

## Equations of Linear Relations Lesson #2: Writing Equations Using $y = mx + b$

**Review**

We have learned that the graph of an equation in the form  $y = mx + b$  is a straight line with slope  $m$  and y-intercept  $b$ .

**Using the Form  $y = mx + b$  to Write the Equation of a Line**

The form  $y = mx + b$  can be used to determine the equation of a line when the following information is given:

- the slope of the line
- the y-intercept of the line.

**Class Ex. #1**

Write the equation of a line passing through the point  $(0, 2)$  with slope  $\frac{5}{2}$ .

$y = mx + b$   
 $2 = \frac{5}{2}(0) + b$  \*solve for b  
 $2 = b$   
 $y = \frac{5}{2}x + 2$

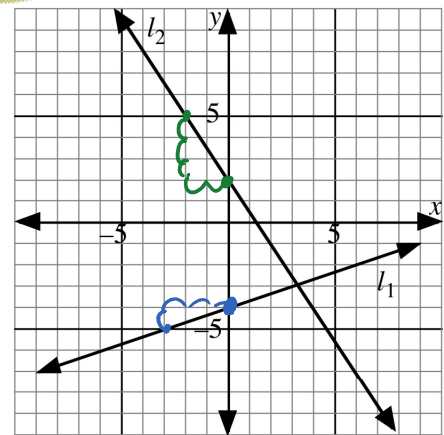
$\swarrow m$   
we don't know b

**Class Ex. #2**

Each line on the grid passes through points with integer coordinates. In each case, state the slope and y-intercept of the line, and determine the equation of the line.

$l_1$ : slope =  $\frac{\uparrow 1}{\rightarrow 3} = \frac{1}{3}$   
 y-int = -4  
 $y = \frac{1}{3}x - 4$

$l_2$ : slope  $\frac{\downarrow 3}{\rightarrow 2} = -\frac{3}{2}$   $y = 2$   
 $y = -\frac{3}{2}x + 2$



**Class Ex. #3**

Determine the equation of the following lines.

- a) The line parallel to  $y = \frac{1}{3}x + 4$ , and with the same y-intercept as  $y = 6x - 7$ .

$y = \frac{1}{3}x - 7$

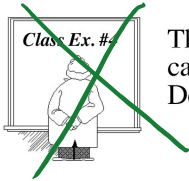
- b) The line passing through  $(0, 9)$ , and perpendicular to the line joining  $(2, -6)$  and  $(-5, 0)$ .

find slope  $\frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - (-6)}{-5 - 2} = \frac{6}{-7} \rightarrow \frac{7}{6}$

$$y = mx + b$$

$$9 = \frac{7}{6} \cdot 0 + b \Rightarrow b = 9 \Rightarrow y = \frac{7}{6}x + 9$$

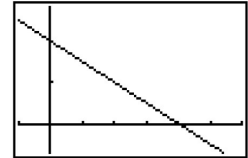
**400** Equations of Linear Relations Lesson #2: Writing Equations Using  $y = mx + b$



The diagram shows the display from a graphing calculator screen. The intercepts are integers. Determine the equation of the line shown.

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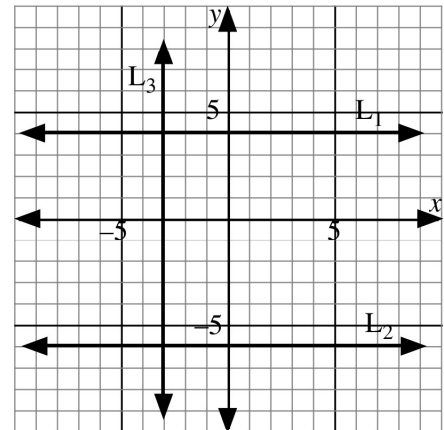
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Xmin=-5
Xmax=30
Xscl=5
Ymin=-5
Ymax=20
Yscl=7
Xres=1
    
```



**Complete Assignment Questions #1 - #5**

**Horizontal and Vertical Lines**

- State the slope and y-intercept of the horizontal line  $L_1$  shown on the grid.
- Use the form  $y = mx + b$  to determine the equation of the horizontal line  $L_1$ .
- Predict the equation of the horizontal line  $L_2$ . Use a graphing calculator to verify.



- State the slope and y-intercept of the vertical line  $L_3$  shown on the grid.
- Why can we not use the form  $y = mx + b$  to determine the equation of the vertical line  $L_3$ ?
- Predict the equation of the vertical line  $L_3$ . Why can we not use a graphing calculator to verify?

**The equation  $y = k$  represents a horizontal line through  $(0, k)$ .**  
**The equation  $x = k$  represents a vertical line through  $(k, 0)$ .**

**The equation  $y = k$  represents a horizontal line through  $(0, k)$ .**  
**The equation  $x = k$  represents a vertical line through  $(k, 0)$ .**

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Class Ex. #5

Determine the equation of the line through the point  $(-2, 8)$  and

- a) parallel to the  $y$ -axis                      b) parallel to the  $x$ -axis

Complete Assignment Questions #6 - #15

HW: 1, 2, 4, 5  
15

## Assignment

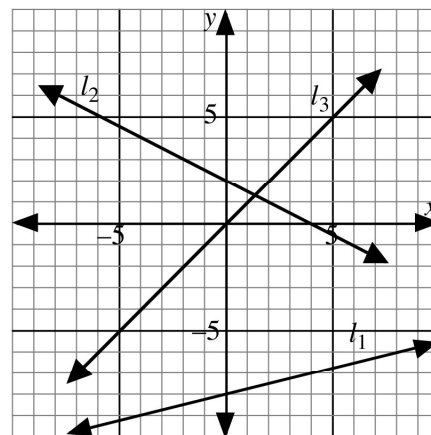
1. Write the equation of each line

- |  |  |
|--|--|
| <p>a) with slope 4 and <math>y</math>-intercept <math>-6</math></p>  | <p>b) with a <math>y</math>-intercept of 3 and a slope of <math>-\frac{4}{3}</math></p>                                      |
| <p>c) passing through the origin with a slope of <math>-\frac{3}{5}</math></p>   | <p>d) with <math>y</math>-intercept <math>-5</math> and parallel to <math>y = x</math></p>                                   |
| <p>e) with a <math>y</math>-intercept of <math>-9</math> and perpendicular to <math>y = -\frac{2}{3}x + 7</math></p>               | <p>f) with the same <math>y</math>-intercept as <math>y = x + 2</math> and parallel to <math>y = \frac{1}{4}x - 6</math></p> |
| <p>g) through the point <math>(0, 1)</math> and perpendicular to <math>y = 4x - 2</math></p>                                       | <p>h) through the point <math>(0, 4)</math> and parallel to <math>y = \frac{1}{10}x + 24</math></p>                          |
| <p>i) with the same <math>y</math>-intercept as <math>y = 2x - 3</math> and perpendicular to <math>y = \frac{7}{3}x - 2</math></p> | <p>j) with the same <math>y</math>-intercept as <math>y = ax + b</math> and perpendicular to <math>y = cx + d</math></p>     |

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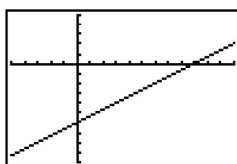
2. Every line on the grid passes through points with integer coordinates. Determine the equation of each line.



3. Each diagram represents the image from the display of a graphing calculator and the window setting used to graph a linear equation. The  $x$ - and  $y$ -intercepts of each graph are integers.

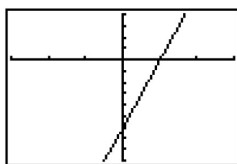
In each case complete the table.

a) WINDOW  
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 Xmax=12  
 Xscl=1  
 Ymin=-10  
 Ymax=5  
 Yscl=1  
 Xres=1



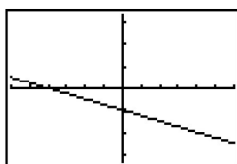
$x$ -intercept	
$y$ -intercept	
slope	
equation	

b) WINDOW  
 Xmin=-6  
 Xmax=6  
 Xscl=2  
 Ymin=-36  
 Ymax=16  
 Yscl=4  
 Xres=1



$x$ -intercept	
$y$ -intercept	
slope	
equation	

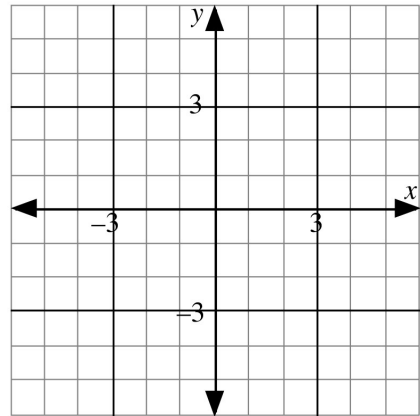
c) WINDOW  
 Xmin=-6  
 Xmax=6  
 Xscl=1  
 Ymin=-10  
 Ymax=10  
 Yscl=3  
 Xres=1



$x$ -intercept	
$y$ -intercept	
slope	
equation	

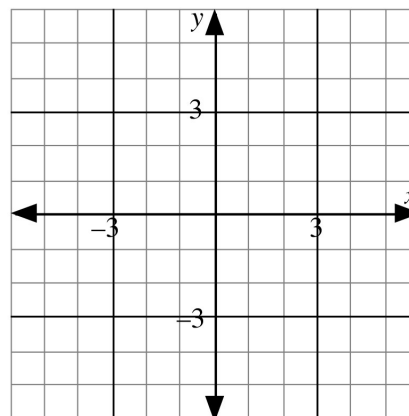
4. Determine the equation of the line which passes through the point  $(0, 16)$  and is parallel to the line which passes through  $(1, 3)$  and  $(4, -6)$ .
  
5. Determine the equation of the line which passes through the point  $(0, -1)$  and is perpendicular to the line which passes through  $(7, -2)$  and  $(12, -3)$ .
  
6. State the equations of the following lines
  - a) through the point  $(-5, 3)$  and parallel to the  $y$ -axis
  - b) through the point  $(-5, 3)$  and parallel to the  $x$ -axis
  - c) through the point  $(1, -1)$  and parallel to the  $x$ -axis
  - d) through the point  $(a, b)$  and parallel to the  $y$ -axis

7. Consider the graph of the function with equation  $y = 2$ .
  - a) State the values of  $m$  and  $b$ .
  - b) Sketch the graph on the grid provided.
  - c) State the  $x$ - and  $y$ -intercepts of the graph.
  - d) Determine the domain and range of the function.
  - e) On the same grid draw the line with equation  $y = 2x - 4$  without using a graphing calculator.
  - f) State the coordinates of the point of intersection of the two lines.
  - g) On the grid, draw the line with equation  $y = -5$ .



8. Consider the graph of the relation with equation  $x = -4$ .

- a) Sketch the graph on the grid provided.  
 b) State the  $x$ - and  $y$ -intercepts of the graph.  
 c) Explain why the relation is not a function.



- d) Explain why the  $\boxed{y=}$  editor key cannot be used to graph  $x = -4$ .

- e) Determine the domain and range of the relation with equation  $x = -4$ .

- f) On the grid draw the line with equation  $x = 2$ .

9. Write the equation of each line

- a) parallel to the  $x$ -axis through  $(3, -9)$       b) parallel to the  $y$ -axis through  $(3, -9)$   
 c) perpendicular to the  $x$ -axis through  $(1, 4)$       d) perpendicular to the  $y$ -axis through  $(1, 4)$   
 e) the  $x$ -axis      f) the  $y$ -axis

**Multiple Choice**

10. A line is parallel to the  $y$ -axis and passes through the point  $(2, -7)$ . The equation of the line is

- A.  $x = 2$   
 B.  $x = -7$   
 C.  $y = 2$   
 D.  $y = -7$

11. A line is parallel to the  $x$ -axis and passes through the point  $(-6, 10)$ .  
The equation of the line is
- A.  $x = 10$
  - B.  $x = -6$
  - C.  $y = 10$
  - D.  $y = -6$
12. The line through the origin, perpendicular to the line with equation  $y = \frac{2}{3}x$ , has equation
- A.  $y = \frac{2}{3}x$
  - B.  $y = \frac{3}{2}x$
  - C.  $y = -\frac{2}{3}x$
  - D.  $y = -\frac{3}{2}x$
13. The point  $(2, -1)$  lies on a line with slope 3. The  $y$ -intercept of the line is
- A.  $-7$
  - B.  $-5$
  - C.  $5$
  - D.  $7$

**Numerical Response**

14. Consider the line which is perpendicular to the line  $y = \frac{1}{3}x + 4$  and has the same  $y$ -intercept as  $y = 6x - 7$ . If the equation of this line is written in the form  $y = mx + b$ , then the exact value of  $m - b$  is \_\_\_\_\_.

(Record your answer in the numerical response box from left to right)

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15. Two perpendicular lines intersect on the  $y$ -axis. One line has equation  $y = 4x + 6$ . If the equation of the other line is  $y = mx + b$ , then the exact value of  $m + b$  is \_\_\_\_\_.

(Record your answer in the numerical response box from left to right)

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**Answer Key**

1. a)  $y = 4x - 6$     b)  $y = -\frac{4}{3}x + 3$     c)  $y = -\frac{3}{5}x$     d)  $y = x - 5$     e)  $y = \frac{3}{2}x - 9$   
 f)  $y = \frac{1}{4}x + 2$     g)  $y = -\frac{1}{4}x + 1$     h)  $y = \frac{1}{10}x + 4$     i)  $y = -\frac{3}{7}x - 3$     j)  $y = -\frac{1}{c}x + b$

2.  $l_1: y = \frac{1}{4}x - 8$      $l_2: y = -\frac{1}{2}x + 2$      $l_3: y = x$

3. a)

x-intercept	9
y-intercept	-6
slope	$\frac{2}{3}$
equation	$y = \frac{2}{3}x - 6$

b)

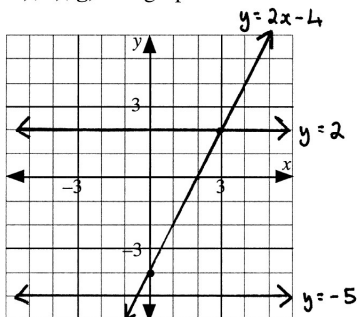
x-intercept	2
y-intercept	-24
slope	12
equation	$y = 12x - 24$

c)

x-intercept	-4
y-intercept	-3
slope	$-\frac{3}{4}$
equation	$y = -\frac{3}{4}x - 3$

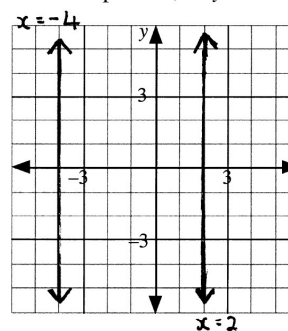
4.  $y = -3x + 16$     5.  $y = 5x - 1$     6. a)  $x = -5$     b)  $y = 3$     c)  $y = -1$     d)  $x = a$

7. a)  $m = 0, b = 2$   
 b), e), g) see graph below



- c) no x-intercept, y-intercept = 2  
 d) domain  $x \in R$ , range  $y = 2$     f) (3, 2)

8. a), f) see graph below  
 b) x-intercept = -4, no y-intercept



- c) When the input = -4, there are multiple values for the output. The graph of the relation does not pass the vertical line test.  
 d) The equation  $x = -4$  cannot be written in the form  $y = \dots$   
 e) domain  $x = -4$ , range  $y \in R$

9. a)  $y = -9$     b)  $x = 3$     c)  $x = 1$     d)  $y = 4$     e)  $y = 0$     f)  $x = 0$

10. A    11. C    12. D    13. A

14. 

4			
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15. 

5	.	7	5
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## Lesson 3: Writing Equations Using $y - y_1 = m(x - x_1)$

Friday, August 31, 2018 2:43 AM

## Equations of Linear Relations Lesson #3: Writing Equations using $y - y_1 = m(x - x_1)$

### Review

In the last lesson we learned how to write the equation of a straight line using slope y-intercept form, namely  $y = mx + b$  where  $m$  is the slope and  $b$  is the y-intercept.

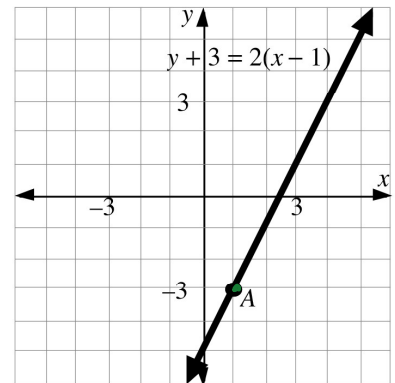
There is another equally important method exists in writing the equation of straight lines. The investigation below explores this method.

### Investigation

*Point-Slope Form*

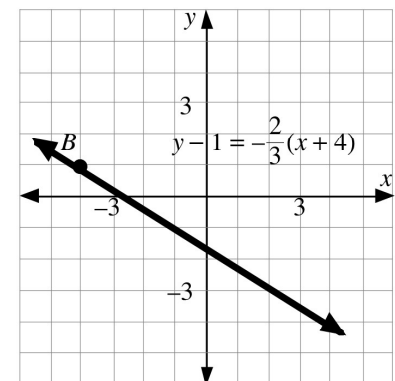
The graph of  $y + 3 = 2(x - 1)$  is shown on the grid.

- a) Determine the slope of the graph of  $y + 3 = 2(x - 1)$ .
- b) List the coordinates of point  $A$  on the line.
- c) Compare your answers in **a)** and **b)** with the numbers in the equation.



The graph of  $y - 1 = -\frac{2}{3}(x + 4)$  is shown on the grid.

- d) Determine the slope of the graph of  $y - 1 = -\frac{2}{3}(x + 4)$ .
- e) List the coordinates of point  $B$  on the line.
- f) Compare your answers in **d)** and **e)** with the numbers in the equation.



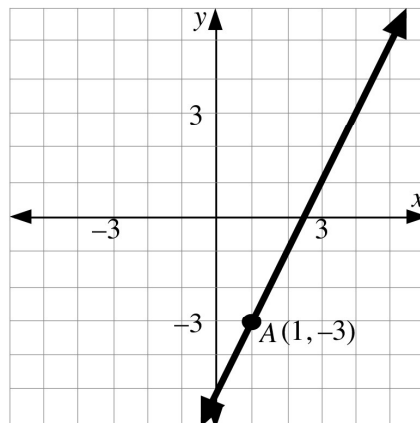
- g) Consider the graph of the linear equation  $y - y_1 = m(x - x_1)$ . Based on your observations in **c)** and **f)**, state the slope of the line, and write the coordinates of one point on the line.





### *Equation of a Line Given the Slope of the Line and a Point on the Line*

Consider the line with slope 2 passing through the point  $A(1, -3)$ . The line is shown on the grid.



Our objective is to determine the equation of the line. In other words, to find a relation between  $x$  and  $y$  which is satisfied by every point  $(x, y)$  on the line.

Let  $P(x, y)$  be any point on the line except  $A$ .

Using the slope formula we have

$$\frac{y_P - y_A}{x_P - x_A} = m_{AP} \quad \frac{y - (-3)}{x - 1} = 2$$

Cross multiply and solve for  $y$  to determine the equation of the line in the form  $y = mx + b$ .

At this point in the exploration, the equation above is valid for all points on the line except  $A$ .

Note that the coordinates of  $A$  also satisfy the equation, and therefore it is the equation of all points on the line.

In the next section we will use the same procedure to develop a formula for the equation of any line, given the slope of the line and the point on the line.

### *The Equation of the Line with slope $m$ through the point $(x_1, y_1)$*

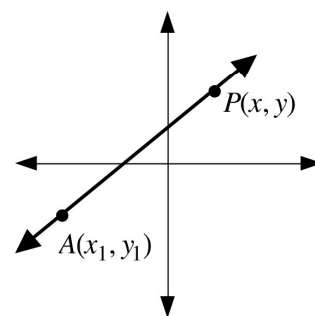
Consider the line with slope  $m$  passing through the point with coordinates  $(x_1, y_1)$ .

We will use the same procedure as above to show that the equation of the line can be expressed in the form  $y - y_1 = m(x - x_1)$ .

Let  $P(x, y)$  be any point on the line distinct from  $A$ .

Using the slope formula we have

$$m_{AP} = \frac{y_P - y_A}{x_P - x_A} \quad \text{so} \quad m = \frac{y - y_1}{x - x_1}$$





**Point-Slope Equation of a Line**  $\rightarrow y - y_1 = m(x - x_1)$

- The point-slope form of the equation of a line is  $y - y_1 = m(x - x_1)$  where  $m$  is the slope of the line, and  $(x_1, y_1)$  represents a point on the line.
- To determine the equation of a line in future math courses, the point-slope equation,  $y - y_1 = m(x - x_1)$ , is used more frequently than the slope-y-intercept equation,  $y = mx + b$ .



- The point-slope equation is used when we have the slope of a line and the coordinates of any point on that line.
- It is customary to give the final equation in slope y-intercept form or in the general form,  $Ax + By + C = 0$  (to be taught in the next lesson).



Class Ex. #1

State the equation, in point-slope form, of the line through the given point and with the given slope.

a)  $(6, 5), 3$       b)  $(1, -1), -4$       c)  $(-9, -8), \frac{1}{2}$

$$y - 5 = 3(x - 5) \quad y - (-1) = -4(x - 1) \quad y - (-9) = \frac{1}{2}(x - (-9))$$

$$y + 1 = -4(x - 1) \quad y + 9 = \frac{1}{2}x + 8$$



Class Ex. #2

In each case the slope of a line and a point on the line are given. Determine the equation of the line in slope y-intercept form,  $y = mx + b$ .

a)  $m = 5$ , point  $(-5, 2)$       b)  $m = -7$ , point  $(-3, 4)$

$$y = mx + b$$

$$2 = 5(-5) + b$$

$$2 = -25 + b$$

$$b = 27 \rightarrow y = 5x + 27$$
  

$$y = mx + b$$

$$4 = -7(-3) + b$$

$$4 = 21 + b$$

$$b = -17 \rightarrow y = -7x - 17$$



Class Ex. #3

John and Nicki were solving the following quiz question:

“Determine the equation, in slope y-intercept form, of a line with slope  $-2$  passing through the point  $(3, -5)$ ”.

John could only remember the slope y-intercept form  $y = mx + b$ , but Nicki remembered the point-slope form  $y - y_1 = m(x - x_1)$ . Complete their work which is started below.

John's work

$$y = mx + b$$

$$y = -2x + b$$

$$-5 = -2(3) + b$$

$$-5 = -6 + b$$

$$b = 1$$

$$y = -2x + 1$$

Nicki's work

$$y - y_1 = m(x - x_1)$$

$$y - y_1 = -2(x - x_1)$$

$$y - (-5) = -2(x - 3)$$

$$y + 5 = -2x + 6 - 5$$

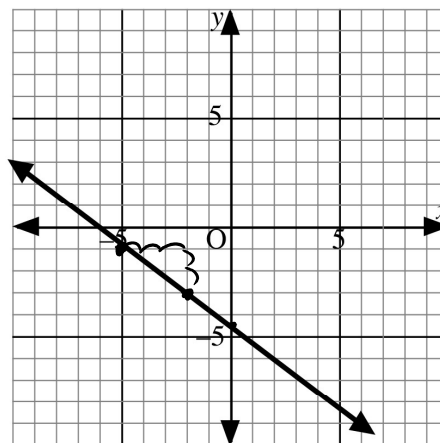
$$y = -2x + 1$$





The line on the grid passes through at least two points with integer coordinates. Determine the equation of the line in slope y-intercept form.

$-\frac{2}{3}$        $-4.5$



In each case, state the slope of the line and write the coordinates of a point on the line.

a)  $y + 11 = \frac{1}{7}(x - 4)$   
 $m = \frac{1}{7}, (4, -11)$

c)  $y = -3(x - 6)$   
 $m = -3, (6, 0)$

b)  $y - 9 = -\frac{5}{3}(x - 7)$   
 $m = -\frac{5}{3}, (7, 9)$

d)  $y = -3x - 6$   
 $m = -3, (0, -6)$   
 ↑  
 y-int

Complete Assignment Questions #1 - #9

**Assignment**

#(1-5)ace 13

1. State the equation, in point-slope form, of the line through the given point and with the given slope.

a)  $(9, 3), 4$

b)  $(8, -2), -3$

c)  $(-5, 7), 1$

d)  $(0, 3), \frac{1}{2}$

e)  $(-7, 0), \frac{1}{4}$

f)  $(-\frac{1}{2}, -\frac{5}{4}), \frac{6}{5}$

2. Write the following equations in slope y-intercept form  $y = mx + b$ .

a)  $y + 1 = 8(x - 2)$

b)  $y - 3 = -2(x - 7)$

c)  $y - 9 = -11(x + 3)$



3. Find the equation, in slope y-intercept form, of the line through the given point and with the given slope.

**a)**  $(2, 4), 6$

**b)**  $(2, -1), 2$

**c)**  $(0, 4), -2$

**d)**  $(-6, 2), \frac{1}{2}$

**e)**  $(-7, -7), 1$

**f)**  $(0, b), m$

4. Find the equation, in slope y-intercept form, of the line through the given point and with the given slope.

**a)**  $(2, -5), \frac{1}{4}$

**b)**  $(-4, 2), -\frac{1}{3}$

**c)**  $(0, -8), -\frac{3}{4}$

5. The point-slope equation of a line is given. State the slope and the coordinates of the point which were used to write the equation.

**a)**  $y - 9 = -\frac{11}{3}(x + 3)$

**b)**  $y + 3 = \frac{1}{2}x$

**c)**  $y - 8 = -2(x - 6)$

**d)**  $y = 3(x + 12)$

**e)**  $y - 9 = -\frac{5}{3}x$

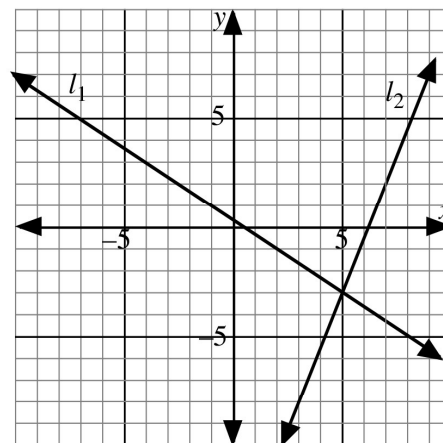
**f)**  $y = \frac{2}{5}x$





6. Two lines have been drawn on the grid. Each line passes through at least two points with integer coordinates.

Determine the equation of each line.



**Multiple Choice**

7. The equation of the line passing through the point  $(4, 2)$  with slope  $-3$  is

- A.  $y = -3x - 14$
- B.  $y = -3x - 10$
- C.  $y + 2 = -3(x + 4)$
- D.  $y - 2 = -3(x - 4)$

8. Which of the following linear equations is equivalent to  $y - 3 = -\frac{3}{4}(x + 7)$ ?

- A.  $y = -\frac{3}{4}x + \frac{9}{4}$
- B.  $y = -\frac{3}{4}x - \frac{9}{4}$
- C.  $y = -\frac{3}{4}x + 10$
- D.  $y = -\frac{3}{4}x - 10$



**Numerical Response** 9. The equation of the line with an  $x$ -intercept of  $-2$  and slope  $12$  can be written in the form  $y - A = C(x - B)$ . The value of  $A + B + C$  is \_\_\_\_\_.

(Record your answer in the numerical response box from left to right)

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**Answer Key**

1. a)  $y - 3 = 4(x - 9)$     b)  $y + 2 = -3(x - 8)$     c)  $y - 7 = 1(x + 5)$   
 d)  $y - 3 = \frac{1}{2}x$     e)  $y = \frac{1}{4}(x + 7)$     f)  $y + \frac{5}{4} = \frac{6}{5}\left(x + \frac{1}{2}\right)$
2. a)  $y = 8x - 17$     b)  $y = -2x + 17$     c)  $y = -11x - 24$
3. a)  $y = 6x - 8$     b)  $y = 2x - 5$     c)  $y = -2x + 4$     d)  $y = \frac{1}{2}x + 5$     e)  $y = x$     f)  $y = mx + b$
4. a)  $y = \frac{1}{4}x - \frac{11}{2}$     b)  $y = -\frac{1}{3}x + \frac{2}{3}$     c)  $y = -\frac{3}{4}x - 8$
5. a)  $m = -\frac{11}{3}, P(-3, 9)$     b)  $m = \frac{1}{2}, P(0, -3)$     c)  $m = -2, P(6, 8)$     d)  $m = 3, P(-12, 0)$   
 e)  $m = -\frac{5}{3}, P(0, 9)$     f)  $m = \frac{2}{5}, P(0, 0)$
6.  $l_1 \Rightarrow 2x + 3y - 1 = 0$  or  $y = -\frac{2}{3}x + \frac{1}{3}$      $l_2 \Rightarrow 5x - 2y - 31 = 0$  or  $y = \frac{5}{2}x - \frac{31}{2}$
7. D    8. B    9. 

1	0		
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