

Lesson 3: Factoring Trinomials of the Form $x^2 + bx + c$ - Part Two

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Factoring Polynomial Expressions Lesson #3:

Factoring Trinomials of the Form $x^2 + bx + c$ - Part Two

Review of Factoring By Inspection

In order to factor $x^2 + bx + c$ by inspection, we need to find two integers which have a product equal to c and a sum equal to b . If no two such integers exist, then the polynomial cannot be factored.

In order to factor $x^2 + 6x + 9$, we need to find two numbers whose product is $\frac{9}{\times}$ and whose sum is $\frac{6}{+}$.
 $\frac{9}{\times} \rightarrow 3, 3$

In order to factor $x^2 + x - 12$, we need to find two numbers whose product is $\frac{-12}{\times}$ and whose sum is $\frac{1}{+}$.
 $\frac{-12}{\times} \rightarrow -4, 3$

Recall the following points from the previous lesson.

- If the **product is positive**, then the two integers must be either **both positive** or **both negative**.
- If the **product is negative**, then one integer is **positive** and the other is **negative**.

$$\begin{matrix} + \cdot - = - \\ + \cdot + = + \text{ OR } - \cdot - = + \end{matrix}$$



Class Ex. #1

Factor the following trinomials by inspection.

a) $x^2 - x - 12$ $\frac{-12}{\times} \frac{-1}{+}$ $(x+3)(x-4)$ $\frac{12}{2 \ 6} \frac{-1}{3 \ -4}$

b) $x^2 + 3x - 18$ $\frac{-18}{\times} \frac{3}{+}$ $(x-3)(x+6)$ $\frac{18}{2 \ 9} \frac{3}{-3 \ 6}$

c) $a^2 - 7a - 8$ $\frac{-8}{\times} \frac{-7}{+}$ $(a+1)(a-8)$ $\frac{8}{2 \ 4} \frac{-7}{-1 \ -7}$



Class Ex. #2

Factor where possible.

a) $-a^2 - 6a + 27$ $\frac{-27}{\times} \frac{6}{+}$ $-(a^2 + 6a - 27)$ $\frac{27}{1 \ 27} \frac{6}{-3 \ 9}$ $-(a-3)(a+9)$

b) $2t^2 - 14t + 20$ $\text{GCF: } 2$ $\frac{10}{\times} \frac{-7}{+}$ $2(t^2 - 7t + 10)$ $\frac{10}{-5 \ 2} \frac{-7}{-7}$ $2(t-2)(t-5)$

c) $x^2 - 3x - 6$ **not possible**

d) $4x^4 - 16x^3 - 20x^2$ $\text{GCF: } 4x^2$ $\frac{-5}{\times} \frac{-4}{+}$ $= 4x^2(x^2 - 4x - 5)$ $\frac{-5}{\times} \frac{-4}{+}$ $= 4x^2(x-5)(x+1)$

Complete Assignment Questions #1 - #5

Factoring Trinomials of the form $x^2 + bxy + cy^2$

Complete the following statements:

i) $(x+2)(x+4)$ can be expanded to $x^2 + 6x + 8$,
 so $x^2 + 6x + 8$ can be factored to $(x+2)(x+4)$.

ii) $(x+2y)(x+4y)$ can be expanded to $x^2 + 4xy + 2xy + 8y^2$
 so $x^2 + 6xy + 8y^2$ can be factored to $(x+2y)(x+4y)$.



Factor.

a) $x^2 + 13xy + 30y^2$
 $(x+3y)(x+10y)$
 $\begin{array}{r|l} 30 & 13 \\ \hline 3 \cdot 10 & 13 \\ 2 & 15 \end{array}$

b) $x^2 + 71xy - 72y^2$

c) $3a^2 - 15ab - 252b^2$ GCF: 3
 $3(a^2 - 5ab - 84b^2)$
 $3(a-12b)(a+7b)$
 $\begin{array}{r|l} -84 & -5 \\ \hline 1 & 84 \\ 2 & 42 \\ 3 & 28 \\ 4 & 21 \\ 7 & 12 \end{array}$

Complete Assignment Questions #6 - #11

$(1, 3, 4)$ every 2nd letter, 5, 6a, 7a

Assignment

1. Complete the table to find two numbers with the given sum and the given product.

	Sum	Product	Integers
a)	8	-20	
b)	-8	-20	
c)	-1	-20	

	Sum	Product	Integers
d)	3	-70	
e)	-11	28	
f)	0	-16	

2. Factor the following trinomials.

a) $x^2 - 2x - 15$

b) $x^2 - 2x - 24$

c) $x^2 + 2x - 24$

d) $x^2 + 2x - 3$

e) $x^2 + x - 30$

f) $x^2 - 3x - 10$

3. Factor where possible.

a) $x^2 + 10x + 16$ b) $x^2 - 11x + 18$ c) $x^2 - 2x - 8$ d) $x^2 + 3x - 18$

e) $x^2 - 4x + 12$ f) $x^2 - 4x - 12$ g) $x^2 - 10x + 25$ h) $x^2 + x - 20$

i) $m^2 + 21m + 38$ j) $a^2 - 17a + 42$ k) $p^2 - 10p - 9$ l) $p^2 - 9p - 10$

4. Factor.

a) $-x^2 - 7x - 12$ b) $4x^2 - 28x - 32$ c) $5x^2 - 20x + 15$

d) $-2a^2 + 2a + 220$ e) $b^2x^2 - 4b^2x - 45b^2$ f) $2x^3 + 2x^2 - 40x$

5. Consider the following in which the each letter represents a whole number.

$x^2 + 4x - 5 = (x + A)(x - O)$ $x^2 - 3x - 54 = (x - E)(x + I)$

$x^3 + 2x^2 - 8x = x(x - Y)(x + P)$ $3x^2 - 48x + 192 = T(x - R)^2$

$-5x^2 + 20x + 105 = -5(x + T)(x - H)$

Determine the value of each letter and hence name the fictional character represented by the following code.

(7) (5) (8) (8) (2) (4) (1) (3) (3) (9) (8)
 - - - - - - - - - - -

6. Factor.

a) $x^2 + 18xy + 45y^2$ b) $x^2 + 10xy - 24y^2$ c) $a^2 - 12ab + 36b^2$

d) $p^2 - 12pq + 11q^2$ e) $x^2 + xy - 72y^2$ f) $x^2 - 54xy - 112y^2$

7. Factor completely.

a) $4x^2 - 80xy + 144y^2$ b) $3b^2 - 15bv - 72v^2$ c) $2c^2 + 66cd - 140d^2$

Multiple Choice

8. When factored, the trinomials $x^2 - 10x + 21$ and $x^2 - 4x - 21$ have one binomial factor in common. This factor is

- A. $x - 7$ B. $x + 7$
C. $x - 3$ D. $x + 3$

9. One factor of $-m^3 - m^2 + 6m$ is

- A. $m - 2$ B. $m + 2$
C. $m - 3$ D. $m - 6$

10. One factor of $3x^2 - 6xy - 9y^2$ is

- A. $3x$ B. $x + 2y$
C. $x + 3y$ D. $x + y$

11. The expression $x^2 - 4x + c$ **cannot** be factored if c has the value

- A. -5
B. 0
C. 4
D. 5

Answer Key

1. a) $-2, 10$ b) $-10, 2$ c) $-5, 4$ d) $-7, 10$ e) $-4, -7$ f) $-4, 4$
2. a) $(x - 5)(x + 3)$ b) $(x - 6)(x + 4)$ c) $(x + 6)(x - 4)$ d) $(x + 3)(x - 1)$
e) $(x + 6)(x - 5)$ f) $(x - 5)(x + 2)$
3. a) $(x + 8)(x + 2)$ b) $(x - 9)(x - 2)$ c) $(x + 2)(x - 4)$ d) $(x + 6)(x - 3)$
e) not possible f) $(x - 6)(x + 2)$ g) $(x - 5)^2$ h) $(x + 5)(x - 4)$
i) $(m + 2)(m + 19)$ j) $(a - 14)(a - 3)$ k) not possible l) $(p - 10)(p + 1)$
4. a) $-(x + 3)(x + 4)$ b) $4(x - 8)(x + 1)$ c) $5(x - 3)(x - 1)$ d) $-2(a - 11)(a + 10)$
e) $b^2(x - 9)(x + 5)$ f) $2x(x + 5)(x - 4)$ 5. HARRY POTTER
6. a) $(x + 15y)(x + 3y)$ b) $(x - 2y)(x + 12y)$ c) $(a - 6b)^2$ d) $(p - q)(p - 11q)$
e) $(x - 8y)(x + 9y)$ f) $(x + 2y)(x - 56y)$
7. a) $4(x - 18y)(x - 2y)$ b) $3(b - 8v)(b + 3v)$ c) $2(c + 35d)(c - 2d)$
8. A 9. A 10. D 11. D