Lesson 1: Common Factors

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Factoring Polynomial Expressions Lesson #1:
Common Factors

Overview of Unit

In this unit, we introduce the process of factoring. This includes factoring by removing a common factor, factoring a trinomial, and factoring a difference of squares. These techniques are illustrated concretely, pictorially, and symbolically. We express a polynomial as a product of its factors and include, for enrichment, polynomial equation solving.

Expanding and Factoring

In the previous unit, we were concerned with multiplying polynomial expressions.
In particular we multiplied

i) a monomial by a polynomial e.g. $2x(x+5)$ = $2x^2+10x$

ii) a binomial by a binomial to form a trinomial e.g. $(x+1)(x+3)$ = x^2+4x+3

iii) a binomial by a binomial to form a binomial e.g. $(x-5)(x+5)$ = x^2-25

In these examples, we have **expanded** a product of polynomials to form a sum or difference of monomials.

In this unit, we are concerned with the opposite process. We want to write a sum or difference of monomials as a product of polynomials. This process is called **factoring**.

We will be studying the following three major types of factoring.

Complete the following using the results obtained above.

i) factoring by removing a common factor e.g. $2x^2+10x$ = $2x(x+5)$

ii) factoring a trinomial. e.g. x^2+4x+3 = _____

iii) factoring a difference of squares e.g. x^2-25 = _____

Greatest Common Factor

In the lesson "Applications of Prime Factors" page 9, we met the concept of the greatest common factor of whole numbers.

The GCF of 48 and 72 was found by using prime factorization.

$48 = 2 \times 2 \times 2 \times 2 \times 3$ and $72 = 2 \times 2 \times 2 \times 3 \times 3$

To determine the greatest common factor of 48 and 72, we found the product of each prime factor (including repeats) which is common to each prime factorization.

GCF of 48 and 72 is $2 \times 2 \times 2 \times 3 = 24$.

The same process can be used to determine the greatest common factor of two monomials like $6a^3$ and $9a^2b$.

$6a^3 = 2 \times 3 \times a \times a \times a$ and $9a^2b = 3 \times 3 \times a \times a \times b$

GCF of $6a^3$ and $9a^2b$ is $3 \times a \times a = 3a^2$.

Write the prime factorization of $8x^2y^2$ and $20xy^3$ and determine the greatest common factor of $8x^2y^2$ and $20xy^3$.

$5 \times 4 \times y \times y \times 2$ $\quad$ $4 \times x \times x \times y \times y$

2×2 $\quad$ 2×2

$4xy^2$

- The greatest common factor of two simple monomials can be determined by inspection, by taking the GCF of any numerical coefficients and multiplying by each common variable to the lowest common exponent.

  The greatest common factor of $10p^3q$ and $15p^2q^2$ is determined by multiplying 5 by p^2 by q, i.e. $5p^2q$.

- If all the monomials are negative, the GCF is usually considered to be negative (see example d) below).

In each case, state the greatest common factor of the following sets of monomials.

a) $12ab,\ 15a^2b^3$ $\quad$ $3ab$

b) $18x^4y^2,\ -24x^3y^5$ $\quad$ $6x^3y^2$

c) $a^3bc^2,\ 2ac^7$ $\quad$ ac^2

d) $-40a^3b,\ -20a^2b^3,\ -10a^2b^2$ $\quad$ $-10a^2b$

Complete Assignment Question #1 - #3

Factoring a Polynomial by Removing the Greatest Common Factor

Factoring is a process in which a sum or difference of terms is expressed as a product of factors.

A polynomial like $8x^2y^2 + 20xy^3$ can be factored by removing (or taking out, or dividing out) the greatest common factor from each term.

We know that

$4xy^2(2x + 5y)$ can be expanded to give $8x^2y^2 + 20xy^3$.

It follows that

$8x^2y^2 + 20xy^3$ can be factored to give $4xy^2(2x + 5y)$.

GCF

In this case, the greatest common factor $4xy^2$ has been removed from each term.

In each case, complete the factoring.

a) $21x + 14y =$ ____ $(3x + 2y)$ **b)** $5x^4 + 15x^3 + 5x^2 =$ ____ $(x^2 + 3x + 1)$ $5x^2$

In each case, the greatest common factor has been removed. Complete the factoring.

a) $5a^2 + 25a = 5a(a + 5)$ (÷5a ÷5a) **b)** $18p - 16q = 2(9p - 8q)$ (÷2 ÷2)

c) $-4mn - 6m^2 = -2m(2n + 3m)$ (÷−2m ÷−2m) **d)** $18x^2y^2 - 45xy + 9x = 9x(2xy^2 - 5y + 1)$ (÷9x ÷9x ÷9x)

Factor each polynomial by removing the greatest common factor.

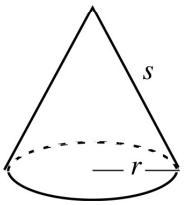
FACTOR = DIVIDE

a) $20x - 6$ $= 2(10x - 3)$ **b)** $16x^4 + 4x^2$ $= 4x^2(4x^2 + 1)$ **c)** $10a^3b^2 + 8ab^3 + 2ab^4$ $= 2ab^2(5a^2 + 4b + b^2)$

d) $12p^3 - 6p^2 + 15p$ $3p(4p^2 - 2p + 5)$ **e)** $25xy^2z^3 - 20x^2y^4z^2 + 30x^4y^2z^5$ $= 5xy^2z^2(5z - 4xy^2 + 6x^3z^3)$

1) Find GCF
2) Divide each part by GCF

The surface area of a cone is given by the formula $A = \pi r^2 + \pi rs$, where r is the radius of the base of the cone and s is the slant height.



i) Determine the surface area of a cone, to the nearest $0.01\ \text{cm}^2$, which has slant height 7.40 cm and base radius 2.60 cm.

ii) Write the formula for A in factored form.

iii) Calculate the surface area of the cone, to the nearest $0.01\ \text{cm}^2$, using the factored form of A.

iv) Which method i) or iii) is simpler to use?

Complete Assignment Questions #4 - #13

Assignment

1. Write the prime factorization of $12a^3$ and $30a^2$ and determine the greatest common factor of $12a^3$ and $30a^2$.

2. Write the prime factorization of $10xy^4$ and $25x^2y^3$ and determine the greatest common factor of $10xy^4$ and $25x^2y^3$.

3. In each case, state the greatest common factor of the following sets of monomials.

a) $7m,\ 14m$ **b)** $6x^2,\ 9x$ **c)** $bc^2,\ bc^7$

d) $ab,\ a^2b^2$ **e)** $4x^4,\ 8x^3$ **f)** $3xyz,\ 9rst,\ 12def$

g) $-8pq^3,\ 18p^2q$ **h)** $-10x^5z^6,\ -15x^5z^4$ **i)** $8ab^2,\ 9ab,\ 6a^2b$

j) $10xy,\ 16xz,\ 20xyz$ **k)** $-2x^3y,\ -4x^3y^4,\ -4x^2y^4$ **l)** $-28pqr^3,\ -56p^2q,\ -64q^2r$

4. Complete the factoring in each case.

a) $12a + 24b =$ ___ $(a + 2b)$ **b)** $4p^2 - 7p =$ ___ $(4p - 7)$

c) $2xy + 3xz =$ ___ $(2y + 3z)$ **d)** $5x^2 + 10x + 15 =$ ___$(x^2 + 2x + 3)$

e) $6cde - 4cd =$ ____$(3e - 2)$ **f)** $3y^3 - 9y =$ ___$(y^2 - 3)$

5. In each case, the greatest common factor has been removed. Complete the factoring.

a) $3a^2 + 15a = 3a(a +$ **b)** $20p - 10q = 10(\quad - q)$

c) $6x^3 - 9x^2 = 3x^2($ **d)** $4a^2b + 8a^3b^2 = 4a^2b($

e) $-15x^2y - 10x^2y^2 = -5x^2y($ **f)** $16xm^2n^3 - 12mn^2 - 4mn = 4mn($

6. Factor the following polynomials by removing the greatest common factor.

a) $6m + 6n$ **b)** $7xy^2 + 49$ **c)** $15pq - 5$ **d)** $8c + 12d$

e) $xy + y$ **f)** $6x^2 - 9x$ **g)** $9ab - 12ac$ **h)** $48y^2 - 72y^5$

7. Factor the following polynomials

a) $12x - 8y + 16z$ **b)** $9pq + 6pr - 15p$ **c)** $t^3 + t^2 + t$

d) $5x^2 - 10xy - 20xz$ **e)** $4abc - 2abd + 8abe$ **f)** $14a^2b^2 + 21a^3b^2 - 35a^2b^3$

8. In each of the following:
i) simplify the expression by combining like terms.
ii) factor the resulting polynomial.

a) $5x^2 - 2x + 7 - 2x^2 + 8x - 7$

b) $6 - 2y + 5y^2 - 10y + 3y^2 - 12$

c) $xy^3 - 2x^3y + 6x^2y^2 - 5xy^3 + 8x^3y$

d) $2(x^3 - 3x) - 4x(x - 6) + 5x^2(x - 2) - 4x$

9. Factor the following polynomials. Expand your answer to verify the factoring.

a) $24x^3 - 60x^2$

b) $-8p^3 - 32p^2 - 8p$

10. An archer standing on the ground fires an arrow vertically upward into the air at a speed of 30 m/s.
The height (h metres) of the arrow above the ground after t seconds can be approximated by the formula $h = 30t - 5t^2$.

a) Write h in factored form.

b) Use the factored form of h to calculate the height for each of the times in the table. Record your answer in the table.

Time (t seconds)	0	1	2	3	4	5	6
Height (h metres)							

c) Explain why the height of the arrow after two seconds is the same as the height of the arrow after four seconds.

d) Calculate h when $t = 7$. Explain why this has no meaning in the context of the question.

Multiple Choice **11.** One factor of $9x^4 - 6x^3 + 3x^2$ is

A. $9x^4$

B. $3x^2 - 2x$

C. $3x^2 - 6x + 3$

D. $3x^2 - 2x + 1$

Numerical Response **12.** When $x^4y^3 - x^2y^3 + x^6y$ is factored, the greatest common factor has degree A and the remaining trinomial factor has degree B. The value of $A + 2B$ is _____.

(Record your answer in the numerical response box from left to right)

13. When the greatest common factor is removed from the binomial $98x^2 - 28x$, the binomial can be written in the form $ax(bx + c)$. The value of $a + b + c$ is _____.

(Record your answer in the numerical response box from left to right)

Answer Key

1. $12a^3 = 2 \cdot 2 \cdot 3 \cdot a \cdot a \cdot a$ $30a^2 = 2 \cdot 3 \cdot 5 \cdot a \cdot a$ $GCF = 2 \cdot 3 \cdot a \cdot a = 6a^2$

2. $10xy^4 = 2 \cdot 5 \cdot x \cdot y \cdot y \cdot y \cdot y$ $25x^2y^3 = 5 \cdot 5 \cdot x \cdot x \cdot y \cdot y \cdot y$ $GCF = 5 \cdot x \cdot y \cdot y \cdot y = 5xy^3$

3. **a)** $7m$ **b)** $3x$ **c)** bc^2 **d)** ab **e)** $4x^3$ **f)** 3 **g)** $2pq$ **h)** $-5x^5z^4$ **i)** ab **j)** $2x$ **k)** $-2x^2y$ **l)** $-4q$

4. **a)** 12 **b)** p **c)** x **d)** 5 **e)** $2cd$ **f)** $3y$

5. **a)** $a+5$ **b)** $2p-q$ **c)** $2x-3$ **d)** $1+2ab$ **e)** $3+2y$ **f)** $4xmn^2-3n-1$

6. **a)** $6(m+n)$ **b)** $7(xy^2+7)$ **c)** $5(3pq-1)$ **d)** $4(2c+3d)$
e) $y(x+1)$ **f)** $3x(2x-3)$ **g)** $3a(3b-4c)$ **h)** $24y^2(2-3y^3)$

7. **a)** $4(3x-2y+4z)$ **b)** $3p(3q+2r-5)$ **c)** $t(t^2+t+1)$ **d)** $5x(x-2y-4z)$
e) $2ab(2c-d+4e)$ **f)** $7a^2b^2(2+3a-5b)$

8. **a)** $3x^2+6x = 3x(x+2)$ **b)** $8y^2-12y-6 = 2(4y^2-6y-3)$
c) $6x^3y+6x^2y^2-4xy^3 = 2xy(3x^2+3xy-2y^2)$ **d)** $7x^3-14x^2+14x = 7x(x^2-2x+2)$

9. **a)** $12x^2(2x-5)$ **b)** $-8p(p^2+4p+1)$

10. **a)** $h = 5t(6-t)$ **b)** 0, 25, 40, 45, 40, 25, 0
c) At 2 sec. the arrow is on the way up and at 4 sec. the arrow is on the way down.
d) $h = -35$. The arrow has already hit the ground at $t = 6$. It does not travel 35m below the ground.

11. D

12.

1	1		

13.

1	9			≤