Equations of Linear Relations Lesson #4: The General Form Equation Ax + By + C = 0

Review

In Lesson #2 and Lesson #3 of this unit we studied two forms of the equation of a straight line. The form of these equations are

- the slope y-intercept form which can also be writen as $y = mx + b$
- the point-slope form which can also be written as $y - y_1 = m(x - x_1)$

General Form of the Equation of a Line → $Ax + By + C = 0$

The **general form** of the equation of a line is an equation where all the terms are collected to the left side of the equation. The right side of the equation is zero.
It has the following characteristics:

- It is written as $Ax + By + C = 0$, where A, B, and C are expressed as **integers** if possible, and A is usually positive.
- It allows us to write equations for oblique lines, horizontal lines, and vertical lines.
- In some texts, the form $Ax + By + C = 0$ is referred to as **standard form**.

Convert the following equations from slope y-intercept form, $y = mx + b$, to general form, $Ax + By + C = 0$, where A, B, and C are integers.

a) $y + 2 = \frac{1}{2}(x - 3)$

- move everything to one side

$y + 2 = \frac{1}{2}x - \frac{3}{2}$ (−2, −y)

$0 = \left(\frac{1}{2}x - \frac{3}{2} - 2 - y\right)^2$

$0 = x - 3 - 4 - 2y$

$0 = x - 2y - 7$

b) $y = \frac{2}{3}x + 7$ (×3, −3y)

$3y = 2x + 21$

$0 = 2x - 3y + 21$

c) $y = \frac{1}{4}x + \frac{3}{5}$ (+¼x, −⅗)

not a fraction

$\left(\frac{1}{4}x + y - \frac{3}{5}\right)^4 = 0$

$\left(x + 4y - \frac{12}{5}\right)^5 = 0$

$5x + 20y - 12 = 0$

Determine the equation, in general form, of the line with slope $-\frac{3}{7}$ passing through $(2, -1)$.

$y - y_1 = m(x - x_1)$ * put it in point slope form

$y + 1 = -\frac{3}{7}(x - 2)$ * rearrange to general form

$y + 1 = -\frac{3}{7}x + \frac{6}{7}$ (×7)

$7y + 7 = -3x + 6$ (+3x −6)

$3x + 7y + 1 = 0$

$3x+7y+1=0$

$y=mx+b$

$y-y_1=m(x-x_1)$

Determining the Slope and y-intercept from Ax + By + C = 0

Given the equation of a line in general form, $Ax+By+C=0$, the slope and y-intercept can be found by converting the equation into slope y-intercept form, $y=mx+b$.

Determine the slope and y-intercept of the graph of the following lines.

a) $2x-5y+25=0$

$-2x \quad -25 \quad -2x \quad -25$

*solve for y

$-5y=-2x-25$

$\frac{-5y}{-5}=\frac{-2x}{-5}-\frac{25}{-5}$

$y=\frac{2}{5}x+5$

b) $6x+2y-15=0$

$-6x \quad +15 \quad -6x+15$

*get y alone

$\frac{2y}{2}=\frac{-6x}{2}+\frac{15}{2}$

$y=-3x+\frac{15}{2}$

Complete Assignment Questions #1 - #4

The lines $3x-4y+8=0$ and $5x-ky-6=0$ have the same y-intercept. Determine the value of k.

*put both into y=mx+b

$3x-4y+8=0$

$-3x \quad -8 \quad -3x-8$

$\frac{-4y}{-4}=\frac{-3x}{-4}-\frac{8}{-4}$

$y=\frac{3}{4}x+2$

$5x-ky-6=0$

$-5x \quad +6 \quad -5x+6$

$\frac{-ky}{-k}=\frac{-5x}{-k}+\frac{6}{-k}$

$y=\frac{5}{k}x-\frac{6}{k}$

*y-int are same

$\frac{2}{2}=\frac{-6}{2k}$

$k=-3$

Which of the following lines is/are perpendicular to the line $4x-2y+9=0$?

i) $6x+3y-1=0$ **ii)** $x+2y-12=0$ **iii)** $5x+10y=0$

#1-4

15

Use the following information to answer Class Ex. #6.

A student made the following statements about the line with equation $2y = 5x + 12$.

Statement 1: The line has a slope of 5.
Statement 2: The line is parallel to $10x - 4y + 13 = 0$.
Statement 3: The line passes through $(-2, 1)$.

Which of the above statement(s) is/are true?

A. 1 and 2 only
B. 1 and 3 only
C. 2 and 3 only
D. some other combination of statements 1, 2, and 3

Complete Assignment Questions #5 - #16

Assignment

1. Convert the following equations to general form $(Ax + By + C = 0)$ where A, B, and C are integers.

a) $y - 4 = 7(x - 1)$ **b)** $y = -2x + 9$ **c)** $y = mx + b$

d) $y = -\frac{3}{4}x + 5$ **e)** $y + 8 = -\frac{3}{2}(x - 5)$ **f)** $y = \frac{5}{3}x - \frac{1}{4}$

2. Find the equation, in general form, of the line through the given point and with the given slope.

a) $(6, 1)$, 3 **b)** $(-9, -2)$, $\frac{2}{5}$ **c)** $(0, 0)$, $\frac{4}{3}$

3. Determine the slope and y-intercept of the graph of the following lines.

a) $x + y - 11 = 0$ **b)** $3x - 2y + 30 = 0$ **c)** $3x + 6y - 7 = 0$

4. Determine the slope, y-intercept, and x-intercept of the graph of the following lines.

a) $2x + y - 6 = 0$ **b)** $5x - 2y + 20 = 0$ **c)** $4x - 5y - 3 = 0$

5. Consider the lines $x - 2y + 1 = 0$ and $4x + ky - 8 = 0$.

a) Determine the value of k if the lines have the same slope.

b) Determine the value of k if the lines have the same y-intercept.

6. Consider the lines $3x - 5y - 15 = 0$ and $ax + 2y - 6 = 0$.

a) Determine the value of a if the lines have the same slope.

b) Determine the value of a if the lines have the same x-intercept.

Multiple Choice **7.** The equation of the line passing through the orgin with slope $-\frac{1}{2}$ is

A. $x + 2y = 0$

B. $x - 2y = 0$

C. $2x + y = 0$

D. $2x - y = 0$

8. Match each equation on the left with the correct characteristic of the graph of the equation on the right. Each characteristic may be used once, more than once, or not at all.

Equation		Characteristic	
i)	$6x - 2y + 5 = 0$	**A.**	Slope $= -\frac{1}{3}$
ii)	$2x - 5y = 0$	**B.**	y-intercept $= -\frac{5}{2}$
iii)	$x + 3y + 6 = 0$	**C.**	Passes through $(-10, -4)$
iv)	$x - 4y + 10 = 0$	**D.**	Slope $= 0$
v)	$2x - y - 5 = 0$	**E.**	y-intercept $= \frac{5}{2}$
		F.	Perpendicular to $y = \frac{5}{2}x - 3$
		G.	x-intercept $= \frac{5}{2}$

9. The slope of the line with equation $6x + 5y - 1 = 0$ is

A. $-\frac{6}{5}$

B. $-\frac{5}{6}$

C. $\frac{6}{5}$

D. $\frac{1}{5}$

10. Which line has a y-intercept of 1?

A. $x + 5y + 1 = 0$
B. $x + 3y + 3 = 0$
C. $x - 2y + 2 = 0$
D. $2y = 3x + 1$

11. The slope of a line perpendicular to the line $x + 3y + 8 = 0$ is

A. -8

B. $-\frac{1}{3}$

C. $\frac{1}{3}$

D. 3

12. The line $2y + 3x + 6 = 0$ intersects the y-axis at P.
The slope of the line joining P to $Q(6, -2)$ is

A. $-\frac{5}{6}$

B. $\frac{1}{6}$

C. $-\frac{1}{6}$

D. $-\frac{2}{3}$

13. The lines with equations $ay = 4x + 9$ and $y = 5x - 7$ are perpendicular.
The value of a is

A. $\frac{4}{5}$

B. $-\frac{4}{5}$

C. $-\frac{5}{4}$

D. -20

Use the following information to answer the next question.

Consider the following statements about all the lines in the form $kx + 4y - 8 = 0$, where $k \in R$.

Statement 1: The lines have the same slope.
Statement 2: The lines have the same y-intercept.
Statement 3: The lines have the same x-intercept.

14. Which of the above statement(s) is/are true?

A. 1, 2, and 3
B. 1 only
C. 2 only
D. 3 only

15. Line L has equation $5x - 3y + 21 = 0$. A is the point $(-6, -3)$, B is $(3, -2)$, and C is $(-3, 2)$. Which of these points lie on line L?

A. A only
B. A and B only
C. A and C only
D. B and C only

Numerical Response

16. Given that the line joining the points $(2, 3)$ and $(8, -q)$, where $q \in W$, is perpendicular to the line $3x - 2y - 5 = 0$, then the value of q is _____ .

(Record your answer in the numerical response box from left to right)

Answer Key

1. **a)** $7x - y - 3 = 0$ **b)** $2x + y - 9 = 0$ **c)** $mx - y + b = 0$
d) $3x + 4y - 20 = 0$ **e)** $3x + 2y + 1 = 0$ **f)** $20x - 12y - 3 = 0$

2. **a)** $3x - y - 17 = 0$ **b)** $2x - 5y + 8 = 0$ **c)** $4x - 3y = 0$

3. **a)** slope $= -1$, y–int $= 11$ **b)** slope $= \frac{3}{2}$, y–int $= 15$ **c)** slope $= -\frac{1}{2}$, y–int $= \frac{7}{6}$

4. **a)** slope $= -2$, y–int $= 6$, x–int $= 3$
b) slope $= \frac{5}{2}$, y–int $= 10$, x–int $= -4$
c) slope $= \frac{4}{5}$, y–int $= -\frac{3}{5}$, x–int $= \frac{3}{4}$

5. **a)** -8 **b)** 16 **6.** **a)** $-\frac{6}{5}$ **b)** $\frac{6}{5}$

7. A
8. **i)** E **ii)** C **iii)** A **iv)** E **v)** G
9. A **10.** C **11.** D **12.** B
13. D **14.** C **15.** C **16.** 1