Lesson 4: Solving Systems of Linear Equations by Elimination

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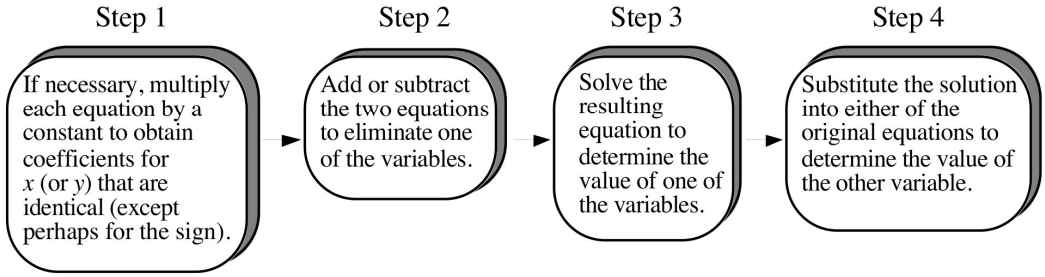
Systems of Linear Equations Lesson #4:
Solving Systems of Linear Equations by Elimination

So far we have used three methods to solve systems of equations: graphing, inspection, and substitution. In this lesson we will learn another algebraic technique: the method of elimination. This method is particularly useful when the equations involve fractions.

Method of Elimination

In using the method of elimination, there are four steps which are shown below.



Consider the system of equations:

$$2x + 7y = 13$$
$$3x - 7y = 2$$

$5x + 0 = 15$

$5x = 15$

$x = 3$

a) Add the two equations.
This will eliminate the variable y.

b) Use the equation in **a)** to determine the value of x and hence solve the system.

$2(3) + 7y = 13$

$6 + 7y = 13$

$7y = 7$

$y = 1$

c) Verify the solution satisfies both equations.

verify $3(3) - 7(1) \stackrel{?}{=} 2$ $\quad 9 - 7 = 2$ ✓

Consider the system of equations:

$$2x + 6y = 6$$
$$-(2x + 3y = 4.5)$$

$0 + 3y = 1.5$

$y = \frac{1.5}{3} = 0.5$

a) Subtract the two equations.
This will eliminate the variable x.

b) Use the equation in **a)** to determine the value of y and hence solve the system.

$2x + 6(0.5) = 6$

$2x + 3 = 6$

$2x = 3$

$x = \frac{3}{2} = 1.5$

c) Verify the solution satisfies both equations.

Complete Assignment Questions #1 - #3

Consider the system of equations: $2x + 3y = 4$
$3(4x - y = 22)$

a) Does adding or subtracting the equations eliminate either of the variables?

b) Multiply the second equation by 3 and then add the two equations.

$12x - 3y = 66$
$+\ 2x + 3y = 4$
$14x + 0 = 70$

c) Solve and verify the system.

$14x = 70$
$x = \frac{70}{14}$
$x = 5$

$2(5) + 3y = 4$
$10 + 3y = 4$
$3y = -6$
$y = \frac{-6}{3} = -2$

d) Consider the original system. Multiply the first equation by an appropriate number which will eliminate x by addition or subtraction. Solve the system.

$2(2x + 3y = 4) = 4x + 6y = 8$
$-4x - y = 22$
$0 + 7y = -14$

$\frac{7y}{7} = \frac{-14}{7}$
$y = -2$

Consider the system of equations: $3(5a + 3b = 3)$
$5(3a - 7b = 81)$

a) Choose appropriate whole numbers to multiply each equation so that the system can be solved by eliminating b.

b) Solve and verify the system by eliminating b.

$35a + 21b = 21$
$+\ 9a - 21b = 243$
$44a + 0 = 264$

$a = \frac{264}{44} = 6$

c) Choose appropriate whole numbers to multiply each equation so that the system can be solved by eliminating a.

d) Solve the system by eliminating a.

$15a + 9b = 9$
$-(15a - 35b = 405)$
$0 + 44b = 396$

$44b = 396$
$b = \frac{396}{44}$
$b = 9$

Solve the following system using elimination. $4x + 2y - 13 = 0$ $\quad 3x = 5y + 26$

$5(4x+2y-13=0)$ $\quad 2(3x-5y-26=0)$

$20x+10y-65=0$

$+\ 6x-10y-52=0$

$26x+0-117=0$

$26x = 117$

$x = \frac{117}{26} =$

Solve the following system using elimination.

$$\frac{x-2}{3} - \frac{y+2}{5} = 2, \qquad \frac{3}{5}(x+1) - \frac{4}{5}(y-3) = \frac{21}{2}$$

#(1, 2, 4)ab, 5b, 6

9

Complete Assignment Questions #4 - #12

Assignment

1. In each of the following systems:

- solve the system using the method of elimination by adding the equations.
- verify the solution satisfies both equations.

a) $8x - y = 10$
$4x + y = 14$

b) $x + 2y = 3$
$-x + 3y = 2$

c) $4a - 3b = 2$
$-4a - b = 6$

2. In each of the following systems:

- solve the system using the method of elimination by subtracting the equations.
- verify the solution satisfies both equations.

a) $7x + y = 15$
$3x + y = 3$

b) $5m + 3n = 10$
$5m - 2n = -15$

c) $4a - 3b = -18$
$-2a - 3b = -9$

3. Solve and verify each of the following systems using the method of elimination.

a) $-10p + 10q = 3$
$10p + 5q = 6$

b) $x + 4y = -0.5$
$5x + 4y = 2.3$

c) $4x + 2y - 31 = 0$
$-4x + 6y - 13 = 0$

4. Solve each of the following systems by elimination. Check each solution.

a) $2a + 5b = 16$
$a - b = 1$

b) $4x - 3y = 9$
$2x - 5y = 1$

c) $5x - 2y = 0.6$
$2x + y = 1.5$

5. Solve each of the following systems by elimination. Check each solution.

a) $2x + 4y = 7, \quad 4x - 3y = 3$

b) $5x = 8y, \quad 4x - 3y + 17 = 0$

c) $7e + 4f - 1 = 0, \quad 5e + 3f + 1 = 0$

d) $3x + 2y - 6 = 0, \quad 9x = 5y + 18$

6. Consider the system of equations $x - 2y + 1 = 0$, $2x + 3y = 12$. Solve the system by

a) elimination **b)** substitution

Which method do you prefer?

7. Consider the system of equations: $11x + 3y + 2 = 0$, $11x - 5y - 62 = 0$.

Solve the system by

a) elimination **b)** substitution

Which method do you prefer?

8. Solve each of the following systems by elimination. Explain the results.

a) $-2x + 6y - 1 = 0,\ \ 5x - 15y + 2.5 = 0$ **b)** $2x - 4y = 7,\ \ -7x + 14y = -21$

9. Solve each of the following systems by elimination.

a) $3x - \frac{1}{2}y = 5$

$\frac{1}{3}x + \frac{1}{4}y = 3$

b) $\frac{m}{2} - \frac{n-4}{4} = 2$

$\frac{3m}{4} - \frac{n}{5} = 5$

Multiple Choice

10. When b is eliminated from the equations $2x + b = 8$ and $5x + 2b = 2$, we obtain

A. $7x = 10$
B. $9x = 18$
C. $x = -14$
D. $3x = -6$

11. The solution to the systems of equations $x + y = 0$, $\frac{1}{2}x + \frac{1}{3}y = 1$ is

A. $x = 6, y = -6$
B. $x = 1, y = -1$
C. $x = 0, y = -0$
D. $x = -6, y = 6$

Numerical Response

12. If $\frac{1}{3}x + 5 = \frac{2}{3}y$ and $\frac{1}{2}x + \frac{1}{3}y = \frac{1}{3}$, then the value of $y - \frac{1}{2}x$, to the nearest tenth, is _____ .

(Record your answer in the numerical response box from left to right)

Answer Key

1. **a)** $x = 2,\ y = 6$ **b)** $x = 1,\ y = 1$ **c)** $a = -1,\ b = -2$

2. **a)** $x = 3,\ y = -6$ **b)** $m = -1,\ n = 5$ **c)** $a = -\frac{3}{2},\ b = 4$

3. **a)** $p = \frac{3}{10},\ q = \frac{3}{5}$ **b)** $x = 0.7,\ y = -0.3$ **c)** $x = 5,\ y = \frac{11}{2}$

4. **a)** $a = 3,\ b = 2$ **b)** $x = 3,\ y = 1$ **c)** $x = 0.4,\ y = 0.7$

5. **a)** $x = \frac{3}{2},\ y = 1$ **b)** $x = -8,\ y = -5$ **c)** $e = 7,\ f = -12$ **d)** $x = 2,\ y = 0$

6. $x - 3,\ y - 2$ **7.** $x - 2,\ y - -8$

8. **a)** There are an infinite number of solutions of the form $x = a, y = \frac{1}{6}(2a + 1), a \in R$ because the equations are identical (the resulting equation reduces to 0 = 0).

b) There are no solutions since the graphs of the equations are parallel lines (the resulting equation reduces to e.g. 0 = 7).

9. **a)** $x = 3,\ y = 8$ **b)** $m = 12,\ n = 20$ **10.** C **11.** A **12.**

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