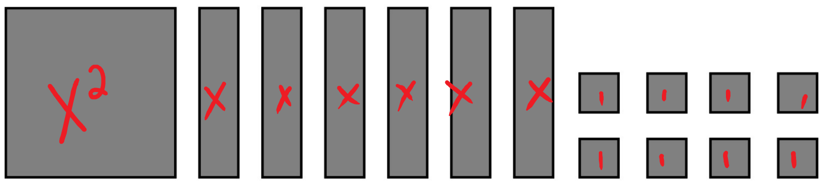
Factoring Polynomial Expressions Lesson #2:
Factoring Trinomials of the Form $x^2 + bx + c$ - Part One

Factoring Trinomials using Algebra Tiles

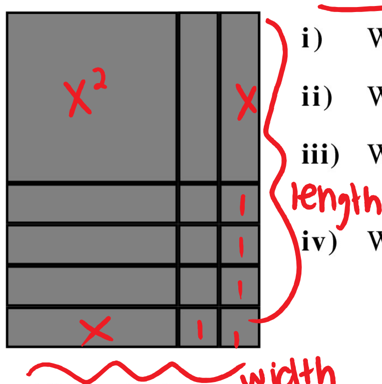
Consider the algebra tile diagram shown.



- Write the polynomial expression which is represented by the algebra tiles.

$x^2 + 6x + 8$

- The algebra tiles can be rearranged into a rectangular form as shown below.



i) Write an expression for the length of the rectangle.

$x+4$

ii) Write an expression for the width of the rectangle.

$x+2$

iii) Write the area of the rectangle as a product of two binomials.

$A = lw = (x+4)(x+2)$

iv) Write the area of the rectangle in expanded form.

$(x+4)(x+2) = x^2 + 2x + 4x + 8$

$= x^2 + 6x + 8$

- The work above provides a method for factoring the trinomial $x^2 + 6x + 8$ into the product of two binomials $(x+2)(x+4)$: i.e. $x^2 + 6x + 8 = (x+2)(x+4)$.

a) Write the polynomial expression which is represented by the algebra tiles.

$x^2 + 5x + 4$

b) Arrange the algebra tiles into a rectangle and write an expression for the length and width of the rectangle.

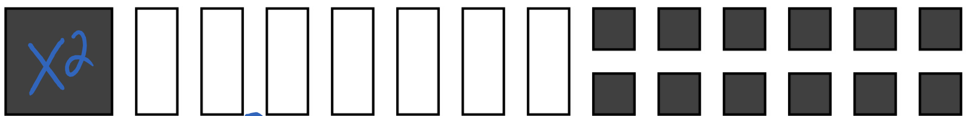
$l = x+1 \quad w = x+4$

c) Use the results above to express the polynomial in factored form.

$x^2 + 5x + 4 = (x+1)(x+4)$

c) Use the results above to express the polynomial in factored form.

a) Write the polynomial expression which is represented by the algebra tiles.



b) Arrange the algebra tiles into a rectangle and express the polynomial in factored form.

Complete Assignment Questions #1 - #3

Investigation: Factoring Trinomials by Inspection

- Expand the following binomials as shown.

$(x + 2)(x + 4) = x^2 + 4x + 2x + 8 = x^2 + 6x + 8$

$(x + 3)(x + 3) =$

$(x + 1)(x + 7) =$

$(x + 5)(x + 2) =$

$(x - 5)(x - 2) =$

$(x + 8)(x - 6) =$

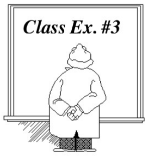
- Consider the expansion $(x + p)(x + q) = x^2 + bx + c$.

In each of the examples above what is the connection between

In each of the examples above what is the connection between

i) the value of b and the values of p and q? $b =$ p+q

ii) the value of c and the values of p and q? $c =$ p·q



Use FOIL to show that $(x + p)(x + q)$ can be written in the form $x^2 + (p + q)x + pq$.

First
Outside
Inside
Last

$x^2 + qx + px + pq$

$x^2 + (q + p)x + pq$

middle term is p+q

last term is p times q

Factoring $x^2 + bx + c$ by Inspection

In order to factor $x^2 + bx + c$ by inspection we need to find two integers which have a product equal to c and a sum equal to b. If no two such integers exist, then the polynomial cannot be factored.

$(x \quad)(x \quad)$

In order to factor $x^2 + 8x + 12$ we need to find two numbers which multiply to 12 and add to 8.

In order to factor $x^2 - 13x + 12$ we need to find two numbers which multiply to 12 and add to -13.

The next example practices this skill.

Complete the tables to find two numbers with the given sum and the given product.

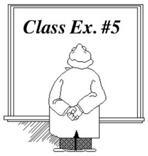
Sum	Product	Integers
12	20	2, 10
9	20	4, 5
4	4	2, 2
–9	18	-3, -6

Sum	Product	Integers
–15	14	-1, -14
–1	–6	2, -3
2	–15	-3, 5
–26	48	-24, -2

Notice that:

- if the product is **positive**, then the two integers must be either **both positive** or **both negative.**
- if the product is **negative**, then one integer is **positive** and the other is **negative**.

For the remainder of this lesson, we will only deal with examples where the product is positive. In the next lesson we will include examples where the product is negative.



Factor the following trinomials where possible.

a) $x^2 + 8x + 12$ $= (x+2)(x+6)$

b) $x^2 + 13x + 12$ $= (x+1)(x+12)$

c) $x^2 - 13x + 12$ $(x-1)(x-12)$

d) $a^2 - 11a + 10$ $= (a-1)(a-10)$

e) $y^2 + 3y + 4$ not possible

f) $x^2 + 27x + 50$ $(x+25)(x+2)$

Factor the polynomial expressions by first removing a common factor.

a) $4x^2 - 32x + 48$ *divide out common factor

$= 4(x^2 - 8x + 12)$

$= 4(x-6)(x-2)$

b) $3x^3 + 21x^2 + 30x$ GCF = 3x

$= 3x(x^2 + 7x + 10)$

$= 3x(x+5)(x+2)$

In this example there were two steps in the factoring process - a common factor followed by a trinomial. If we are asked to factor a polynomial expression, it is understood this means to continue factoring until no further factoring is possible. This is sometimes written as "factor completely ...". The operation "factor" means "factor completely".

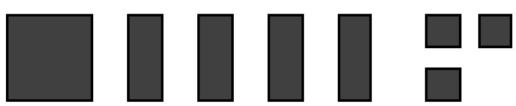
Complete Assignment Questions #4 - #15

Assignment

#2, 3, (4, 5, 6, 10) every 2nd letter 19

8a, 9 extra

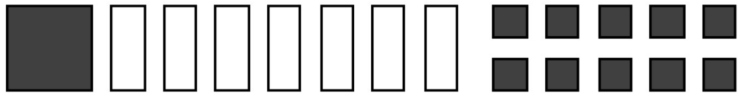
1. a) Write the polynomial expression which is represented by the algebra tiles.



b) Arrange the algebra tiles into a rectangle and write an expression for the length and width of the rectangle.

c) Use the results above to express the polynomial in factored form.

2. a) Write a polynomial expression for the group of algebra tiles shown.



b) Arrange the algebra tiles into a rectangle.

c) State the length and width of the rectangle and hence express the polynomial in factored form.

3. Use algebra tiles to factor the following trinomials.

a) $x^2 + 5x + 6$ **b)** $x^2 + 6x + 5$ **c)** $x^2 - 6x + 8$

4. Complete the tables to find two numbers with the given sum and the given product.

	Sum	Product	Integers
a)	5	6	
b)	8	7	
c)	11	30	
d)	–11	30	

	Sum	Product	Integers
e)	11	10	
f)	–8	15	
g)	–15	56	
h)	–18	56	

5. Complete the following.

a) $x^2 + 7x + 12 = (x + 3)(x + \quad)$ **b)** $x^2 + 9x + 8 = (x + \quad)(x + \quad)$

c) $x^2 - 7x + 10 = (x - 2)(x - \quad)$ **d)** $t^2 - 14t + 24 = (t - \quad)(t - \quad)$

e) $z^2 + 8z + 15 = (z + 5)(\qquad)$ **f)** $b^2 - 12b + 20 = (b - 2)(\qquad)$

6. Factor the following.

a) $x^2 + 3x + 2$ **b)** $x^2 - 3x + 2$ **c)** $x^2 + 9x + 18$

d) $x^2 + 8x + 12$ **e)** $x^2 - 10x + 21$ **f)** $x^2 - 11x + 24$

7. Factor where possible.

a) $x^2 + 11x + 10$ **b)** $x^2 + 10x + 11$ **c)** $n^2 + 12n + 32$

d) $y^2 - 11y + 28$ **e)** $y^2 + 17y + 42$ **f)** $f^2 - 10f + 21$

g) $p^2 - 16p + 28$ **h)** $x^2 + 24x + 80$ **i)** $c^2 - 32c + 60$

j) $a^2 - 12a + 24$ **k)** $d^2 + 18d + 45$ **l)** $p^2 - 29p + 100$

m) $m^2 + 22m + 121$ **n)** $n^2 - 23n + 102$ **o)** $q^2 - 28q + 115$

8. **a)** The expression $x^2 + bx + 12$ can be factored over the integers. Determine all possible values of b.

b) If the expression $x^2 + 6x + c$, where $c > 0$, can be factored over the integers, determine all possible values of c.

9. A volleyball court has an area of $x^2 + 15x + 36$ square metres.

a) Factor $x^2 + 15x + 36$ to find binomials that represent the length and width of the court.

b) If $x = 3$, determine the length and width of the court.

10. Factor.

a) $2x^2 + 6x + 4$ **b)** $4x^2 - 48x + 128$ **c)** $-2a^2 - 30a - 108$

d) $5x^2 - 20x + 15$ **e)** $ax^2 - 14ax + 45a$ **f)** $-10a^4 + 100a^3 - 240a^2$

11. Consider the following in which each letter represents a whole number.

$x^2 + 5x + 6 = (x + A)(x + B)$ $x^2 + 10x + 21 = (x + B)(x + G)$

$x^2 - 9x + 20 = (x - T)(x - L)$ $2x^2 - 16x + 32 = 2(x - T)^2$

$x^3 + 10x^2 + 9x = x(x + S)(x + E)$ $6x^2 - 54x + 48 = 6(x - I)(x - S)$

Determine the value of each letter and hence name the famous person represented by the following code.

(3) (8) (5) (5) (7) (2) (4) (9) (1)

_ _ _ _ _ _ _ _ _

Multiple Choice

12. Which of the following is **not** a factor of $3m^2 - 27m + 54$?

A. $m - 3$
B. $m - 6$
C. $m - 9$
D. 3

13. For which of the following trinomials is $a + 5$ **not** a factor?

A. $a^2 + 6a + 5$
B. $a^2 + 11a + 30$
C. $a^2 + 10a + 50$
D. $a^2 + 10a + 25$

14. The expression $t^2 + kt + 12$ **cannot** be factored if k has the value

A. -13 **B.** -8 **C.** 7 **D.** 11

Numerical Response

15. The largest value of b for which $x^2 + bx + 32$ can be factored over the integers is ____.

(Record your answer in the numerical response box from left to right)

Answer Key

1. a) $x^2 + 4x + 3$ **b)** $x + 3,\ x + 1,$ **c)** $x^2 + 4x + 3 = (x + 3)(x + 1)$
2. a) $x^2 - 7x + 10$ **c)** $x - 2, x - 5,$ $x^2 - 7x + 10 = (x - 2)(x - 5)$
3. a) $(x + 2)(x + 3)$ **b)** $(x + 1)(x + 5)$ **c)** $(x - 4)(x - 2)$
4. a) 2, 3 **b)** 1, 7 **c)** 5, 6 **d)** –5, –6
e) 1, 10 **f)** –3, –5 **g)** –7, –8 **h)** –4, –14
5. a) $(x + 3)(x + 4)$ **b)** $(x + 1)(x + 8)$ **c)** $(x - 2)(x - 5)$
d) $(t - 2)(t - 12)$ **e)** $(z + 5)(z + 3)$ **f)** $(b - 2)(b - 10)$

6. a) $(x + 1)(x + 2)$ **b)** $(x - 1)(x - 2)$ **c)** $(x + 3)(x + 6)$
d) $(x + 2)(x + 6)$ **e)** $(x - 3)(x - 7)$ **f)** $(x - 3)(x - 8)$

7. a) $(x + 1)(x + 10)$ **b)** not possible **c)** $(n + 4)(n + 8)$
d) $(y - 4)(y - 7)$ **e)** $(y + 3)(y + 14)$ **f)** $(f - 3)(f - 7)$
g) $(p - 2)(p - 14)$ **h)** $(x + 4)(x + 20)$ **i)** $(c - 2)(c - 30)$
j) not possible **k)** $(d + 3)(d + 15)$ **l)** $(p - 4)(p - 25)$
m) $(m + 11)(m + 11)$ OR $(m + 11)^2$ **n)** $(n - 6)(n - 17)$ **o)** $(q - 5)(q - 23)$
8. a) 7, 8, 13,–7, –8, –13 **b)** 5, 8, 9 **9. a)** $(x + 12)(x + 3)$ **b)** 15m, 6m

10. a) $2(x + 1)(x + 2)$ **b)** $4(x - 4)(x - 8)$ **c)** $-2(a + 6)(a + 9)$
d) $5(x - 1)(x - 3)$ **e)** $a(x - 5)(x - 9)$ **f)** $-10a^2(a - 4)(a - 6)$
11. BILL GATES **12.** C **13.** C **14.** D **15.**

3	3		