

Lesson 4: The Slope Formula

Friday, August 31, 2018 2:41 AM

Characteristics of Linear Relations Lesson #4: The Slope Formula

Review

Complete the following statements.

- Slope is the measure of the steepness of a line.
- Slope is the ratio of the vertical change (called the rise) over the horizontal change (called the run).
- A line segment which rises from left to right has a positive slope.
- A line segment which falls from left to right has a negative slope.
- A horizontal line segment has a slope of 0.
- A vertical line segment has an undefined slope.
- The slopes of all line segments on a line are equal.

Developing the Slope Formula

- Calculate the slope of line segment

$$AB \text{ using slope} = \frac{\text{rise}}{\text{run}}$$

$$\frac{3}{6} = \frac{1}{2}$$

- List the coordinates of the endpoints of line segment AB .

$$A(1, 8) \quad B(7, 11)$$

- How can the rise of line segment AB be determined using y_B and y_A ?

$$11 - 8 = 3$$

- How can the run of line segment AB be determined using x_A and x_B ?

$$7 - 1 = 6$$

- Use your results from **c)** and **d)** to write a formula which describes how the slope of line segment AB can be calculated using its endpoints.

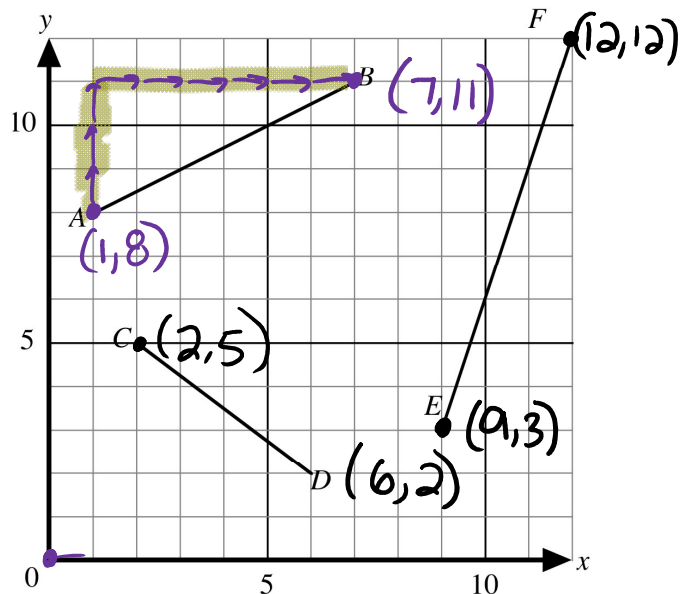
$$\text{slope } AB = \frac{y_B - y_A}{x_B - x_A}$$

- Calculate the slope of line segment AB using the formula in **e)**.

$$\frac{11 - 8}{7 - 1} = \frac{3}{6} = \frac{1}{2}$$

- Calculate the slope of the line segments CD and EF using the method in **a)** and verify using the formula from **e)**.

$$\text{slope } CD = \frac{y_D - y_C}{x_D - x_C} = \frac{2 - 5}{6 - 2} = \frac{-3}{4}$$



$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 2}{6 - 2} = \frac{-2}{4}$$

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$$\text{slope EF} = \frac{12 - 3}{12 - 9} = \frac{9}{3} = 3$$

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The Slope Formula

M = midpoint
 m = slope

In mathematics the letter “ m ” is used to represent slope. If the graph of a linear relation passes through the points $P(x_1, y_1)$ and $Q(x_2, y_2)$, then the slope of this line can be calculated using

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad m_{PQ} = \frac{y_Q - y_P}{x_Q - x_P}$$



Find the slope of a line which passes through the points $G(-3, 8)$ and $H(7, -2)$.

$$m_{GH} = \frac{y_H - y_G}{x_H - x_G} = \frac{-2 - 8}{7 - (-3)} = \frac{-10}{10} = -1$$



Eleanor, Bonnie, and Carl are calculating the slope of a line segment with endpoints $E(15, 8)$ and $F(-10, 6)$. Their work is shown below.

	Eleanor	Bonnie	Carl
Step 1:	$m_{EF} = \frac{x - 10 - 15}{y \quad 6 - 8}$	$m_{EF} = \frac{6 - 8}{15 - (-10)}$	$m_{EF} = \frac{8 - 6}{15 - 10}$
Step 2:	$= \frac{-25}{-2}$	$= \frac{-2}{25}$	$= \frac{2}{5}$
Step 3:	$m_{EF} = \frac{25}{2}$	$m_{EF} = -\frac{2}{25}$	$m_{EF} = \frac{2}{5}$

Since their answers are all different, at least two of the students have made errors in their calculations. Describe all the errors which have been made and determine the correct slope.

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 8}{-10 - 15} = \frac{-2}{-25} = \frac{2}{25}$$

1, (2, 3) every 2nd one, 4ab, 5

1, (2,3) every 2nd one, 4ab, 5

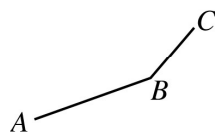
Complete Assignment Questions #1 - #5

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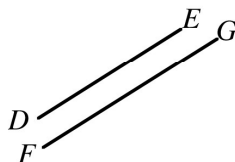
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Collinear Points

Two lines in a plane can either be



at an angle



parallel and distinct



parallel and form a straight line.

Points that lie on the same straight line are said to be **collinear**, i.e. P , Q , and R are collinear.

If three points P , Q , and R are collinear then $m_{PQ} = m_{QR} = m_{PR}$.

Proving that any two of these three slopes are equal is sufficient for the third to be equal and for the points to be collinear.



Consider points $A(5, -3)$, $B(2, 6)$, and $C(-7, 33)$.

a) Prove that the points A , B , and C are collinear.

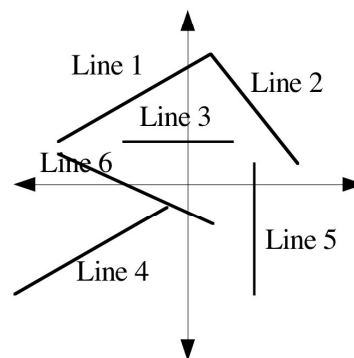
b) Find the value of y if the point $D(-4, y)$ lies on line segment AC .

Complete Assignment Questions #6 - #12

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Assignment

1. State whether the slope of each line is positive, negative, zero, or undefined.



2. Use the slope formula to calculate the slope of the line segment with the given endpoints.

a) $A(12, -2)$ and $B(0, 3)$

b) $C(-2, 3)$ and $D(2, -2)$

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A} =$$

c) $P(-15, -2)$ and $O(0, 0)$

d) $S(36, -41)$ and $T(-20, -27)$

e) $U(-172, -56)$ and $V(-172, 32)$

f) $K(8, -41)$ and $L(397, -41)$

3. Use the slope formula to calculate the slope of the line passing through the given points.

a) $(3, -6)$ and $(8, 4)$

b) $(-12, 7)$ and $(0, -2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} =$$

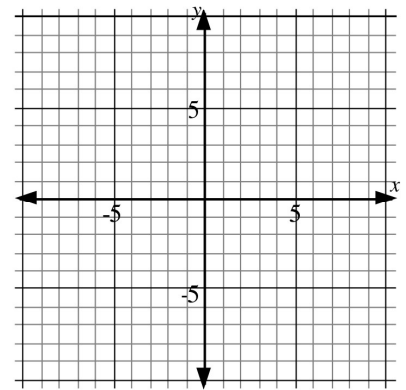
c) $(-3, -8)$ and $(1, 5)$

d) $(21, 1)$ and $(-4, -9)$

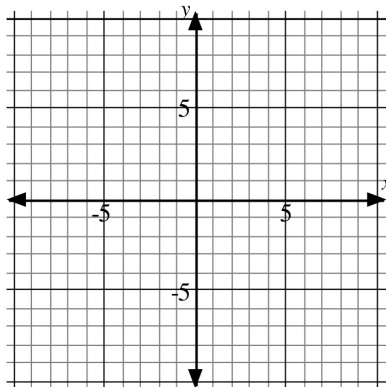
4. A coordinate grid is superimposed on a cross-section of a hill. The coordinates of the bottom and the top of a straight path up the hill are, respectively, (3, 2) and (15, 47), where the units are in metres.
- a) Calculate the slope of the hill.
 - b) Calculate the coordinates of the midpoint of the path up the hill.
 - c) Calculate the length of the path to the nearest tenth of a metre.

5. The line segment joining each pair of points has the given slope. Determine each value of k and draw the line segment on the grid.

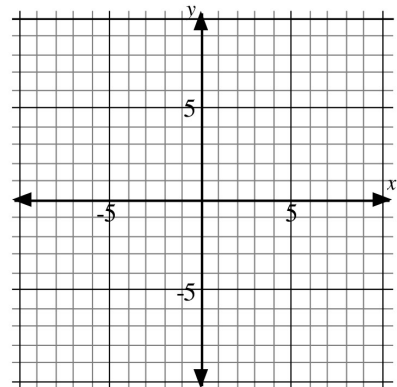
a) $S(4, 6)$ and $T(5, k)$ slope = 3



b) $L(k, -2)$ and $M(3, -7)$ slope = $-\frac{1}{2}$



c) $U(2, 5)$ and $V(k, 3)$ slope = $\frac{2}{7}$



8. A private jet has crashed in the desert at the point $P(-10, 17)$. A search party sets out in an all terrain vehicle from A_1 , passing in a straight line through A_2 . A helicopter sets out from B_1 and flies in a straight line through B_2 .
If the search parties continue in these directions, will either of them discover the crashed plane?

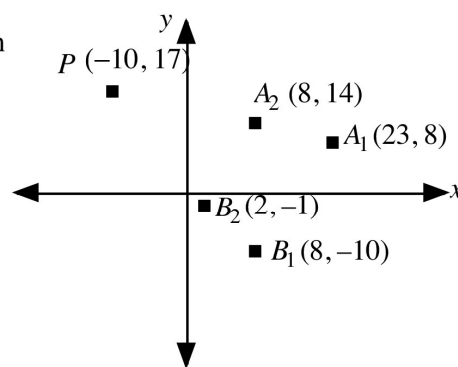


Diagram not to scale

Multiple Choice

9. The slope of the line segment joining $E(5, -1)$ and $F(3, 7)$, is
- A. -3
 - B. -4
 - C. $-\frac{1}{3}$
 - D. $-\frac{1}{4}$
10. If the line segment joining $(2, 3)$ and $(8, k)$ has slope $-\frac{2}{3}$, then $k =$
- A. -1
 - B. -3
 - C. -6
 - D. 7
11. One endpoint of a line segment is $(1, 6)$. The other endpoint is on the x -axis. If the slope of the line segment is -3 , then the midpoint of the line segment is
- A. $(4, 6)$
 - B. $(2, 3)$
 - C. $(-10, 3)$
 - D. $\left(\frac{1}{2}, \frac{15}{2}\right)$

Numerical Response

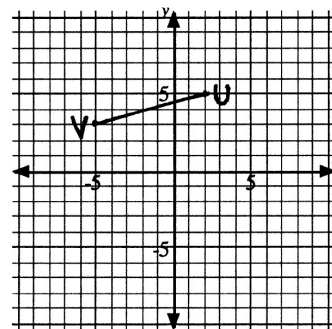
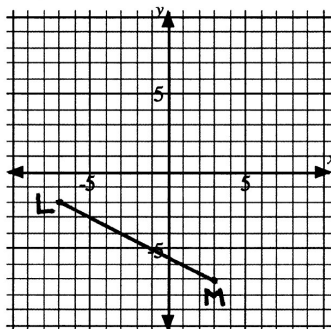
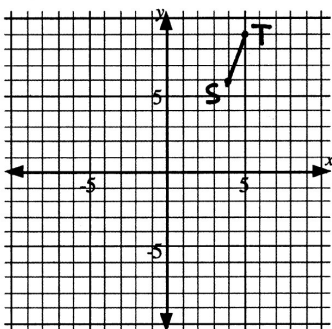
12. $P(3, 6)$, $Q(8, -2)$, and $R(-6, 0)$, are the vertices of a triangle. T is the midpoint of QR . The slope of the line PT , to the nearest tenth, is _____.

(Record your answer in the numerical response box from left to right)

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Answer Key

1. Line 1 - positive, Line 2 - negative, Line 3 - zero, Line 4 - positive, Line 5 - undefined, Line 6 - negative
 2. a) $-\frac{5}{12}$ b) $-\frac{5}{4}$ c) $\frac{2}{15}$ d) $-\frac{1}{4}$ e) undefined f) 0
 3. a) 2 b) $-\frac{3}{4}$ c) $\frac{13}{4}$ d) $\frac{2}{5}$
 4. a) $\frac{15}{4}$ b) $(9, \frac{49}{2})$ c) 46.6 m.
 5. a) $k = 9$ b) $k = -7$ c) $k = -5$



6. a) $m_{PQ} = -\frac{2}{5}$, $m_{QR} = -\frac{2}{5}$. Since $m_{PQ} = m_{QR}$, the points P , Q and R are collinear.
 b) $PQ = \sqrt{29}$, $QR = 2\sqrt{29}$, $PR = 3\sqrt{29}$. Since $PQ + QR = PR$, the points P , Q and R are collinear.
 7. a) $m_{AB} = -\frac{1}{4}$, $m_{BC} = -\frac{1}{4}$. Since $m_{AB} = m_{BC}$, the points A , B and C are collinear.
 b) $m_{AD} = -\frac{3}{10}$. Since $m_{AD} \neq m_{AB}$, the point D does not lie on line segment AC . c) $k = -4$
 8. $m_{A_1A_2} = -\frac{2}{5}$, $m_{A_2P} = -\frac{1}{6}$. Since $m_{A_1A_2} \neq m_{A_2P}$, the search party in the all terrain vehicle will not discover the plane.
 $m_{B_1B_2} = -\frac{3}{2}$, $m_{B_2P} = -\frac{3}{2}$. Since $m_{B_1B_2} = m_{B_2P}$, the search party in the helicopter will discover the plane.
 9. B 10. A 11. B 12.

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