Lesson 3: Slope of a Line Segment

Friday, August 31, 2018 2:41 AM

Characteristics of Linear Relations Lesson #3:
Slope of a Line Segment

A trucker driving up a hill with a heavy load may be concerned with the steepness of the hill. When building a roof, a builder may be concerned with the steepness (or pitch) of the roof. A skier going down a hill may be concerned with the steepness of the ski hill.

In mathematics, the term **slope** is used to describe the steepness of a line segment.

Slope of a Line Segment

The **slope** of a line segment is a measure of the steepness of the line segment.

It is the ratio of **rise** (the change in vertical height between the endpoints) over **run** (the change in horizontal length between the endpoints).

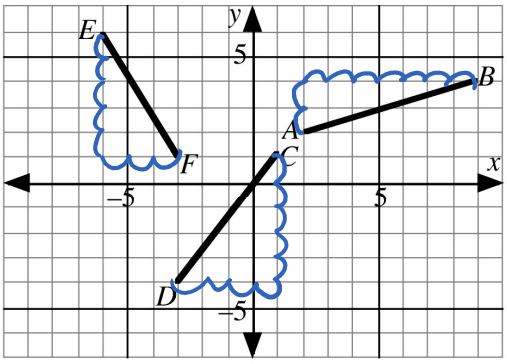
$$\textbf{Slope} = \frac{\textbf{rise}}{\textbf{run}}$$

+ ↑ − ↓

- the **rise** is POSITIVE if we count UP, and NEGATIVE if we count DOWN.
- the **run** is POSITIVE if we count RIGHT, and NEGATIVE if we count LEFT.

→ ←

Each line segment on the grid has endpoints with integer coordinates. Complete the table below.

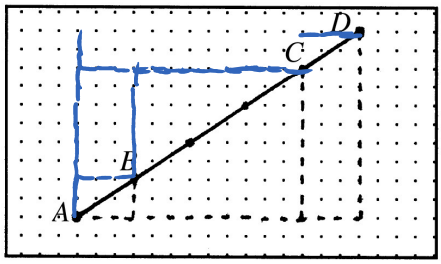


Line Segment	Rise	Run	Slope = $\frac{\text{Rise}}{\text{Run}}$
AB	2	7	$\frac{2}{7}$
CD	−5↓	−4←	$\frac{-5}{-4} = \frac{5}{4}$
EF	−5	3	$\frac{-5}{3}$

Investigation #1 *Investigating the Slope of Line Segments*

a) Complete the chart. Write the slopes in simplest form.

Line Segment	Rise	Run	Slope = $\frac{\text{Rise}}{\text{Run}}$
AB	2	3	$\frac{2}{3}$
AC	8	12	$\frac{8}{12} = \frac{2}{3}$
AD	10	15	$\frac{10}{15} = \frac{2}{3}$
BC	6	9	$\frac{6}{9} = \frac{2}{3}$



b) How are the slopes of the line segments related?

They are the same

Slope of a Line

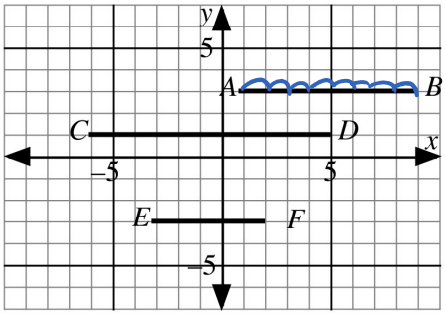
The slopes of all line segments on a line are equal.
The slope of a line representing the graph of a linear relation can be found using

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$ for any two points on the line.

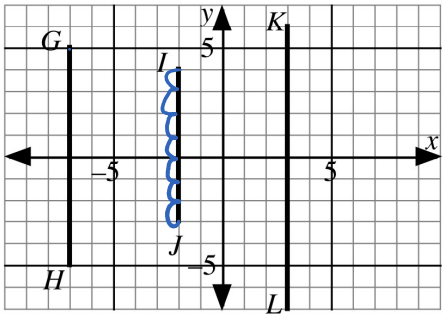
Investigation #2 *Slopes of Horizontal and Vertical Line Segments*

Consider the line segments in Grid 1 and Grid 2 below.

Grid 1



Grid 2



a) Determine the slopes of all the line segments in Grid 1. $\frac{\text{rise}}{\text{run}}$ $\frac{0}{8} = 0$

b) Determine the slopes of all the line segments in Grid 2. $\frac{7}{0}$ undefined

c) Complete the following statements.

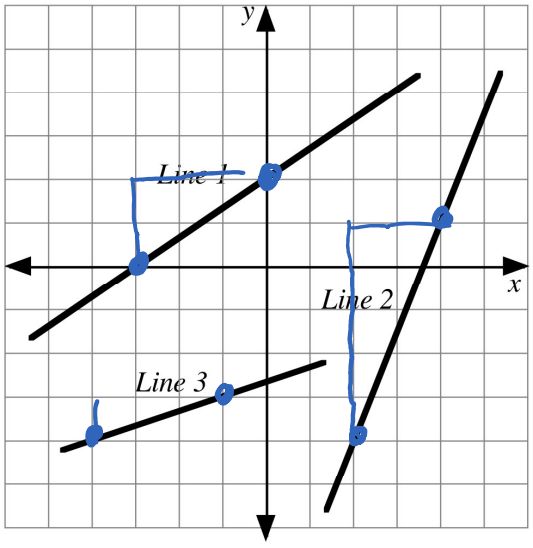
- Horizontal line segments have a slope of zero .
- Vertical line segments have an undefined slope.

Investigation #3 — *Positive and Negative Slopes*

a) Each line on the grids passes through at least two points with integer coordinates. Calculate the slope of each of the lines.

Remember on a Cartesian Plane
- the **rise** is POSITIVE if we count UP, and NEGATIVE if we count DOWN
- the **run** is POSITIVE if we count RIGHT, and NEGATIVE if we count LEFT

Grid 1



Grid 2

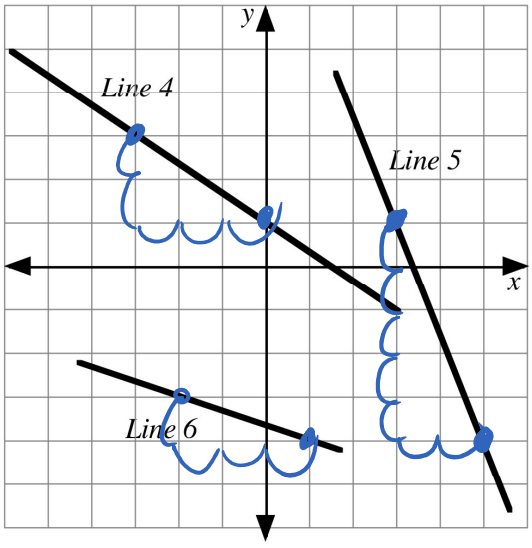


Table For Grid 1

Line	Slope
1	$\frac{2}{3}$
2	$\frac{5}{2}$
3	$\frac{1}{3}$

Table For Grid 2

Line	Slope
4	$-\frac{2}{3}$
5	$-\frac{5}{2}$
6	$-\frac{1}{3}$

b) Compare the slopes of:

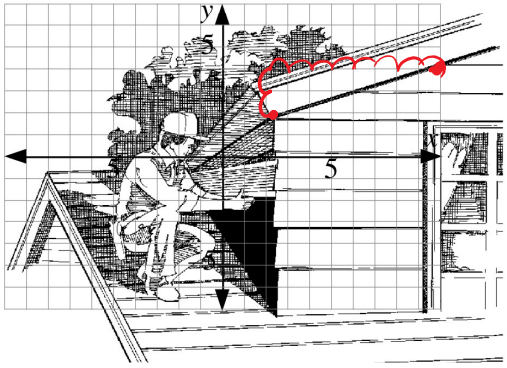
- Line 1 and Line 4
- Line 2 and Line 5
- Line 3 and Line 6

c) Complete the following statements.

- A line which rises from left to right has a __positive__ slope.
- A line which falls from left to right has a __negative__ slope.

Complete Assignment Questions #1 and #2

A grid has been superimposed on the sketch.



a) Estimate the pitch (slope) of the roof to the right of the worker's head.

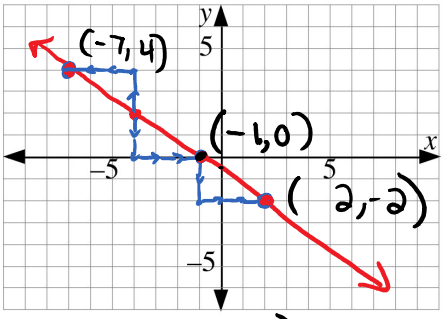
$\frac{2}{3}$ ... positive ↑ / positive →

b) Could the grid be used to estimate the pitch of the roof the worker is standing on? Explain.

Draw a line segment on the grid which passes through the point $(-4, 2)$ and has a slope of $-\frac{2}{3}$.

The line segment must be long enough to cross both the x-axis and the y-axis.

Write the coordinates of three other points on the line segment which have integer coordinates.



Slope $-\frac{2}{3}$ → neg ↓ / pos →

pos ↑ / neg ←

- plotted (-4, 2)
- down 2, right 3
- up 2, left 3
- connect points

A line segment has a slope of $-\frac{5}{7}$ and a rise of 12. Calculate the run as an exact value.

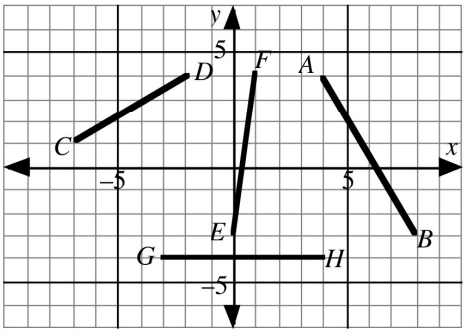
$-\frac{5}{7} = \frac{-5\downarrow}{7\rightarrow} = \frac{5\uparrow}{-7\leftarrow}$

HW #2-4 T3

Complete Assignment Questions #3 - #13

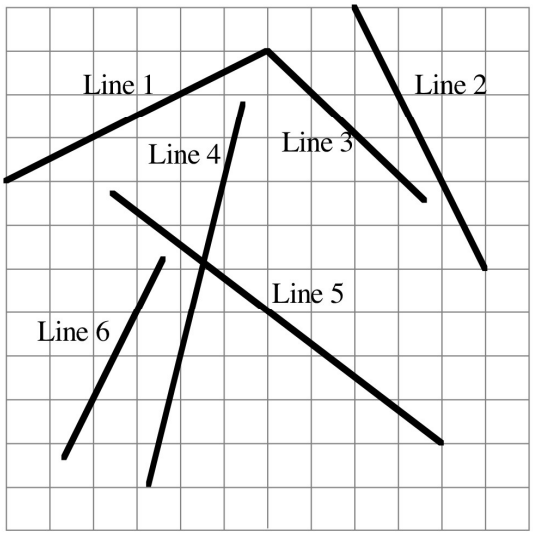
Assignment

1. Each line segment on the grid has endpoints with integer coordinates. Complete the table.



Line Segment	Rise	Run	Slope = $\frac{\text{Rise}}{\text{Run}}$
AB			
CD			
EF			
GH			

2. Every line on the grid passes through at least two points with integer coordinates. Calculate the slope of each of the lines.



slope of Line 1 :

slope of Line 2:

slope of Line 3:

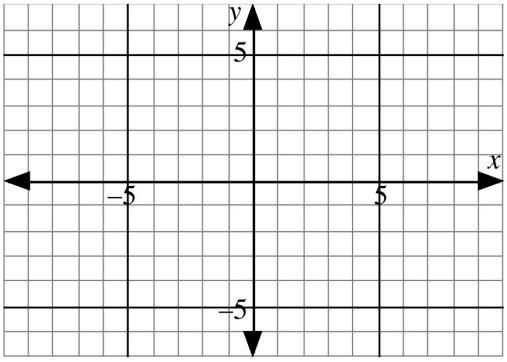
slope of Line 4:

slope of Line 5:

slope of Line 6:

3. Draw a line segment on the grid which passes through the point $(-5, -2)$ and has a slope of $\frac{2}{3}$. The line segment must be long enough to cross both the *x*-axis and the *y*-axis.

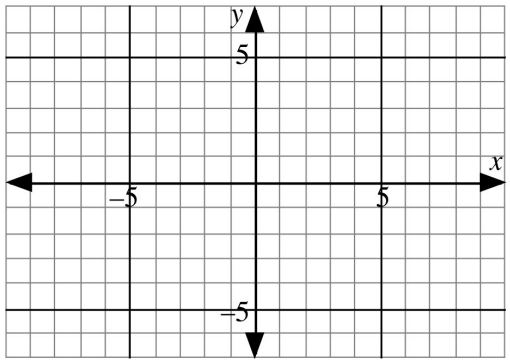
Write the coordinates of three other points on the line segment which have integer coordinates.



4. Repeat question #3 for line segments with the given slope passing through the given point.

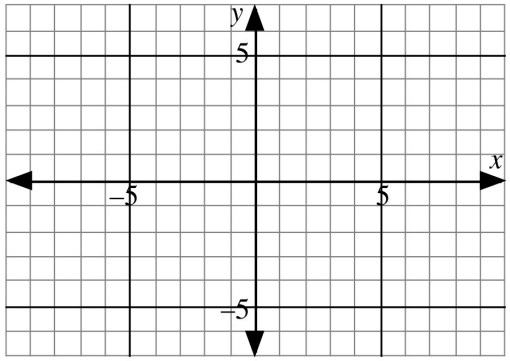
a) slope $= \frac{2}{5}$, $(2, 1)$

b) slope $= -\frac{1}{3}$, $(6, -3)$



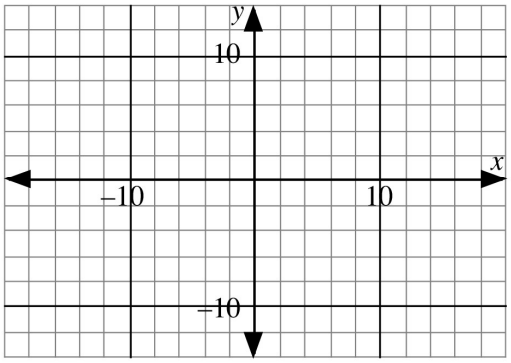
c) slope $= -\frac{4}{3}$, $(-9, 6)$

d) slope $= 4$, $(0, -7)$



e) slope $= -2$, $(4, -12)$

f) slope $= 0$, $(0, 6)$



5. P has coordinates $(-1, 2)$. Find two positions for point Q so that the slope of PQ is

a) 2 **b)** -3 **c)** $\frac{1}{3}$

d) $-\frac{2}{5}$ **e)** 0 **f)** undefined

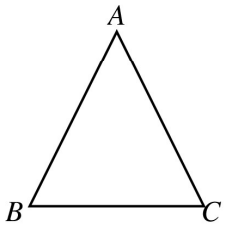
6. Two of three measures are given for rise, run, and slope.
Calculate the value of the third measure in each of the following.

a) slope $= \frac{5}{7}$ and run $= 49$ **b)** slope $= -\frac{3}{8}$ and rise $= 15$

c) slope $= -\frac{6}{11}$ and run $= 33$ **d)** slope $= \frac{3}{4}$ and rise $= 15$

7. A ramp which has been set up by skateboarders has a slope of $\frac{2}{3}$. Calculate the height of the ramp if the it has a base length of 1.5 metres.

8. Triangle ABC is isosceles with $AB = AC$ and $BC = 6.8$ cm. Calculate the area of the triangle if the slope of $AC = -\frac{5}{4}$.



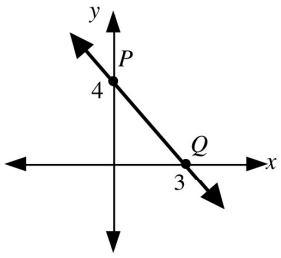
Multiple Choice **9.** The slope of $\overline{PQ}$ is

A. $\frac{3}{4}$

B. $-\frac{3}{4}$

C. $\frac{4}{3}$

D. $-\frac{4}{3}$



10. The point $(-4, 0)$ is on a line which has a slope of $-\frac{2}{5}$. The next point with integer coordinates on the line to the right of $(-4, 0)$ is

A. $(-9, -2)$

B. $(-9, 2)$

C. $(1, -2)$

D. $(-2, -5)$

11. P is a point in quadrant I, Q is a point in quadrant II, R is a point in quadrant III, and S is a point in quadrant IV.

Which one of the following statements must be true?

A. Line segment PQ has a positive slope.

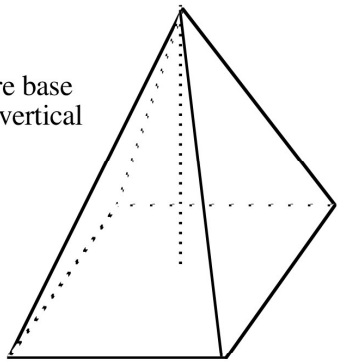
B. Line segment QR has a positive slope.

C. Line segment PR has a positive slope.

D. Line segment QS has a positive slope.

Use the following information to answer questions #12 and #13.

A pyramid has a square base of length 12 m and a vertical height of 15 m.



Numerical Response

12. A beetle starts to climb the pyramid starting from the midpoint of one of the faces. To the nearest tenth, the slope of the beetle's climb is _____.

(Record your answer in the numerical response box from left to right)

13. A fly starts to climb the pyramid along one of the edges. To the nearest tenth, the slope of the fly's climb is _____.

(Record your answer in the numerical response box from left to right)

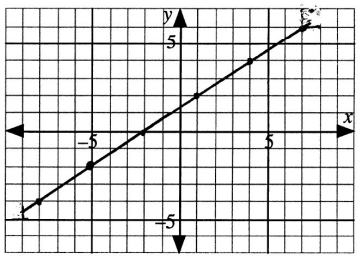
Answer Key

1.

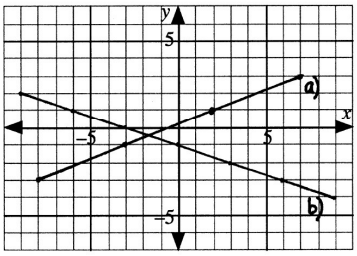
Line Segment	Rise	Run	Slope = $\frac{\text{Rise}}{\text{Run}}$
AB	−7	4	$-\frac{7}{4}$
CD	3	5	$\frac{3}{5}$
EF	7	1	$\frac{7}{1} = 7$
GH	0	7	$\frac{0}{7} = 0$

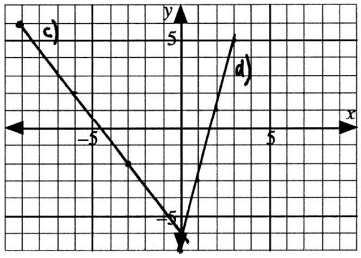
2. slope of line 1 = $\frac{1}{2}$, slope of line 2 = –2, slope of line 3 = –1
slope of line 4 = 4, slope of line 5 = $-\frac{3}{4}$, slope of line 6 = 2

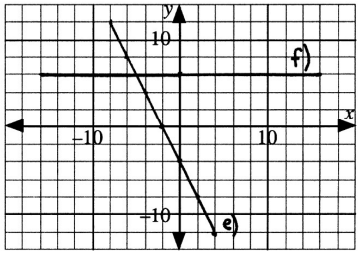
3. Any three of (–8, –4), (–2, 0), (1, 2), (4, 4)



4.







a) (–8, –3), (–3, –1), (7, 3)

b) Any 3 of (–9, 2), (–6, 1), (–3, 0) (0, –1), (3, –2), (9, –4)

c) (–6, 2), (–3, –2), (0, –6)

d) (1, –3), (2, 1), (3, 5)

e) Many possible answers including (2, –8), (0, –4), (–2, 0)

f) Many possible answers including (1, 6), (2, 6), (3, 6)

5. Many possible answers, including any two from:
a) (–3, –2), (–2, 0), (0, 4), (1, 6) **b)** (–3, 8), (–2, 5), (0, –1), (1, –4),
c) (2, 3), (5, 4), (–4, 1), (–7, 0) **d)** (–11, 6), (–6, 4), (4, 0), (9, –2)
e) (–3, 2), (–2, 2), (0, 2), (1, 2) **f)** (–1, 1), (–1, 0), (–1, –1), (–1, 3)

6. a) rise = 35 **b)** run = –40 **c)** rise = −18 **d)** run = 20

7. 1 metre **8.** 14.45 cm^2 **9.** D **10.** C **11.** C

12.

2	.	5	

13.

1	.	8	