

Lesson 3: Solving Systems of Linear Equations by Inspection and by Substitution

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Systems of Linear Equations Lesson #3: Solving Systems of Linear Equations by Inspection and by Substitution

Method of Inspection

In some simple cases, a system of linear equations can be solved by mentally trying different values for the variables until a correct solution is reached. This is called the method of inspection and is really only practical if the equations are very simple.



Class Ex. #1

The sum of two numbers is 14 and the difference between the numbers is 2.
Form two equations in two variables and determine the numbers by inspection.

$$\begin{aligned}x + y &= 14 \\x - y &= 2\end{aligned}$$

test

$$\begin{aligned}1 + 13 &= 14 \\1 - 13 &= -12 \quad \times \\8 + 6 &= 14 \\8 - 6 &= 2 \quad \checkmark\end{aligned}$$

$$x = 8, y = 6$$



Class Ex. #2

Solve the system $x + 2y = 12$ and $x + 3y = 17$ by inspection.

$$\begin{aligned}x + 2y &= 12 \\x + 3y &= 17\end{aligned}$$

$$\begin{aligned}4 + 2(4) &= 12 \\4 + 3(4) &= 16 \quad \times \\6 + 2(3) &= 12 \\6 + 3(3) &= 15 \quad \times\end{aligned}$$

$$\begin{aligned}2 + 2(5) &= 12 \\2 + 3(5) &= 17 \quad \checkmark\end{aligned}$$

$$x = 2 \\y = 5$$

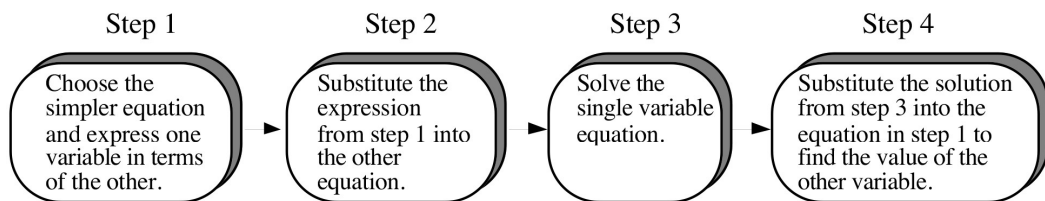
Complete Assignment Questions #1 - #3

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Method of Substitution

If the equations are too complex to be solved by inspection, then algebraic procedures such as the method of substitution and the method of elimination (next lesson) may be used.

When using the method of substitution, there are four steps which are shown in the flowchart below.



Consider the following system of equations:
 $x = \underline{\hspace{2cm}}$
 $y = \underline{\hspace{2cm}}$

① $x + 4y = 17$
 ② $2x - y = 7$

a) Solve the system using the method of substitution by rewriting the first equation in the form $x = \dots$

Handwritten solution for part a):

$x + 4y = 17$
 $x = 17 - 4y$ (step 1)
 $2(17 - 4y) - y = 7$ (step 2: ① into ②)
 $34 - 8y - y = 7$
 $34 - 9y = 7 - 34$
 $-9y = -27$
 $y = 3$ (step 3: solve for y)
 $x = 17 - 4(3)$ (step 4: sub y into ①)
 $x = 17 - 12$
 $x = 5$
solution $x = 5$
 $y = 3$

b) ~~Solve the system using the method of substitution by rewriting the first equation in the form $y = \dots$~~

c) Which method, a) or b), was simpler?

d) Verify that the solution satisfies both equations.

Handwritten verification:

① $x + 4y = 17$
 $5 + 4(3) \stackrel{?}{=} 17$
 $5 + 12 = 17 \checkmark$
 ② $2x - y = 7$
 $2(5) - 3 \stackrel{?}{=} 7$
 $10 - 3 = 7 \checkmark$

e) Check the solution using a graphing calculator.



Consider the following system of equations:

$$\begin{aligned} x + 3y &= 0 & 8x - 9y &= 66 \\ 4x + 3y &= 0 & 8x - 9y &= 5 \end{aligned}$$

a) Solve and verify the system using the method of substitution.

$$\textcircled{1} x + 3y = 0 \quad \textcircled{2} 8x - 9y = 66$$

$$x = -3y$$

$$8(-3y) - 9y = 66$$

$$-24y - 9y = 66$$

$$-33y = 66$$

$$y = -2$$

$$\begin{aligned} x &= -3(-2) \\ x &= 6 \end{aligned}$$

1. Rearrange $\textcircled{1}$

2. Sub $\textcircled{1}$ into $\textcircled{2}$

3. solve for y

4. Sub y into $\textcircled{1}$

verify $x=6, y=-2$

$$\textcircled{1} 6 + 3(-2) \stackrel{?}{=} 0$$

$$6 - 6 = 0 \quad \checkmark$$

$$\textcircled{2} 8(6) - 9(-2) \stackrel{?}{=} 66$$

$$48 + 18 = 66 \quad \checkmark$$

verified \checkmark

~~Check the solution using a graphing calculator.~~



Consider the following system of equations:

$$\begin{aligned} 5(2a - 3) + b &= 5 & 6a - 2(b - 4) &= 20 \end{aligned}$$

a) Solve the system using the method of substitution.

$$\textcircled{1} y = x + 2 \quad \textcircled{2} 3x + 4y = 1$$

*sub $\textcircled{1}$ into $\textcircled{2}$

*solve for x

$$3x + 4(x + 2) = 1$$

$$3x + 4x + 8 = 1$$

$$7x + 8 = 1$$

$$7x = -7$$

$$x = -1$$

*sub x into $\textcircled{1}$

$$y = -1 + 2$$

$$y = 1$$

b) Verify algebraically that the solution satisfies both equations.

$$\textcircled{1} 1 \stackrel{?}{=} -1 + 2 \quad \checkmark$$

$$\textcircled{2} 3(-1) + 4(1) \stackrel{?}{=} 1$$

$$-3 + 4 = 1 \quad \checkmark$$

Complete Assignment Questions #4 - #10

1ac, 3, 4b, 5ad



Assignment

- Solve the following linear systems by method of inspection.
a) $x + y = 9$, $x - y = 1$ b) $x + y = 12$, $x - y = 0$ c) $x + y = 4$, $x - y = 6$
- At the Little River Pow Wow, a vendor sells a salmon burger and two cans of cola for \$8. If two salmon burgers and two cans of cola sell for \$14, then determine the cost of
a) a salmon burger b) a can of cola
- Tickets are on sale for a music concert. Three adult tickets and two child tickets cost \$90. Three adult tickets and four child tickets cost \$120.
a) Write a system of equations in two variables to represent the above information.

b) Determine the total cost of two adult tickets and three child tickets.
- In each of the following systems:
 - solve the system using the method of substitution
 - verify the solution satisfies both equations
 - check the solution by graphinga) $y = x + 2$, $3x + 4y = 1$ b) $x - 2y = 10$, $x + 5y + 4 = 0$

5. Solve each of the following systems by substitution. Check each solution.

a) $4p + q = 0$, $7p + 4q = 3$

b) $6u - 3v + 4 = 0$, $3u = 3v - 5$

c) $2x - 5y = -7$
 $\frac{1}{2}x - y = 3$

d) $2(x + 2) + y = 8$
 $7x - 2(y - 3) + 24 = 0$

6. The straight line $px + qy + 14 = 0$ passes through the points $(-3, 1)$ and $(-4, 6)$.
- a) Substitute the x and y -coordinates of the two points into the equation of the line to form two equations in p and q .
 - b) Solve this system of equations by substitution to determine the values of p and q and write the equation of the line.
 - c) Verify the equation in **b)** using the slope formula and the point-slope equation of a line formula.

7. Solve the following systems by substitution. Explain the results.

a) $y = 3x - 7$
 $6x - 2y = 14$

b) $x = 3y + 2$
 $2x - 6y = 5$

- Multiple Choice** 8. If $x + 2y = 10$ and $x - 2y = 2$, then $x + y$ is equal to

- A. 8
- B. 12
- C. 13
- D. -2

9. When solving a system of equations, one of which is $\frac{x}{2} - \frac{y}{3} = 1$, a substitution which can be made is

- A. $x = \frac{1}{3}(2y + 1)$
- B. $y = \frac{1}{2}(3x - 1)$
- C. $x = \frac{1}{2}(3y + 6)$
- D. $y = \frac{1}{2}(3x - 6)$

Numerical Response

10. If $s - 8t + 20 = 5s - 7t + 1 = 0$, then the value of $s + t$, to the nearest tenth, is _____ .

(Record your answer in the numerical response box from left to right)

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Answer Key

- 1. a) $x = 5, y = 4$ b) $x = 6, y = 6$ c) $x = 5, y = -1$
- 2. a) \$6 b) \$1
- 3. a) $3x + 2y = 90, 3x + 4y = 120$ b) \$85
- 4. a) $x = -1, y = 1$ b) $x = 6, y = -2$
- 5. a) $p = -\frac{1}{3}, q = \frac{4}{3}$ b) $u = \frac{1}{3}, v = 2$ c) $x = 44, y = 19$ d) $x = -2, y = 8$
- 6. a) $-3p + q + 14 = 0, -4p + 6q + 14 = 0$ b) $p = 5, q = 1$ $5x + y + 14 = 0$
- 7. a) There are an infinite number of solutions of the form $x = a, y = 3a - 7, a \in R$ because the equations are identical, (the resulting equation reduces to $0 = 0$).
- b) There are no solutions since the graphs of the equations are parallel lines, (the resulting equation reduces to $4 = 5$).

8. A

9. D

10.

7	.	0	
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