ **d)** $g^2 - 64h^2$

e) $25x^2 - 144$ **f)** $16a^2 - 9b^2$ **g)** $4x^2 + z^2$ **h)** $121a^2 - 36b^2$

i) $49 - 4h$ **j)** $100 - 81b^2$ **k)** $1 - 25z^2$ **l)** $225a^2 - b^2$

m) $169z^2 - 4q^2$ **n)** $256 - y^2$ **o)** $t^2 + 36z^2$ **p)** $49a^2 - 400$

5. The floor of a classroom is rectangular with an area of $81m^2 - 4n^2$ square metres.

a) Write expressions in m and n for the length and width of the floor.

b) If the perimeter of the floor is 72 metres, form an equation in m and n and solve for m.

c) Determine the length and width of the floor if the length is 25% greater the width.

6. Factor.

a) $8x^2 - 32$ **b)** $4a^2 - 100y^2$ **c)** $3t^2 + 27s^2$

d) $7x^2 - 7y^2$ **e)** $9a^2b^2 - 36$ **f)** $8 - 50p^2q^2$

g) $xy^2 - x^3$ **h)** $20a^2b^2 - 5a^4b^4$

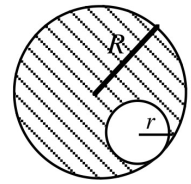
7. Factor.

a) $a^2b^2 - 9$ **b)** $c^2 - d^2e^2$ **c)** $100x^2 - y^2z^2$

d) $p^2q^2 - r^2s^2$ **e)** $25x^2y^2 - 1$ **f)** $c^2d^2 - 4f^2$

g) $4x^2a^2 - 49z^2t^2$ **h)** $16a^2c^2 - 225b^2d^2$

8. The diagram shows a circle of radius R with a circle of radius r removed.



a) Write an expression for the shaded area.

b) Write the expression in a) in factored form.

c) Determine the shaded area (as a multiple of π) if $R = 8.5$ and $r = 1.5$.
Do not use a calculator.

9. The expression $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$ occurs in physics.

a) Write the expression in factored form.

b) Determine the value of the expression when $m = 10$, $v = 75$, and $u = 25$.
Do not use a calculator.

10. Consider the following in which each letter represents a whole number.

$64x^2 - y^2 = (Hx - y)(Hx + y)$ $\qquad$ $16x^2 - 4 = C(Ix + 1)(Ix - 1)$

$7x^2 - 252y^2 = P(x - Ey)(x + Ey)$ $\qquad$ $Lx^2 - Ny^2 = (3x - 5y)(Sx + Ay)$

Determine the value of each letter and hence name the country represented by the following code.

(4) (8) (2) (9) (6)

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11. Susan was showing Rose how the difference of squares method can be used to multiply certain numbers without using a calculator. She showed Rose the following:

$$38 \times 42$$
$$= (40 - 2)(40 + 2) \quad = (40^2 - 2^2) \quad = (1600 - 4) \quad = 1596$$

a) Use the above process to evaluate:

i) 27×33 ii) 61×59

b) Explain why this process is more difficult to determine the product 66×72.

c) Make up your own multiplication question which can be answered using this process.

Multiple Choice

12. One factor of $16 - 4m^2$ is

A. $4 - m$
B. $8 - 2m$
C. $4 + m$
D. $2 + m$

13. Given that $x^2 - y^2 = 45$ and $x + y = 9$, the value of x is

A. 2
B. 5
C. 7
D. impossible to determine

Numerical Response

14. $3x + 2y$ is a factor of the binomial $a^2x^2 - b^2y^2$.
The value of $a^2 + b^2$ is _____ .
(Record your answer in the numerical response box from left to right)

Answer Key

1. a) $(x+6)$ **b)** $(c-11)$ **c)** $(j+k)$

2. a) $(x-7)(x+7)$ **b)** $(x-1)(x+1)$ **c)** $(x-15)(x+15)$ **d)** $(x-20)(x+20)$

3. A difference of squares can be regarded as a trinomial of the form x^2+bx+c in which $b=0$ and c is negative. We need to find two numbers which multiply to c and add to zero.

4. a) $(m-n)(m+n)$ **b)** $(c-7)(c+7)$ **c)** $(1-k)(1+k)$
d) $(g-8h)(g+8h)$ **e)** $(5x-12)(5x+12)$ **f)** $(4a-3b)(4a+3b)$
g) not factorable **h)** $(11a-6b)(11a+6b)$ **i)** not factorable using whole number exponent.
j) $(10-9b)(10+9b)$ **k)** $(1+5z)(1-5z)$ **l)** $(15a+b)(15a-b)$
m) $(13z-2q)(13z+2q)$ **n)** $(16-y)(16+y)$ **o)** not factorable **p)** $(7a+20)(7a-20)$

5. a) $(9m+2n)$ metres, $(9m-2n)$ metres **b)** $2(9m+2n)+2(9m-2n)=72, \quad m=2$
c) Length = 20 metres, Width = 16 metres.

6. a) $8(x-2)(x+2)$ **b)** $4(a-5y)(a+5y)$ **c)** $3(t^2+9s^2)$ **d)** $7(x-y)(x+y)$
e) $9(ab-2)(ab+2)$ **f)** $2(2-5pq)(2+5pq)$ **g)** $x(y-x)(y+x)$ **h)** $5a^2b^2(2-ab)(2+ab)$

7. a) $(ab-3)(ab+3)$ **b)** $(c-de)(c+de)$ **c)** $(10x-yz)(10x+yz)$
d) $(pq-rs)(pq+rs)$ **e)** $(5xy-1)(5xy+1)$ **f)** $(cd-2f)(cd+2f)$
g) $(2xa-7zt)(2xa+7zt)$ **h)** $(4ac-15bd)(4ac+15bd)$

8. a) $A=\pi R^2-\pi r^2$ **b)** $\pi(R-r)(R+r)$ **c)** 70π

9. a) $\frac{1}{2}m(v-u)(v+u)$ **b)** 25 000 **10.** CHILE

11. a) i) 891 **ii)** 3599
b) 66×72 expressed as a difference of squares $(69^2 - 3^2)$ cannot easily be evaluated without a calculator or long multiplication.

12. D **13.** C **14.**

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