Polynomial Operations Lesson #1:
Review and Preview

Overview of Unit

In this unit we study algebraic expressions called polynomials. We review the classification of polynomials, addition and subtraction of polynomials, and multiplication by a monomial. We introduce the product of two binomials (concretely, pictorially, and symbolically) and extend this to the multiplication of polynomials. We also solve problems involving polynomial expressions.

Review

In algebra, a letter that represents one or more numbers is called a **variable**.

Expressions like $2a - b + 4$ or $\frac{5}{x} + 3$ are called **algebraic expressions**. not equations because no "="

Certain algebraic expressions are called **polynomials** as explained below.

A **monomial** is a number or a variable or the product of numbers and variables. (Note that the exponent of any variable must be a positive integer in the numerator of the monomial.)

eg. 6, x, $6x$, $-\frac{1}{2}xy$, $0.25x^3$, abc, $2p^4q^2$ etc. are all monomials.

The number that multiplies the variable is called the **numerical coefficient**.

6x (6: coefficient, x: variable)

A **polynomial** is a monomial or a sum or difference of monomials.

- 6, x, $6+x$, $2y+7z$, x^2-5x-9 etc. are all examples of polynomials.

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Explain why $\frac{5}{x} + 3$ is not a polynomial.

$5x^{-1} + 3$ not a polynomial because of the negative exponent (no bottom variable)

Classifying Polynomials

Polynomials may be classified in two different ways as shown below.

Ways to Classify Polynomials

- number of terms in the polynomial
- degree of the polynomial

Continued on the next two pages

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Classifying Polynomials by The Number of Terms

A polynomial may be classified by the number of **terms** it contains.

- A term can be a number, a variable, or the product of a number and variable(s).
- When there is more than one term, the terms are connected by + or – signs.

A polynomial with 1 term is a **monomial** (eg. $4x$).

A polynomial with 2 terms is a **binomial** (eg. $x + 4$).

A polynomial with 3 terms is a **trinomial** (eg. $x^2 + x + 4$).

A polynomial with 4 or more terms is simply called a polynomial when classifying by the number of terms.

Consider the following algebraic expressions. In each case:

- State whether the expression represents a polynomial or not.
- If the expression does not represent a polynomial, explain why.
- If the expression does represent a polynomial, state whether the polynomial is a monomial, a binomial, or a trinomial.

a) $\frac{1}{4}xy - 10$

→ yes, polynomial

→ binomial

b) $3pq^{\frac{1}{2}}$

→ no, fraction for exponent

c) $\sqrt{7}\,x^4 - x^3 + 1$

→ yes, polynomial

→ trinomial

d) $3x^2 + 9x - 4x^{0.2}$

→ no, decimal exponent

e) $\frac{7}{a} = 7a^{-1}$

no, negative exponent

Complete the following table.

Polynomial Expression	# Variables	# Terms	Classification by # Terms
$4x + 3yz$	3 → x, y, z	2	binomial
$2a - 4b + 7c$	3 → a, b, c	3	trinomial
$x^2 + 3x + 4$	1 → x	3	trinomial
$\sqrt{2}\,x$	1 → x	1	monomial
$2x^3 + 3x^2y + 3y^2 - 8$	2 → x y	4	polynomial

Classifying Polynomials by The Degree of The Polynomial

Polynomials can also be classified according to **degree** of the polynomial .

The **degree of a monomial** is the sum of the exponents of its variable(s).

eg. $2x^5$ has degree 5 $-\frac{2}{3}ab^3c^2$ has degree 6 (1+3+2)

Recall from the lesson "Whole Number Exponents" on page 20 that a variable raised to the power zero is equal to 1.

For example, the monomial $7x^0$ can be written as $7(1)$ or 7.
Therefore the degree of a monomial with no variable present is 0.

The **degree of a polynomial** is given by the **term** or **monomial** with highest degree.

eg. $3x^2y^2 - 2x^4 + xy^4 - 2$ has degree 5 (4, 4, 5, 0)

If a polynomial has a term with no variable present, this term is called a **constant term**.

In the polynomial $3x^2y^2 - 2x^4 + xy^4 - 2$, the constant term is -2. (just a #, no letter)

State the degree of the following polynomials.

a) $3x^2 - 10x^4 - 9$ (2, 4, 0) constant term — degree 4

b) $7p^2q^3 - 8p^7q - 2q^7$ (5, 8, 7) no constant term — degree 8

Give an example of

a) a binomial of degree 1 in one variable. $2+x$, $2+x$, $x+3$

b) a trinomial in two variables with a constant term. $4x+3y-6$

c) a monomial of degree 6 with a (numerical) coefficient of 9. $9x^6$

d) a binomial of degree 8 with each term containing two variables. x^6y^2+ab, x^7y+ab

The following list classifies polynomials by using the degree of the polynomial.

- A **Constant** Polynomial has a degree of 0 eg. 8
- A **Linear** Polynomial has a degree of 1 eg. $x+3$
- A **Quadratic** Polynomial has a degree of 2 eg. $x^2 - 2x + 5$
- A **Cubic** Polynomial has a degree of 3 eg. $x^3 - 8x^2 + x + 1$
- A **Quartic** Polynomial has a degree of 4 eg. $x^4 - 61x + 9$
- A **Quintic** Polynomial has a degree of 5 eg. $x^5 - 17$

There are names for polynomials of higher degree that are beyond the scope of this course.

Complete the following table.

Polynomial Expression	Degree	Classification by Degree	Constant Term
$4xy - 6$	2	quadratic poly	-6
$9y^2 - 8y^3$	3	cubic poly	0

Polynomials in a Single Variable

Polynomials in a single variable are usually arranged in **ascending** or **descending** order of the powers of the variable.

The **leading coefficient** of a polynomial in a single variable is the coefficient of the term with highest power of the variable.

Class Ex. #7

Consider the polynomial expression $2x - 4x^3 - 7 + \frac{6x^2}{5}$.

a) Write the polynomial in descending powers of x.

b) Write the polynomial in ascending powers of x.

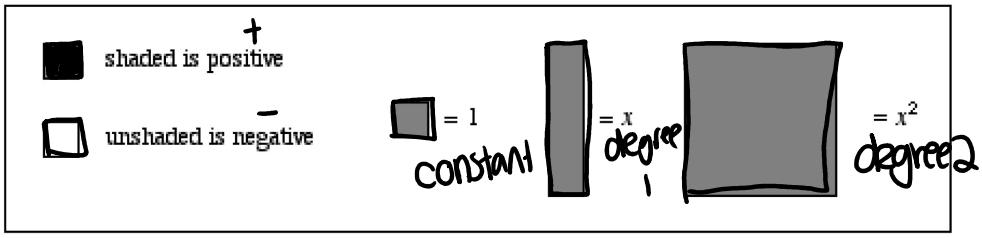
c) State the leading coefficient and the constant term.

d) State the numerical coefficient of the term in x^2.

Complete Assignment Questions #1 - #9

Representing Polynomials Using Algebra Tiles

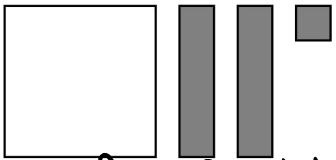
The following legend will be used for algebra tiles in this workbook.



Class Ex. #8

State the polynomial expression which describes each diagram.

a)



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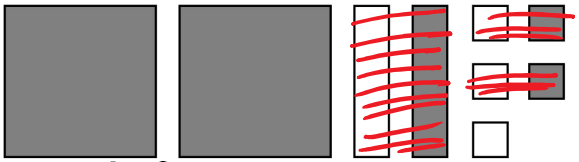
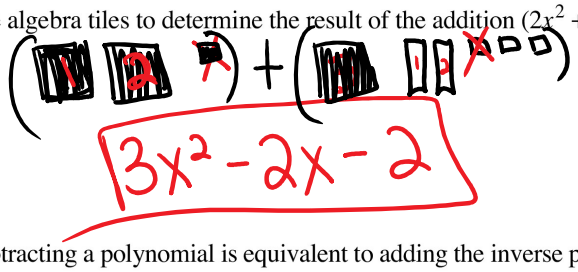


Addition and Subtraction Using Algebra Tiles

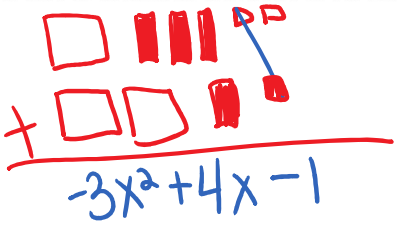
Use algebra tiles to determine the result of the addition $(2x^2 + 1) + (x^2 - 2x - 3)$.



Subtracting a polynomial is equivalent to adding the inverse polynomial,

eg. $(4x + 3) - (2x - 5)$ is equivalent to $(4x + 3) + (-2x + 5)$

Use algebra tiles to determine the result of the subtraction $(-x^2 + 3x - 2) - (2x^2 - x - 1)$.



Addition and Subtraction of Polynomial Expressions

Like terms are terms with the same variable raised to the same exponent.

eg. $3a$, $7a$ and a are like terms. $2x^3$, $\frac{1}{5}x^3$ and $-4x^3$ are like terms.

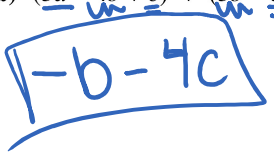
Unlike terms have different variables or the same variable raised to different exponents.

eg. $2x^3$, $\frac{1}{5}x^2$ and $-4x$ are unlike terms. $4x$ and $4y$ are unlike terms.

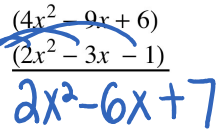
Like terms can be added or subtracted to produce a single term.

Simplify the following polynomials by collecting like terms.

a) $(3a - 4b + c) + (3b - 5c - 3a)$



b) $(4x^2 - 9x + 6) - (2x^2 - 3x - 1)$



Simplify

a) $4x - 2x^2 + 3 - 6x^2 + 5 - x$ $-8x^2+3x+8$

b) $a^2b - ab^2 + 4a^3b - 7ab^2 + 5a^2b$ $6a^2b-8ab^2+4a^3b$

Complete Assignment Questions #10 - #20

Assignment #4-7, 10-14

1. Identify as a monomial, a binomial, or a trinomial.

a) $x + 1$ **b)** $3x^3$ **c)** $2x^2 + 2x + 2$

2. State the degree of each monomial.

a) 5a **b)** $3x^3y$ **c)** 10 **d)** $-2a^2b^2$ **e)** $3xy^2z^3$

3. State whether or not the following are polynomial expressions. If they are not polynomial expressions, explain why not.

a) $\frac{1}{2}x^2 - 3x$ **b)** $8m^{-2}$ **c)** $\sqrt{6}$

d) $\frac{7}{x^3}$ **e)** $\frac{8x^2}{3}$ **f)** $x^4 + 3x^{1.5}$

4. Complete the following table.

Polynomial Expression	# Variables	# Terms	Classification by Number of Terms	Degree
$2y^3 + y^4 - y + 13$				
$9ab - 4x + 11c$				
25				
$\frac{3}{5}x^3yz^5 + 3x^2yz^4$				

5. Complete the following table for the single variable polynomials.

Polynomial Expression	Leading Coefficient	Constant Term	Degree	Classification by Degree
$y^4 - y + 13$				
$0.2t^3 - 0.3t^2 + 0.4t - 0.5$				
$\sqrt{7} - x^5$				
$\pi x^2 - 7 - 3x$				
$-\frac{1}{10}$				
$9x + 12$				

6. Give an example of

a) a trinomial of degree 2 in one variable.

b) a binomial in four variables with a constant term of 6.

c) a monomial of degree 3 in two variables with a negative numerical coefficient.

d) a monomial with a degree of 0.

7. Arrange the following in descending powers of the variable.

a) $6w^2 - 9w + 5 + 2w^3$

b) $\frac{1}{4}a^2 - \frac{2}{3}a^3 - 1 - a$

c) $z - 3 - 4z^6 + z^3$

8. Arrange the following in ascending powers of the variable.

a) $6w^2 - 9w + 5 - 2w^3$

b) $3x^2 - 4x^5 - 2x^4 - 4x^3 + 9x - 7$

c) $8x^3 - 8x + 8$

9. State which of the following are true and which are false.

a) -54 is a polynomial.

b) The degree of the polynomial $3x^3y^3$ is 9.

c) The numerical coefficient of $\frac{6x}{5}$ is 6.

d) A polynomial may have 1000 terms.

e) $\frac{2}{a^3} - 1$ is a binomial.

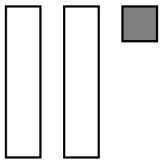
f) The degree of the polynomial 0 is 0.

g) The polynomial $x^3 + 2x^2 + 3x + 4$ is written in ascending powers of x.

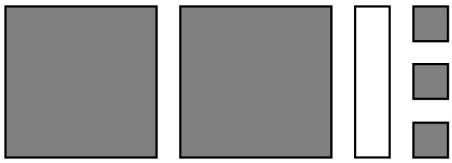
h) The polynomials $3x^2 - 9x + 1$ and $1 - 9x + 3x^2$ are equivalent.

10. State the polynomial expression which describes each diagram.

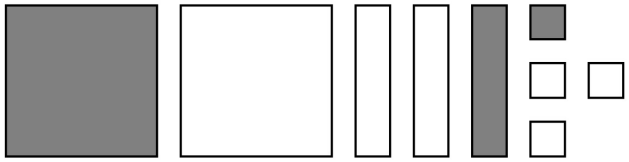
a)



b)



c)



11. Use algebra tiles to determine the result of the addition of :

a) $(x^2 - x - 3) + (x^2 - 2x - 3)$ **b)** $(3x + 1) + (2x^2 - 3x - 2)$

12. Use algebra tiles to determine the result of the subtraction of:

a) $(x^2 - 3) - (2x^2 + 4x + 1)$ **b)** $(2 - x - x^2) - (1 - 2x + x^2)$

13. Simplify

a) $6p - 7q - 3q - 2p$ **b)** $5x - 3x^2 + 2x - 8x^2$ **c)** $\frac{1}{2}x - 3 + \frac{3}{2}x + 18$

d) $4a^3 + 7a - 2a^2 - 6a - 4a^3 - a^2$ **e)** $3 - 2x + 7y + 4y - 2x + 8z - 9$

14. Simplify the following polynomial expressions by collecting like terms.

a) $(5a - 9b - 2c) + (c - 7b - 3a)$ **b)** $(3 - a - 2a^2) + (9 - 4a + 5a^2)$

c) $(2x^2 + 5x - 1) + (3x - 6 - 6x^2) + (4 - 5x + x^2)$ **d)** $(4a - 6b) - (5a - 2b)$

e)
$$\begin{array}{rl} & (5x^2 - 8x + 3) \\ - & \underline{(3x^2 - 3x - 1)} \end{array}$$

f)
$$\begin{array}{rl} & (7x^2 + 2x - 1) \\ - & \underline{(-5x^2 - 3x - 1)} \end{array}$$

g)
$$\begin{array}{rl} & (-4x^2 + 2x - 6) \\ - & \underline{(3x + 6 - 2x^2)} \end{array}$$

15. **a)** Subtract $3x^2 - 2x + 7$ from $6x^2 - 5x - 2$.

b) Subtract the sum of $2x^3 - 7x^2 - 6x + 1$ and $8 - 3x + 5x^2 - 4x^3$ from $2x^3 - 7x + 9$.

16. A triangle has a perimeter of $(6m + n)$ cm. One side measures $(2m - 3n)$ cm and another side measures $(3n + 2m)$ cm.

a) Write and simplify an expression for the length of the third side of the triangle.

b) Determine the measure of each side when $m = 4$ and $n = -1$.

Multiple Choice

17. Which of the following is a polynomial expression of degree 4?

A. $4x^4 - 4x^7$

B. $5x^4 - 3x^3 + 2x^{-2} + x - 1$

C. $\dfrac{4x^4 - 3x}{x}$

D. $9 + 3x - \frac{1}{3}x^2 - x^3 + \frac{2}{5}x^4$

18. Which of the following polynomial expressions, when simplified, is equal to $5x$?

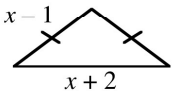
A. $(3x^2 - 3x) - (2x + 3x^2)$

B. $5x - (2x^2 - 2x) + (2x^2 + 2x)$

C. $8 + (4 - 2x) - (12 - 7x)$

D. $(2x^2 - 2x + 6) - (2x^2 - 2x) + (9x - 6)$

19. The perimeter of the isosceles triangle shown can be represented by



A. a monomial

B. a binomial

C. a trinomial

D. none of the above

Numerical Response **20.** If the polynomial $4 - 7x + 2x^2 - 5x^3$ has degree a, leading coefficient b, and constant term c, then the value of $3a - 2b - c$ is _____.

(Record your answer in the numerical response box from left to right) | | | | |

Answer Key

1. **a)** binomial **b)** monomial **c)** trinomial **2.** **a)** 1 **b)** 4 **c)** 0 **d)** 4 **e)** 6

3. **a)** yes **b)** no, negative exponent **c)** yes

d) no, $\frac{7}{x^3} = 7x^{-3}$, which is a negative exponent. **e)** yes

f) no, the exponent 1.5 is not a positive integer.

4.

Polynomial expression	*# variables*	*# terms*	*Classification by Number of Terms*	*degree*
$2y^3 + y^4 - y + 13$	1	4	polynomial	4
$9ab - 4x + 11c$	4	3	trinomial	2
25	0	1	monomial	0
$\frac{3}{5}x^3yz^5 + 3x^2yz^4$	3	2	binomial	9

5.

Polynomial expression	*leading coefficient*	*constant term*	*degree*	*Classification by Degree*
$y^4 - y + 13$	1	13	4	Quartic
$0.2t^3 - 0.3t^2 + 0.4t - 0.5$	0.2	-0.5	3	Cubic
$\sqrt{7} - x^5$	-1	$\sqrt{7}$	5	Quintic
$\pi x^2 - 7 - 3x$	π	-7	2	Quadratic
$-\frac{1}{10}$	$-\frac{1}{10}$	0	0	Constant
$9x + 12$	9	12	1	Linear

6. answers may vary **a)** $x^2 - x + 30$ **b)** $abcd + 6$ **c)** $-2xy^2$ **d)** 10

7. **a)** $2w^3 + 6w^2 - 9w + 5$ **b)** $-\frac{2}{3}a^3 + \frac{1}{4}a^2 - a - 1$ **c)** $-4z^6 + z^3 + z - 3$

8. **a)** $5 - 9w + 6w^2 - 2w^3$ **b)** $-7 + 9x + 3x^2 - 4x^3 - 2x^4 - 4x^5$ **c)** $8 - 8x + 8x^3$

9. **a)** true **b)** false **c)** false **d)** true **e)** false **f)** true **g)** false **h)** true

10. **a)** $-2x + 1$ **b)** $2x^2 - x + 3$ **c)** $-x - 2$ **11.** **a)** $2x^2 - 3x - 6$ **b)** $2x^2 - 1$

12. **a)** $-x^2 - 4x - 4$ **b)** $1 + x - 2x^2$

13. **a)** $4p - 10q$ **b)** $-11x^2 + 7x$ **c)** $2x + 15$ **d)** $-3a^2 + a$ **e)** $-4x + 11y + 8z - 6$

14. **a)** $2a - 16b - c$ **b)** $3a^2 - 5a + 12$ **c)** $-3x^2 + 3x - 3$ **d)** $-a - 4b$

e) $2x^2 - 5x + 4$ **f)** $12x^2 + 5x$ **g)** $-2x^2 - x - 12$

15. **a)** $3x^2 - 3x - 9$ **b)** $4x^3 + 2x^2 + 2x$

16. **a)** $(2m + n)$ cm **b)** 11 cm, 5cm, and 7 cm

17. D **18.** C **19.** A **20.**

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