

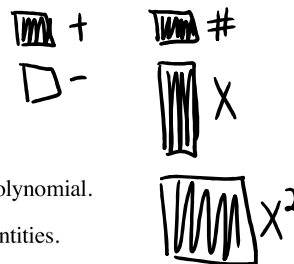
Polynomial Operations Lesson #2: Multiplying a Polynomial by a Monomial

Using Algebra Tiles

In previous math courses, we learned how to multiply
 i) two monomials, and ii) a monomial and a binomial or trinomial.

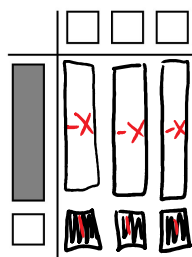
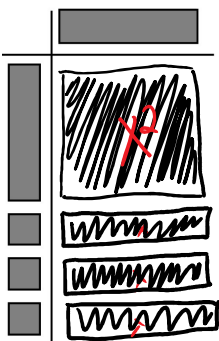
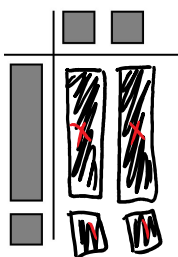
We can use algebra tiles to illustrate the process of multiplying a monomial by a polynomial.

Shaded tiles represent positive quantities and unshaded tiles represent negative quantities.

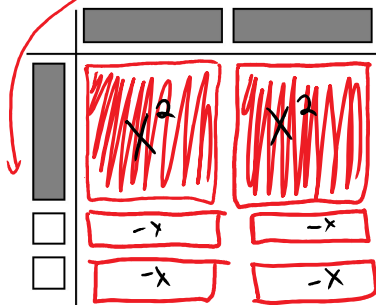


Complete the diagram to determine the product.

- a) $2(x+1) = 2x+2$ b) $x(x+3) = x^2+3x$ c) $-3(x-1) = -3x+3$



d) $2x(x-2) = 2x^2 - 4x$



e) $(x-1)(2x) = 2x^2 - 2x$

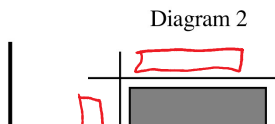
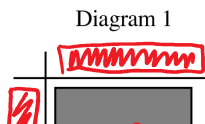


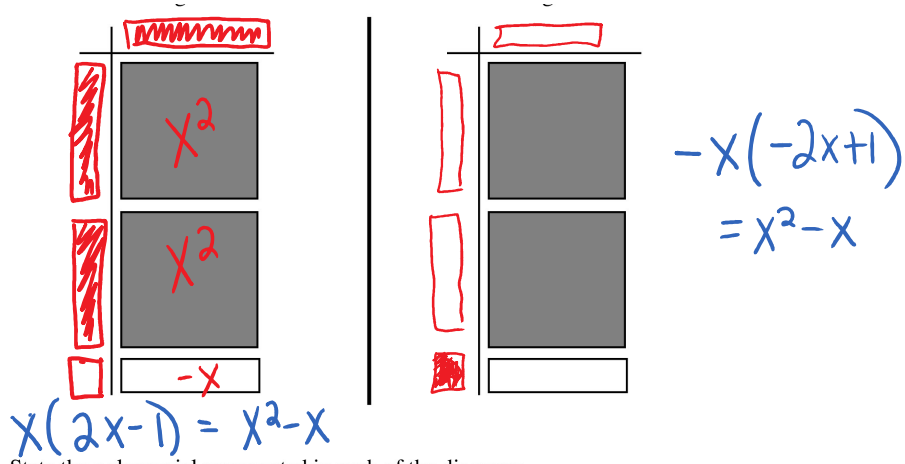
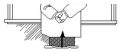
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Each diagram below illustrates the result of the product of a monomial and a binomial.





a) State the polynomial represented in each of the diagrams.

b) Complete the left side and the top of Diagram 1 and write the polynomial product.

c) Complete Diagram 2 to illustrate and write a different polynomial product than in b).

d) Write each product as a sum or difference of terms.

e) Verify the polynomial products in d) when $x = 3$.

$$x(2x-1) \quad x=3$$

$$= 3(2 \cdot 3 - 1)$$

$$= 3(6-1) = 3(5) = 15$$

$$-x(-2x+1), \quad x=3$$

$$-3(-2(3)+1)$$

$$= 15$$

Complete Assignment Questions #1 - #3

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The Distributive Property

In Class Example #1 we have shown that:

$$2(x+1) = 2x+2, \quad x(x+3) = x^2+3x, \quad 2x(x-2) = 2x^2-4x,$$

$$-3(x-1) = -3x+3, \quad \text{and} \quad (x-1)(2x) = 2x^2-2x.$$

These above are examples of the **distributive property**

$$a(b+c) = ab+ac \quad \text{or} \quad (b+c)(a) = ba+ca \Rightarrow ab+ac.$$

The distributive property can be extended to any number of terms.

Using Numerical Values to Verify the Distributive Property

Consider the expression $-2(3-5)$.

i) Evaluate $-2(3-5)$ by calculating the value inside the brackets first and then multiplying by -2 .

$$-2(-2) = 4$$

ii) Evaluate $-2(3-5)$ by using the distributive property.

$$-6+10 = 4$$

iii) Comment on your results from i) and ii).

same answer



Use the distributive property to determine the following products.

a) $4(3x+1)$
 $= 12x+4$

b) $-5(2x^2+x-6)$
 $= -10x^2-5x+30$

c) $(x^3-2)x^2$
 $= x^5-2x^2$

d) $-3x(7x-2y+z)$
 $= -21x^2+6xy-3xz$

In the example above we have written a product of polynomials as a sum or difference of terms.

In this process we **expanded** the polynomial expressions by using the distributive property, $a(b+c) = ab+ac$ and the exponent rule, $x^a \times x^b = x^{a+b}$.

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Expand and simplify.

a) $6-4(8x+1)$
 $= 6-32x-4$ *expanded
 $= -32x+2$ *simplified

b) $4(2x-3)-2(x-6)$
 $= 8x-12-2x+12$ *expanded (no more brackets)
 $= 6x$ *simplified

$= -32x + 2$ *simplified $= 6x$ *simplified (no more brackets)

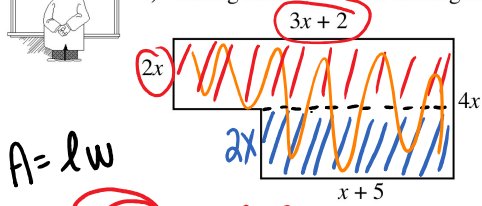
c) $5x(3x^2 - 7x + 1) - (4x + 3x^2)$

$15x^3 - 35x^2 + 5x - 4x - 3x^2$ *expanded ✓

$= 15x^3 - 38x^2 + x$ *combined like terms to simplify

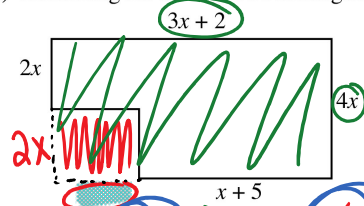


Determine a simplified expression for the area of the given shape by
 i) adding the areas of two rectangles. ii) subtracting the areas of two rectangles.



$A = lw$

$(3x+2)2x = 6x^2 + 4x$
 $(x+5)2x = 2x^2 + 10x$
 $8x^2 + 14x$



side
 $(3x+2) - (x+5)$
 $3x+2-x-5$
 $= 2x-3$

$4x(3x+2) - 2x(2x-3)$
 $12x^2 + 8x - 4x^2 + 6x$
 $= 8x^2 + 14x$

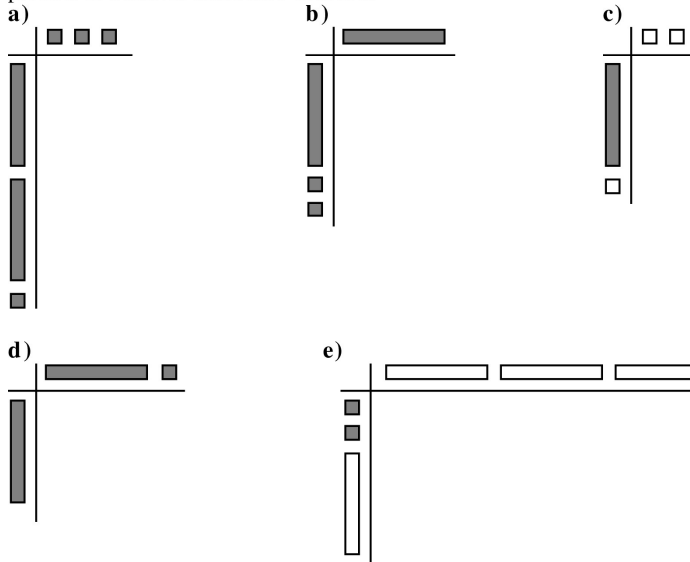
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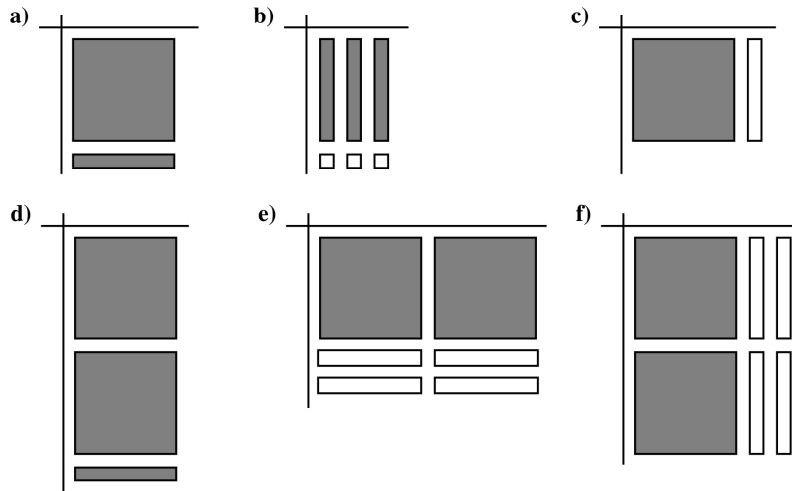
Complete Assignment Questions #4 - #11

Assignment

1. In each case complete the diagram, state the polynomial product in x , and express the product as a sum or difference of terms.



2. In each case state the polynomial product in x which is indicated by the algebra tile diagram. Express the product as a sum or difference of terms.



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3. For each of the following:

- i) Draw an algebra tile diagram to model the product.
- ii) Express the product as a sum or difference of terms.
- iii) Verify the polynomial product when $x = 4$.

a) $2x(2x - 1)$

b) $-3x(2 - x)$

4. Expand.

a) $6(7x - 3)$ b) $-4(4x + 9)$ c) $4x(2y + 8z)$ d) $-x(x - 5y)$

e) $3(x - 2y + 3z)$ f) $-2a(b - c + 5d)$ g) $(x + 3)3x$

h) $2x(x - 5y + 4z)$ i) $x(x - 2x^2 + 3x^3)$ j) $(2x^2 + x - 6)(-4x)$

5. Expand and simplify.

a) $3(x + 5) - 7$

b) $8 - 2(5x + 11)$

c) $6(x - 2) + x$

d) $2(x + 3) + 4(2x - 1)$

e) $2(x + 3) - 4(2x - 1)$

f) $-2(x + 1) + 7(3x - 2)$

g) $5(-x + 12) + 5(x - 8)$

h) $(2 - x) - 2(2x - 10)$

i) $6(-x + 4) - (x - 15)$

6. Identify the errors in the following and provide the correct simplification.

a) $3x(2x + y) = 6x + 3xy$

b) $x^2(x^3 - 2x + 7) = x^6 - 2x^3 + 7x^2$

c) $4(x - 2) - 2(x - 3)$
 $= 4x - 8 - 2x - 6$
 $= 2x - 14$

d) $2(2t - 3) - 4(t + 5)$
 $= 4t - 3 - 4t - 5$
 $= -8$

e) $5(a + b) - (a + b)$
 $= 5a + 5b - a + b$
 $= 4a + 6b$

7. Expand and simplify.

a) $2a(a + 3) - 4a(2a - 1)$ **b)** $4(x^2 + 3) - (2x^2 - 1)$ **c)** $2(x + 3) - x - 1$

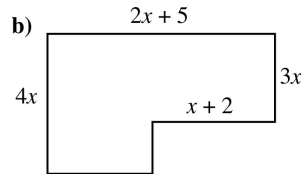
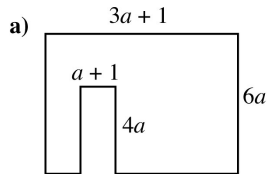
d) $z(z^3 + 3) - (3z + 7)$ **e)** $5(8x - 3y) + 2(4y + x)$

f) $-2x(x^4 + 3x^3) - 7x(2x^4 - x^3)$ **g)** $3a(2a^2b - ab + b^2) - 6b(a^3 + 3ab - 5b^2)$

h) $3x(x - 3) - 2x(x - 1) + x(2x - 2)$ **i)** $(p^2 - 3p)(4p) - (3 + 5p)(-2p^2)$

j) $a(b - c) + b(c - a) + c(a - b)$ **k)** $20x^3y^3 - 4x^3y^2(3x + 5y - xy)$

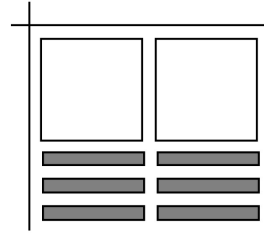
8. Determine a simplified expression for the area of the given shape.



Multiple Choice

9. The algebra tile diagram represents the expansion of:

- A. $2x(x + 3)$
- B. $-2x(x + 3)$
- C. $2x(x - 3)$
- D. $-2x(x - 3)$



10. Which of the following expansions is incorrect?

- A. $-2x^2(3x + 2) = -6x^3 - 4x^2$
- B. $-4x(2 - x) = -8x + 4x^2$
- C. $-5x(x^2 - 3) = -5x^3 - 15x$
- D. $7x^2(x^2 + 3) = 7x^4 + 21x^2$

Numerical Response

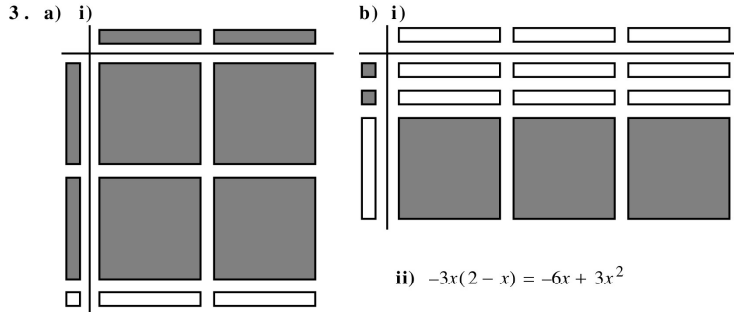
11. The expression $2x(4 - 3x) + 5x(2x - 1) - 3(4x + 2)$ can be written in the form $ax^2 + bx + c$. The value of $a + b - c$ is _____.

(Record your answer in the numerical response box from left to right)

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Answer Key

1. a) $3(2x + 1) = 6x + 3$ b) $x(x + 2) = x^2 + 2x$ c) $-2(x - 1) = -2x + 2$
 d) $(x + 1)(x) = x^2 + x$ e) $-3x(2 - x) = -6x + 3x^2$
2. a) $x(x + 1) = x^2 + x$ b) $3(x - 1) = 3x - 3$ c) $(x - 1)(x) = x^2 - x$
 d) $x(2x + 1) = 2x^2 + x$ e) $2x(x - 2) = 2x^2 - 4x$ f) $(x - 2)(2x) = 2x^2 - 4x$



ii) $2x(2x - 1) = 4x^2 - 2x$

iii)

Left Side	Right Side
$(2x \times 4)(2x \times 4) - 1$	$4(4^2) - 2(4)$
$= (8)(7)$	$= 64 - 8$
$= 56$	$= 56$

ii) $-3x(2 - x) = -6x + 3x^2$

iii)

Left Side	Right Side
$(-3 \times 4)(2 - 4)$	$(-6 \times 4) + 3(4^2)$
$= (-12)((-2))$	$= -24 + 48$
$= 24$	$= 24$

4. a) $42x - 18$ b) $-16x - 36$ c) $8xy + 32xz$
 d) $-x^2 + 5xy$ e) $3x - 6y + 9z$ f) $-2ab + 2ac - 10ad$
 g) $3x^2 + 9x$ h) $2x^2 - 10xy + 8xz$ i) $x^2 - 2x^3 + 3x^4$ j) $-8x^3 - 4x^2 + 24x$
5. a) $3x + 8$ b) $-10x - 14$ c) $7x - 12$
 d) $10x + 2$ e) $-6x + 10$ f) $19x - 16$
 g) 20 h) $-5x + 22$ i) $-7x + 39$
6. a) $3x(2x) = 6x^2$, not $6x$. $3x(2x + y) = 6x^2 + 3xy$
 b) $x^2(x^3) = x^5$ not x^6 . $x^2(x^3 - 2x + 7) = x^5 - 2x^3 + 7x^2$
 c) $-2(-3) = 6$, not -6 . $4(x - 2) - 2(x - 3) = 4x - 8 - 2x + 6 = 2x - 2$
 d) The monomials 2 and -4 multiply both terms in the binomials.
 $2(2t - 3) - 4(t + 5) = 4t - 6 - 4t - 20 = -26$.
 e) The negative multiplies both a and b . $5(a + b) - (a + b) = 5a + 5b - a - b = 4a + 4b$.
7. a) $-6a^2 + 10a$ b) $2x^2 + 13$ c) $x + 5$ d) $z^4 - 7$
 e) $42x - 7y$ f) $-16x^5 + x^4$ g) $-3a^2b - 15ab^2 + 30b^3$ h) $3x^2 - 9x$
 i) $14p^3 - 6p^2$ j) 0 k) $-12x^4y^2 + 4x^4y^3$

8. a) $14a^2 + 2a$ b) $7x^2 + 18x$ 9. D 10. C 11.

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