Lesson 5: Parallel and Perpendicular Lines

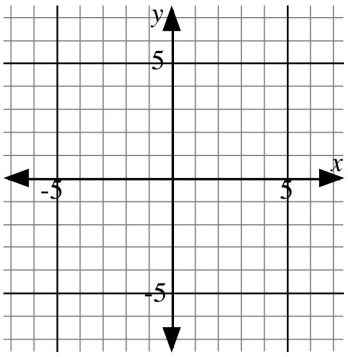
Friday, August 31, 2018 2:41 AM

Characteristics of Linear Relations Lesson #5:
Parallel and Perpendicular Lines

Review of Transformations

In earlier mathematics courses we studied transformations: translations, reflections, and rotations. In order to investigate parallel and perpendicular line segments, we will review translations and rotations.

On the grid, show the image of the point $A(2, 5)$ after the following transformations. In each case write the coordinates of the image.



a) A translation 3 units right and 2 units up.

$A(2, 5) \rightarrow \quad B(\quad , \quad)$

b) A 90° clockwise rotation about the origin.

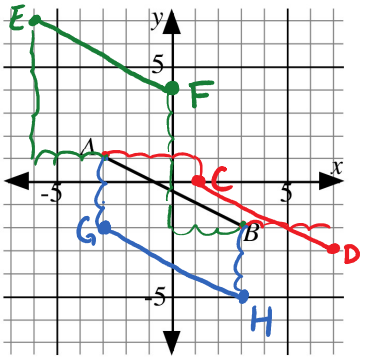
$A(2, 5) \rightarrow \quad C(\quad , \quad)$

c) A 90° counterclockwise rotation about the origin.

$A(2, 5) \rightarrow \quad D(\quad , \quad)$

Investigating Parallel Line Segments

a) On the grid, show the image of line segment AB after the following transformations.



shift/move

i) A translation 4 units right and 1 unit down to form line segment CD. ✓

ii) A translation 3 units left and 6 units up to form line segment EF. ✓

iii) A translation 3 units down to form line segment GH. ✓

b) Calculate the slope of each of the line segments.

$m_{AB} = \frac{-3}{6} = \frac{-1}{2}$ $m_{CD} = \frac{-1}{2}$ $m_{EF} = \frac{-1}{2}$ $m_{GH} = \frac{-1}{2}$

c) The four line segments are parallel. Make a conjecture about the slopes of parallel line segments.

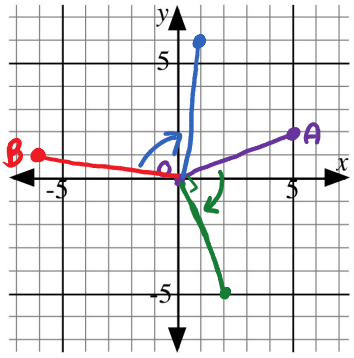
parallel lines have the same slope

Investigating Perpendicular Line Segments

a) i) On the grid, plot the point $A(5, 2)$ and draw the line joining the point to the origin, O.

ii) Rotate the line through an angle of 90° clockwise about O and show the image on the grid.

iii) Find the slopes of the two perpendicular lines and multiply them together.



$m_{AO} = \frac{2}{5}$ $m = \frac{-5}{2}$

$\frac{2}{5} \times \frac{-5}{2} = \frac{-10}{10} = -1$

b) Repeat part **a)** for the point $B(-6, 1)$.

$m_B = \frac{1}{-6}$ $m = \frac{6}{1}$

$\frac{1}{-6} \times \frac{6}{1} = \frac{6}{-6} = -1$

c) Make a conjecture about the slopes of perpendicular line segments.

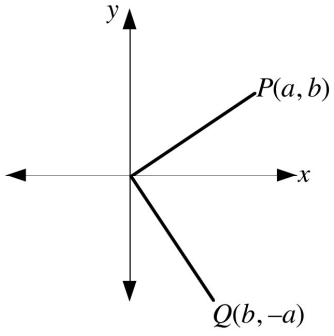
- multiplying perpendicular slopes equals -1
- slopes are negative reciprocals ei. $\frac{2}{3} \rightarrow \frac{-3}{2}$ flipped and negative

d) Complete the following to prove the conjecture in **c)**.
Under a rotation of 90° clockwise about O, $P(a, b) \rightarrow Q(b, -a)$.

$$m_{OP} = \frac{y_P - y_O}{x_P - x_O} = \frac{b - 0}{a - 0} =$$

$$m_{OQ} = \frac{y_Q - y_O}{x_Q - x_O} = \frac{-a - 0}{b - 0} =$$

$$m_{OP} \times m_{OQ} =$$



Parallel Lines and Perpendicular Lines

Recall that the slope of any line segment within a line represents the slope of the line.

Consider then two lines with slopes m_1 and m_2.

- The lines are **parallel** if they have the same slope, i.e. $m_1 = m_2$.
- The lines are **perpendicular** if the product of the slopes is -1, multiply
  i.e. $m_1 \times m_2 = -1$ or $m_1 m_2 = -1$ or $m_1 = -\frac{1}{m_2}$
- For perpendicular lines, each slope is the negative reciprocal of the other provided neither slope is equal to zero.

$\frac{2}{3} \rightarrow -\frac{3}{2}$, $-4 \rightarrow \frac{1}{4}$

Consider line segment AC with a slope of $\frac{3}{4}$.

a) Write the slope of line segment GH which is parallel to AC.

$m_{GH} = \frac{3}{4}$

b) Write the slope of line segment BF which is perpendicular to AC.

$m_{BF} = \frac{-4}{3}$ (flip it and make negative)

The slopes of two lines are given.
Determine if the lines are parallel, perpendicular, or neither.

a) $m_1 = \frac{1}{4}$, $m_2 = \frac{3}{12} = \frac{1}{4}$ parallel

b) $m_1 = \frac{5}{7}$, $m_2 = \frac{14}{10} = \frac{7}{5}$ neither (flipped but not negative)

Complete Assignment Questions #1 - #6

If P is the point $(4, 7)$ and Q is the point $(6, -2)$, find the slope of a line segment

a) parallel to line segment PQ **b)** perpendicular to line segment PQ

lesson 4 $m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$

$m_{PQ} = \frac{-2-7}{6-4} = \frac{-9}{2}$ parallel same $\rightarrow \frac{2}{9}$ perpendicular flip and make positive

$\triangle LMN$ has coordinates $L(-4, 2)$, $M(-2, 7)$, and $N(1, 0)$. Use slopes to show that the triangle is right-angled at L.

Two lines have slopes of $-\frac{3}{4}$ and $\frac{k}{5}$ respectively. Find the value of k if the lines are

a) parallel

↳ slopes are equal

$\frac{-3}{4} = \frac{k}{5}$ *solve for k

$\frac{-15}{4} = k$

b) perpendicular

↳ multiply slopes = -1

$\frac{-3}{4} \times \frac{k}{5} = -1$ *solve for k

$\frac{-3k}{20} = -1$ (20)

$-3k = -20$

$\frac{-3k}{-3} = \frac{-20}{-3}$

$k = \frac{20}{3}$

Complete Assignment Questions #7 - #16

Assignment

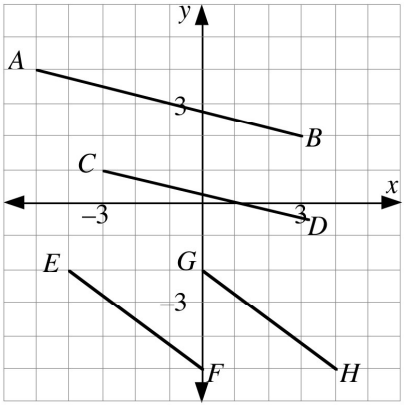
Quiz Tues Unit 7
HW due Tues

#1, 2, 3, 7, (10, 11)ac /31

1. AB is parallel to CD. EF is parallel to GH.

a) Determine the slopes of the following pairs of parallel line segments using $m = \frac{\text{rise}}{\text{run}}$.

Line Segment	Slope	Line Segment	Slope
AB		EF	
CD		GH	



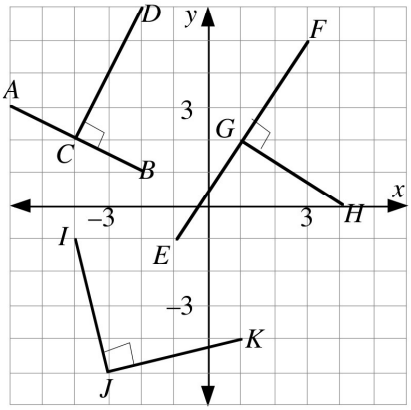
b) Observe the slopes of the pairs of parallel lines in **a)**.
Write a rule in reference to the slopes of parallel lines.

2.a) Determine the slopes of the following pairs of perpendicular line segments using $m = \frac{\text{rise}}{\text{run}}$.

Line Segment	Slope
AB	
CD	

Line Segment	Slope
EF	
GH	

Line Segment	Slope
IJ	
JK	



b) Multiply the slopes of the pairs of perpendicular line segments.

$m_{AB} \times m_{CD}$	$m_{EF} \times m_{GH}$	$m_{IJ} \times m_{JK}$
$-\frac{1}{2} \times 2 =$		

c) Write a rule in reference to the slope of two lines which are perpendicular to each other.

3. The slopes of two line segments are given. Determine if the lines are parallel, perpendicular, or neither.

a) $m_{AB} = \frac{8}{20}$, $m_{PQ} = \frac{2}{5}$ **b)** $m_{AB} = \frac{3}{2}$, $m_{PQ} = -\frac{2}{3}$ **c)** $m_{AB} = \frac{1}{6}$, $m_{PQ} = \frac{2}{12}$

d) $m_{AB} = \frac{7}{8}$, $m_{PQ} = \frac{8}{7}$ **e)** $m_{AB} = \frac{9}{3}$, $m_{PQ} = -\frac{1}{3}$ **f)** $m_{AB} = -5$, $m_{PQ} = \frac{1}{5}$

g) $m_{AB} = \frac{4}{8}$, $m_{PQ} = 2$ **h)** $m_{AB} = -\frac{12}{2}$, $m_{PQ} = -6$ **i)** $m_{AB} = -\frac{5}{2}$, $m_{PQ} = -\frac{2}{5}$

4. The slopes of some line segments are given.

$m_{AB} = 6$ $m_{CD} = \frac{1}{6}$ $m_{EF} = -6$ $m_{GH} = 6$ $m_{IJ} = -6$ $m_{KL} = \frac{1}{6}$

Which pairs of lines are parallel to each other?

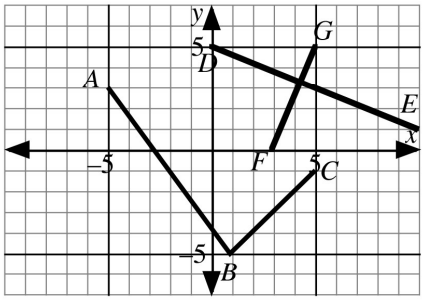
5. The slopes of some line segments are given.

$m_{RS} = -2$ $\quad m_{UV} = \frac{1}{4}$ $\quad m_{EF} = 0.5$ $\quad m_{ZT} = 2$

$m_{PQ} = -4$ $\quad m_{KL} = -\frac{1}{2}$ $\quad m_{MN} = 4$ $\quad m_{XY} = -\frac{1}{4}$

Which pairs of lines are perpendicular to each other?

6. The four line segments have endpoints with integer coordinates. In each case determine whether the two intersecting line segments are perpendicular.



7. A, B, and C are the points $(0, 4), (-3, 1)$, and $(5, -2)$ respectively. Determine the slope of a line

a) parallel to line segment AB

b) perpendicular to line segment AB

c) parallel to line segment BC

d) perpendicular to line segment AC

8. ΔABC has vertices $A(3, 5)$, $B(-2, -5)$, $C(-5, 1)$.

a) Explain how we can determine if $\measuredangle ABC$ is a right triangle.

b) Determine if $\measuredangle ABC$ is a right triangle.

9. The vertices of two triangles are given.
Determine if either of the triangles is right-angled.

a) $\Delta PQR \rightarrow P(-3,3),\ Q(-1,1),\ R(-5,-1)$ **b)** $\Delta ABC \rightarrow A(-7,9),\ B(3,13),\ C(7,3)$

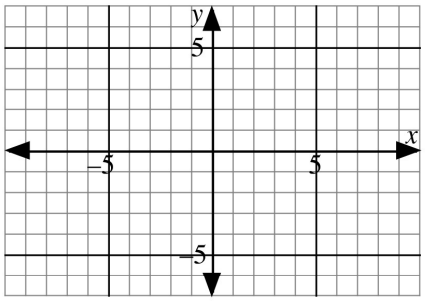
10. The slopes of parallel lines are given.
Determine the value of the variable.

a) $4, \frac{k}{3}$ **b)** $-2, \frac{2}{n}$ **c)** $\frac{5}{6}, 3m$ **d)** $\frac{3}{4}, -\frac{w}{6}$

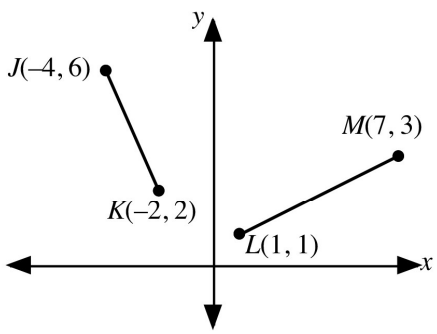
11. The slopes of perpendicular lines are given.
Determine the value of the variable.

a) $\frac{1}{3}, 3h$ **b)** $4, \frac{8}{p}$ **c)** $-5, \frac{s}{2}$ **d)** $-\frac{3}{4}, -\frac{q}{6}$

12. $P(-4,0)$ and $R(1,-3)$ are opposite vertices of a rhombus $PQRS$. Find the slope of diagonal QS.



13. a) Show that when line segments JK and ML are extended until they intersect, they will not meet at right angles.



b) If the y-coordinate of M is changed, the line segments, when extended, will meet at right angles. To what value should the y-coordinate of M be changed?

14. Given that $A, B,$ and C are the points $(-3, 3)$, $(0, 6)$, and $(5, 1)$ respectively, prove that triangle ABC is right angled by using

a) the slope formula

b) the distance formula

Multiple Choice **15.** A and B are the points $(1, 2)$ and $(-2, 3)$ respectively. A line perpendicular to AB will have slope

A. -3

B. $-\frac{1}{3}$

C. 3

D. $\frac{1}{3}$

Numerical Response **16.** The line segment joining $U(-3, p)$ and $V(-6, 5)$ is perpendicular to the line segment joining $X(4, 2)$ and $Y(9, 0)$. The value of p, to the nearest tenth, is _____.

(Record your answer in the numerical response box from left to right)

Answer Key

1. a) slope $AB = -\frac{1}{4}$ slope $CD = -\frac{1}{4}$ slope $EF = -\frac{3}{4}$ slope $GH = -\frac{3}{4}$

b) Lines which are parallel have the same slope

2. a) slope $AB = -\frac{1}{2}$ slope $EF = \frac{3}{2}$ slope $IJ = -4$

slope $CD = 2$ slope $GH = -\frac{2}{3}$ slope $JK = \frac{1}{4}$

b) All the products are –1. **c)** The product of the slopes is –1.

3. a) parallel **b)** perpendicular **c)** parallel

d) neither **e)** perpendicular **f)** perpendicular

g) neither **h)** parallel **i)** neither

4. AB and GH, CD and KL, EF and IJ.

5. RS and EF, UV and PQ, ZT and KL, MN and XY.

6. AB and BC are not perpendicular. DE and FG are perpendicular.

7. a) 1 **b)** –1 **c)** $-\frac{3}{8}$ **d)** $\frac{5}{6}$

8. a) Determine the slope of each side of the triangle. If two of the slopes are negative reciprocals of each, then the triangle is a right triangle.

b) $m_{BC} = -2$, $m_{AC} = \frac{1}{2}$. Since the slopes are negative reciprocals, the triangle is a right triangle.

9. a) ΔPQR is <u>not</u> a right triangle **b)** ΔABC is a right triangle

10. a) $k = 12$ **b)** $n = -1$ **c)** $m = \frac{5}{18}$ **d)** $w = -\frac{9}{2}$

11. a) $h = -1$ **b)** $p = -32$ **c)** $s = \frac{2}{5}$ **d)** $q = -8$

12. $m_{QS} = \frac{5}{3}$

13. a) $M_{JK} = -2$, $M_L = \frac{1}{3}$. The product of the slopes does not equal –1. **b)** $y_M = 4$

14. a) $m_{AB} = 1$, $m_{BC} = -1$ Since the product of the slopes = –1, AB and BC are perpendicular. Triangle ABC is right angled at B.

b) $AB = \sqrt{18}$, $BC = \sqrt{50}$, $AC = \sqrt{68}$. $AC^2 = 68$. $AB^2 + BC^2 = 68$.
$AC^2 = AB^2 + BC^2$ so the Pythagorean theorem is satisfied and the triangle is right angled at B.

15. C **16.**

1	2	.	5