Lesson 1: The Equation of a Line in Slope y-intercept Form $y = mx + b$

Friday, August 31, 2018 2:33 AM

Equations of Linear Relations Lesson #1:
The Equation of a Line in
Slope y-intercept Form → y = mx + b

Overview of Unit

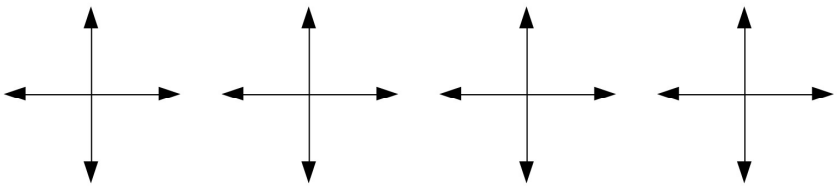
In this unit we express the equation of a linear relation in three different forms: slope *y*-intercept form, point-slope form, and general form. We relate linear relations expressed in these forms to their graphs.

We also determine the linear relation given: a graph, a table of data points, a point and the slope, two points, a point and the equation of a parallel or perpendicular line.

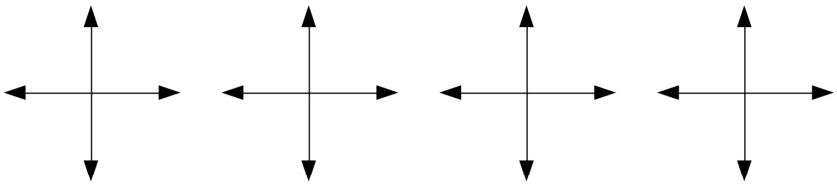
Investigating the Graphs of Linear and Non-Linear Relations

a) The equations of the graphs of some relations are given. In each case, use a graphing calculator to sketch the graph of the relation and make a rough sketch of the graph on the grid provided. Do not list any *x*- or *y*-intercepts.

$y = 3x + 1$ $y = -2x + 3$ $y = -x^3 + 1$ $y = x^2 - 3$



$y = \frac{1}{x}$ $y = \frac{1}{2}x - 2$ $y = 4x$ $y = 2^x$



b) List the equations of the graphs as linear or non-linear.

LINEAR:

NON-LINEAR:

c) Compare the lists. Write a rule from the equation which can be used to determine whether the graph is a straight line or not.

Linear Equation

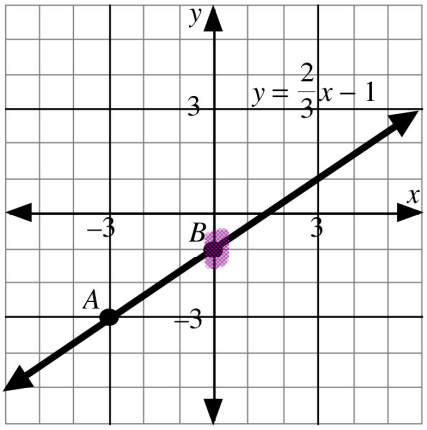
A **linear equation** is an equation of the form $y = mx + b$, where $m, b \in R$.
The graph of a linear equation is a straight line.

slope ↓ y-intercept ↘

Investigating m and b in the equation $y = mx + b$

Part One

Jenine used a graphing calculator to sketch the graph of the linear equation $y = \frac{2}{3}x - 1$.
Her sketch is shown on the grid.

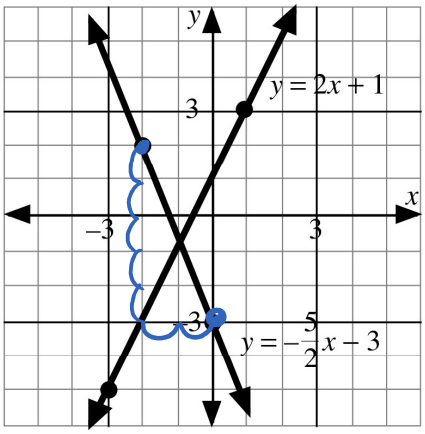


a) Use the sketch and points A and B to find the slope and y-intercept of the graph of $y = \frac{2}{3}x - 1$.

slope: $\frac{rise}{run} : \frac{2}{3}$ y-int $x=0, y=-1$

b) Compare the values found in **a)** with the coefficient of x and the constant term in the equation $y = \frac{2}{3}x - 1$.

c) Jenine sketched the graphs of two more linear equations. Use the grid to determine the slope and y-intercept of each graph.



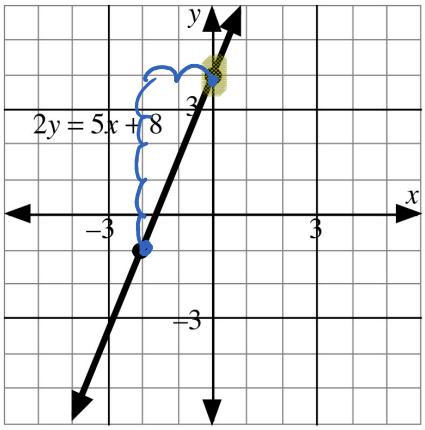
equation	slope	y-intercept
$y = 2x + 1$	$\frac{8}{4} = 2$	1
$y = -\frac{5}{2}x - 3$	$\frac{-5}{2} = \frac{5}{-2}$	−3

d) Make a conjecture about the slope and y-intercept of the graph of the linear equation $y = mx + b$

↑ m = slope ↖ y-intercept

Part Two

Hashib used a graphing calculator to graph the linear equation $2y = 5x + 8$.
The graph is shown on the grid.



y = mx+b

a) Use the sketch to determine the slope and y-intercept of the graph of $2y = 5x + 8$.

m = 5/2 y-int = 4

b) Explain why, in this case, the slope is not 5 (the coefficient of x) and the y-intercept is not 8 (the constant term).

the y needs to be alone

$\frac{2y}{2} = \frac{5x+8}{2}$ $y = \frac{5}{2}x + \frac{8}{2} = \frac{5}{2}x + 4$

Slope y-intercept Form of the Equation of a Line → $y = mx + b$

The graph of an equation in the form $y = mx + b$ (or a function in the form $f(x) = mx + b$) is a straight line with **slope** m and **y-intercept** b.

The equation $y = mx + b$ is known as the **slope y-intercept form** of the equation of a line.

The graph of an equation in this form can be drawn without making a table of values.

y=mx+b

Determine the slope and y-intercept of the graph of each linear equation listed below:

m=slope b=y-int

a) $y = 3x + 2$

m=3 (slope)
b=2 (y-int)

b) $y = 7 - \frac{2}{3}x$

slope: $\frac{-2}{3}$ (in front of the x)
y-int: 7 (constant)

c) $\frac{1}{6}y = \frac{8}{6}x + \frac{1}{6}$

$y = \frac{8x}{6} + \frac{1}{6}$
slope: $\frac{8}{6} = \frac{4}{3}$
y-int: $\frac{1}{6}$

Graphing an Equation of the Form y = mx + b

In this section, we will look at two ways of sketching the graph of a linear equation without using a graphing calculator or a table of values.

Consider the equation $y = 2x - 5$.

a) State the slope and y-intercept.

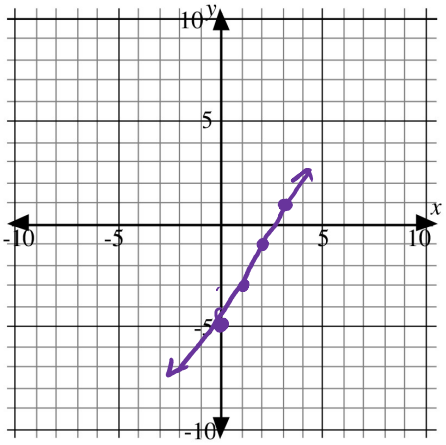
slope: 2 → $\frac{2}{1}$ rise/run $\frac{+2}{+1}$

y-int: -5

b) Mark the *y*-intercept on the grid. ✓

c) Use the *y*-intercept and the formula $\text{slope} = \frac{\text{rise}}{\text{run}}$ to mark three other points on the grid. ✓
Join the points together, and extend the line. ✓

d) Verify the graph using a graphing calculator.



Consider the equation $y = \frac{2}{3}x - 6$.

a) State the *y*-intercept.

-6

b) Determine the *x*-intercept algebraically.

x-int, y = 0

$0 \overset{+6}{=} \frac{2}{3}x - 6 \overset{+6}{}$

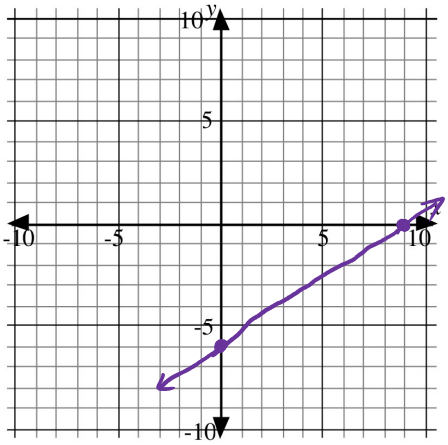
$\frac{3}{2}6 = \frac{2}{3}x \times \frac{3}{2}$

* solve for x

$x = \frac{18}{2} = 9$

c) Mark the *x*- and *y*-intercepts on the grid.
Join the points together, and extend the line.

d) Verify the graph and the intercepts using a graphing calculator.



Complete Assignment Questions #1 - #14

2ace, 3ac, 4, 5 / 13

Assignment

1. Each equation represents a relation.

a) $y = 6x + 1$ **b)** $y = x^2$ **c)** $y = 3x^4 + 5$ **d)** $y = -\frac{1}{4}x - 8$

e) $y = 1 - x$ **f)** $y = \frac{2}{1-x}$ **g)** $y = 4x$ **h)** $y = 4^x$

Without sketching the graph of the relation, list the letters a) through h) as linear or non-linear

LINEAR:

NON-LINEAR:

2. State the slope and *y*-intercept of the graph of each linear equation.

a) $y = 7x - 2$ **b)** $y = \frac{4}{3}x + 3$ **c)** $y = 6 - \frac{1}{6}x$ **d)** $4y = 6x + 8$ **e)** $y = ax + b$

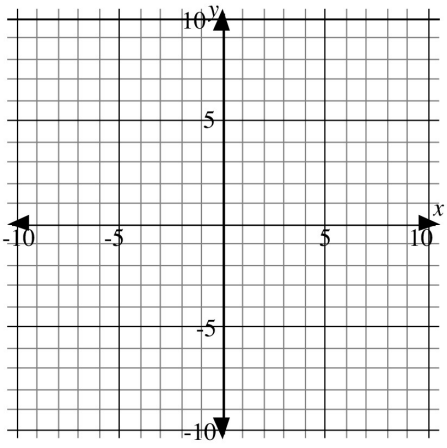
3. Write the equation of each line with the given slope and *y*-intercept.

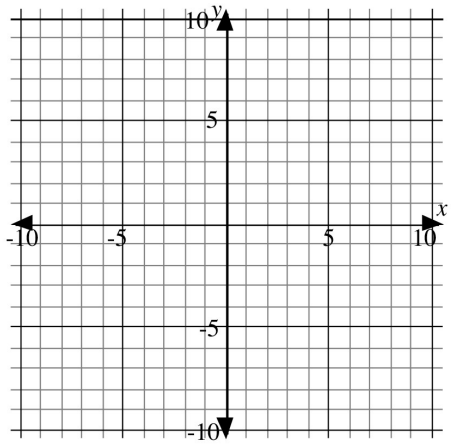
a) slope = 4 **b)** slope = $\frac{1}{5}$ **c)** slope = –3 **d)** slope = *m*

y-intercept = –9 *y*-intercept = $\frac{1}{2}$ *y*-intercept = 0 *y*-intercept = *b*

4. For each line, state the slope and the *y*-intercept. Graph the equation without using a graphing calculator.

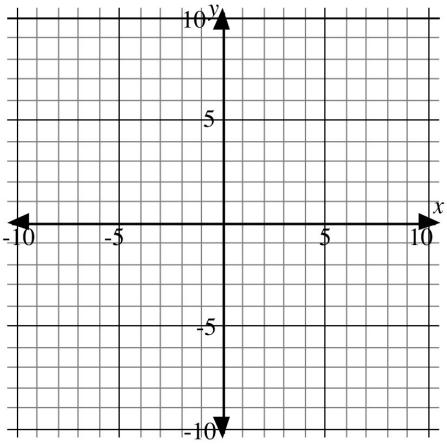
a) $y = \frac{1}{4}x + 2$ **b)** $y = -x - 1$ **c)** $y = -\frac{4}{3}x$ **d)** $y = 5$

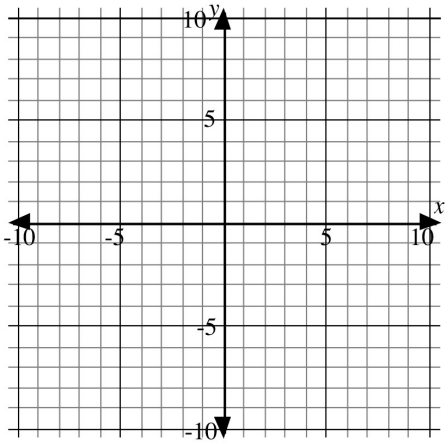




5. For each line, state the y-intercept. Determine the x-intercept algebraically, and graph the equation without using a graphing calculator.

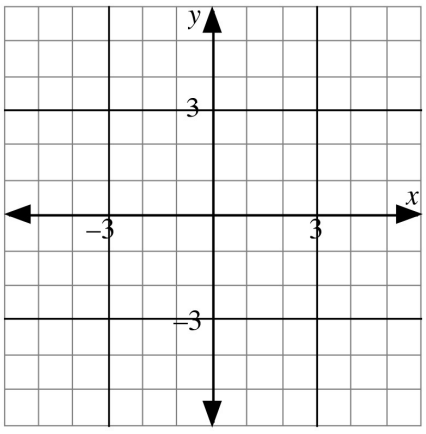
a) $y = 2x + 6$ **b)** $y = -x - 4$ **c)** $y = \frac{6}{7}x - 6$ **d)** $y = -\frac{1}{2}x + 1$





6. Explain why the linear equation $y = 5x$ can be graphed using the method in question 4 but not by the method in question 5.

7. Consider the graph of the function with equation $y = x$.



a) State the values of m and b.

b) Determine the x- and y-intercepts.

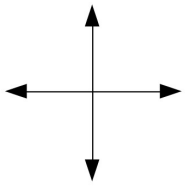
c) Sketch the graph on the grid provided without using a graphing calculator.

d) Determine the domain and range of the function.

e) Use a graphing calculator to graph the line $y = -x$, and sketch the graph on the grid.

8. Use a graphing calculator to sketch the graph of each of the following linear equations. Complete the table giving the x-intercept to the nearest hundredth.

i) $y = 7x - 8$

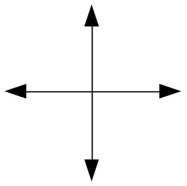


slope	
x-intercept	
y-intercept	

Graphing Window which includes both intercepts:

```
WINDOW
 Xmin=
 Xmax=
 Xscl=
 Ymin=
 Ymax=
 Yscl=
 Xres=1
```

ii) $y = -\frac{31}{2}x - 25$

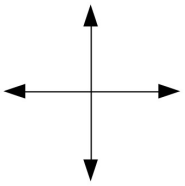


slope	
x-intercept	
y-intercept	

Graphing Window which includes both intercepts:

```
WINDOW
 Xmin=
 Xmax=
 Xscl=
 Ymin=
 Ymax=
 Yscl=
 Xres=1
```

iii) $y = 75 - \frac{5}{3}x$



slope	
x-intercept	
y-intercept	

Graphing Window which includes both intercepts:

```
WINDOW
 Xmin=
 Xmax=
 Xscl=
 Ymin=
 Ymax=
 Yscl=
 Xres=1
```

Multiple Choice

9. Which of the following does not represent the equation of a straight line?

A. $y = 3x$

B. $y = 11 - 3x$

C. $y = \frac{x}{3}$

D. All of the above represent the equation of a straight line.

10. Which of the following statements is false for the line $y = -\frac{1}{2}x + 1$?

A. The graph of the line falls from left to right.

B. The x-intercept is 2.

C. The graph passes through the point $(8, -3)$.

D. The line is perpendicular to the line $y = -2x + 4$.

11. Which of the following statements is true for the line $2y = \frac{1}{4}x + 6$?

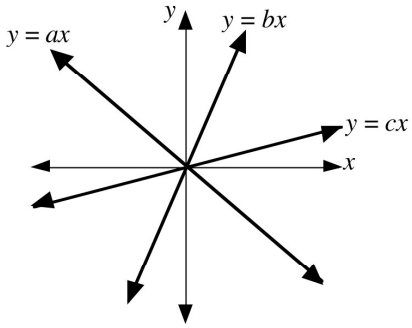
A. The x-intercept is 24.

B. The y-intercept is 6.

C. The slope is $\frac{1}{8}$.

D. The graph passes through the point $(-4, 5)$.

12. The lines $y = ax$, $y = bx$, and $y = cx$ are shown. Which of the following statements is true?



A. $a < b < c$

B. $a < c < b$

C. $c < a < b$

D. $c < b < a$

Use the following information to answer questions 13 and 14.

Consider the line with equation $y = 3x + 5$. The line intersects the x-axis at P and the y-axis at Q. Triangle POQ is formed where O is the origin.

Numerical Response

13. The area of $\triangle POQ$, in square units, to the nearest tenth, is _____.

(Record your answer in the numerical response box from left to right)

14. To the nearest tenth, the perimeter of $\triangle POQ$ is _____.

(Record your answer in the numerical response box from left to right)

Answer Key

1. LINEAR a), d), e), g). NON-LINEAR b), c), f), h).

2. **a)** slope = 7, y-int = –2 **b)** slope = $\frac{4}{3}$, y-int = 3

c) slope = $-\frac{1}{6}$, y-int = 6 **d)** slope = $\frac{3}{2}$, y-int = 2 **e)** slope = a, y-int = b

3. **a)** $y = 4x - 9$ **b)** $y = \frac{1}{5}x + \frac{1}{2}$ **c)** $y = -3x$ **d)** $y = mx + b$

4. **a)** slope = $\frac{1}{4}$, y-int = 2 **b)** slope = –1, y-int = –1

c) slope = $-\frac{4}{3}$, y-int = 0 **d)** slope = 0, y-int = 5

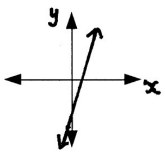
5. **a)** y-int = 6, x-int = –3 **b)** y-int = –4, x-int = –4 **c)** y-int = –6, x-int = 7 **d)** y-int = 1, x-int = 2

6. The method in #4 needs a point and a slope. We have point (0, 0) and slope = 5. The method in #5 needs two points to be joined. Since the x- and y-intercepts are the same point, the line cannot be drawn.

7. **a)** $m = 1$, $b = 0$ **b)** x-int = 0 and y-int = 0. **d)** $D = x \in R$ $R = y \in R$

8.

i) $y = 7x - 8$

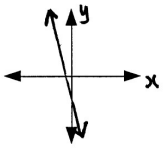


slope	7
x-intercept	1.14
y-intercept	–8

Graphing Window which includes both intercepts:

WINDOW
Xmin= -4
Xmax= 4
Xscl= 1
Ymin= -10
Ymax= 5
Yscl= 2
Xres=1

ii) $y = -\frac{31}{2}x - 25$

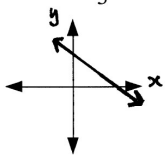


slope	$-\frac{31}{2}$
x-intercept	–1.61
y-intercept	–25

Graphing Window which includes both intercepts:

WINDOW
Xmin= -4
Xmax= 4
Xscl= 1
Ymin= -40
Ymax= 10
Yscl= 10
Xres=1

iii) $y = 75 - \frac{5}{3}x$



slope	$-\frac{5}{3}$
x-intercept	45
y-intercept	75

Graphing Window which includes both intercepts:

WINDOW
Xmin= -10
Xmax= 60
Xscl= 10
Ymin= -20
Ymax= 100
Yscl= 10
Xres=1

9. D **10.** D **11.** C **12.** B

13.

4	.	2	

14.

1	1	.	9